



Linear Relations and Functions

BIG Ideas

- Analyze relations and functions.
- Identify, graph, and write linear equations.
- Find the slope of a line.
- Draw scatter plots and find prediction equations.
- · Graph special functions, linear inequalities, and absolute value inequalities.

Key Vocabulary

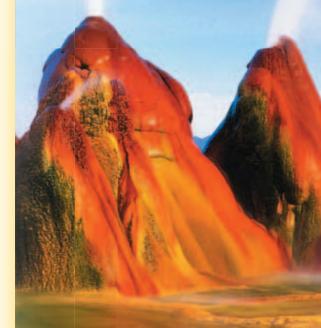
dependent variable (p. 61)

domain (p. 58)

function (p. 58)

independent variable (p. 61)

relation (p. 58)





Real-World Link

Underground Temperature Linear equations can be used to model relationships between many real-world quantities. The equations can then be used to make predictions such as the temperature of underground rocks.



Linear Relations and Functions Make this Foldable to help you organize your notes. Begin with four



10 Fold in half along the width and staple along the fold.



Turn the fold to the left and write the title of the chapter on the front. On each left-hand page of the booklet, write the title of a lesson from the chapter.



GET READY for Chapter 2

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Take the Online Readiness Quiz at algebra2.com.

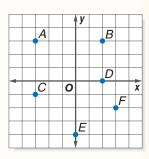
Option 1

Take the Quick Check below. Refer to the Quick Review for help.

OUICKC

Write the ordered pair for each point. (Prerequisite Skill)

- **1.** *A*
- **2.** *B*
- **3.** C
- **4.** D
- **5.** E
- 6. F



ANIMALS A blue whale's heart beats 9 times a minute.

- **7.** Make a table of ordered pairs in which the *x*-coordinate represents the number of minutes and the *y*-coordinate represents the number of heartbeats. (Prerequisite Skill)
- **8.** Graph the ordered pairs. (Prerequisite Skill)

Evaluate each expression if a = -1, b = 3, c = -2, and d = 0. (Prerequisite Skill)

9.
$$c + d$$

10.
$$4c - b$$

11.
$$a^2 - 5a + 3$$

12.
$$2b^2 + b + 7$$

13.
$$\frac{a-b}{c-a}$$

$$14. \ \frac{a+c}{b+c}$$

Simplify each expression. (Prerequisite Skill)

15.
$$x - (-1)$$

16.
$$x - (-5)$$

17.
$$2[x - (-3)]$$

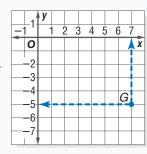
18.
$$4[x - (-2)]$$

19. TRAVEL Joan travels 65 miles per hour for
$$x$$
 hours on Monday. On Tuesday she drives 55 miles per hour for $(x + 3)$ hours. Write a simplified expression for the sum of the distances traveled. (Prerequisite Skill)

OUICKReview

Example 1 Write the ordered pair for point *G*.

Step 1 Follow a vertical line through the point to find the *x*-coordinate on the x-axis. The *x*-coordinate is 7.



- **Step 2** Follow a horizontal line through the point to find the y-coordinate on the *y*-axis. The *y*-coordinate is -5.
- The ordered pair for point G is (7, -5). Step 3 It can also be written as G(7, -5).

Example 2 Evaluate $d(a^2 + 2ab + b^2) - c$ if a = -1, b = 3, c = -2, and d = 0.

$$0[(-1)^2 + 2(-1)(3) + 3^2] - (-2)$$
 Substitute -1 for a , 3 for b , -2 for c , and 0 for d .

$$= 0 - (-2)$$
 Multiplication Property of Zero.
= 2 Subtract.

Example 3 Simplify $\frac{2}{5}[x - (-10)]$.

$$\frac{2}{5}[x - (-10)]$$

$$= \frac{2}{5}(x + 10)$$
 Simplify.
$$= \frac{2}{5}(x) + \frac{2}{5}(10)$$
 Distributive Property
$$= \frac{2}{5}x + 4$$
 Simplify.





Relations and Functions

Main Ideas

- Analyze and graph relations.
- Find functional values.

New Vocabulary

ordered pair Cartesian coordinate plane quadrant relation domain range function mapping one-to-one function discrete function continuous function vertical line test independent variable dependent variable function notation

GET READY for the Lesson

The table shows average and maximum lifetimes for some animals. The data can also be represented as

the ordered pairs (12, 28), (15, 30), (8, 20), (12, 20), and (20, 50). The first number in each ordered pair is the average lifetime, and the second number is the maximum lifetime.

he first number in	Allillidi	(years)	(years)
red pair is the	Cat	12	28
fetime, and the	Cow	15	30
imber is the	Deer	8	20
ı lifetime.	Dog	12	20
(12, 28)	Horse	20	50
↑ ↑ mavimum	Source: The III	orld Almanac	

maximum Source: The World Almanac

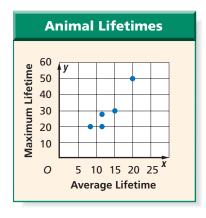
Average

Maximum

average maximum lifetime

Graph Relations You can graph the ordered pairs above on a *coordinate system*. Remember that each point in the coordinate plane can be named by exactly one ordered pair and every ordered pair names exactly one point in the coordinate plane.

The graph of the animal lifetime data lies in the part of the Cartesian coordinate plane with all positive coordinates. The **Cartesian coordinate plane** is composed of the *x-axis* (horizontal) and the *y-axis* (vertical), which meet at the *origin* (0, 0)

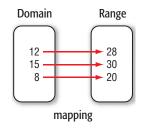


and divide the plane into four **quadrants**. In general, any ordered pair in the coordinate plane can be written in the form (x, y).

A **relation** is a set of ordered pairs, such as the one for the longevity of animals. The **domain** of a relation is the set of all first coordinates (x-coordinates) from the ordered pairs, and the **range** is the set of all second coordinates (y-coordinates) from the ordered pairs. The domain of the function above is $\{8, 12, 15, 20\}$, and the range is $\{20, 30, 28, 50\}$.

A **function** is a special type of relation in which each element of the domain is paired with *exactly one* element of the range. A **mapping** shows how the members are paired. A function like the one represented by the mapping in which each element of the range is paired with exactly one element of the domain is called a **one-to-one function**.

{(12, 28), (15, 30), (8, 20)}



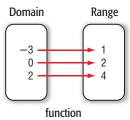


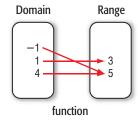
The first two relations shown below are functions. The third relation is not a function because the -3 in the domain is paired with both 0 and 6 in the range.

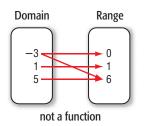
$$\{(-3, 1), (0, 2), (2, 4)\}$$

$$\{(-1, 5), (1, 3), (4, 5)\}$$

$$\{(5, 6), (-3, 0), (1, 1), (-3, 6)\}$$







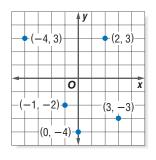
EXAMPLE Domain and Range



State the domain and range of the relation shown in the graph. Is the relation a function?

The relation is $\{(-4, 3), (-1, -2), (0, -4), (2, 3), (3, -3)\}.$ The domain is $\{-4, -1, 0, 2, 3\}$. The range is $\{-4, -3, -2, 3\}$.

Each member of the domain is paired with exactly one member of the range, so this relation is a function.



CHECK Your Progress

1. State the domain and range of the relation $\{(-2, 2), (1, 4), (3, 0), (-2, -4),$ (0,3)}. Is the relation a function?

A relation in which the domain is a set of individual points, like the relation in Example 1, is said to be **discrete**. Notice that its graph consists of points that are not connected. When the domain of a relation has an infinite number of elements and the relation can be





Discrete Relation

Continuous Relation

graphed with a line or smooth curve, the relation is **continuous**. With both discrete and continuous graphs, you can use the vertical line test to determine whether the relation is a function.

Study Tip

Continuous Relations

You can draw the graph of a continuous relation without lifting your pencil from the paper.

KEY CONCEPT

If no vertical line intersects a graph in more than one point, the graph represents a function.

Models

Words



Vertical Line Test

If some vertical line intersects a graph in two or more points, the graph does not represent a function.



In Example 1, there is no vertical line that contains more than one of the points. Therefore, the relation is a function.

EXAMPLE Vertical Line Test

GEOGRAPHY The table shows the population of the state of Kentucky over the last several decades. Graph this information and determine whether it represents a function. Is the relation discrete or continuous?

Year	Population (millions)
1960	3.0
1970	3.2
1980	3.7
1990	3.7
2000	4.0

Source: U.S. Census Bureau

Population of Kentucky			
3.5 3.5 3.7 3.7 3.7 3.7 3.7 3.7 3.7 3.7			

Use the vertical line test. Notice that no vertical line can be drawn that contains more than one of the data points. Therefore, this relation is a function. Because the graph consists of distinct points, the relation is discrete.

CHECK Your Progress

2. The number of employees a company had in each year from 1999 to 2004 were 25, 28, 34, 31, 27, and 29. Graph this information and determine whether it represents a function. Is the relation discrete or continuous?

Equations of Functions and Relations Relations and functions can also be represented by equations. The solutions of an equation in x and y are the set of ordered pairs (x, y) that make the equation true.

Consider the equation y = 2x - 6. Since x can be any real number, the domain has an infinite number of elements. To determine whether an equation represents a function, it is often simplest to look at the graph of the relation.

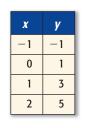
EXAMPLE Graph a Relation

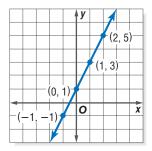


🛂 Graph each equation and find the domain and range. Then determine whether the equation is a function and state whether it is *discrete* or continuous.

a.
$$y = 2x + 1$$

Make a table of values to find ordered pairs that satisfy the equation. Choose values for *x* and find the corresponding values for *y*. Then graph the ordered pairs.





Since *x* can be any real number, there is an infinite number of ordered pairs

that can be graphed. All of them lie on the line shown. Notice that every real number is the *x*-coordinate of some point on the line. Also, every real number is the *y*-coordinate of some point on the line. So the domain and range are both all real numbers, and the relation is continuous.

This graph passes the vertical line test. For each *x*-value, there is exactly one y-value, so the equation y = 2x + 1 represents a function.



Study Tip

Vertical Line Test

You can use a pencil to represent a vertical

line. Slowly move the

pencil to the right

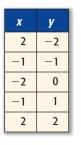
across the graph to

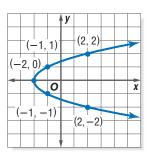
one point.

see if it intersects the graph at more than

b.
$$x = y^2 - 2$$

Make a table. In this case, it is easier to choose y values and then find the corresponding values for x. Then sketch the graph, connecting the points with a smooth curve.





Every real number is the *y*-coordinate of some point on the graph, so the range is all real numbers. But, only

real numbers greater than or equal to -2 are

x-coordinates of points on the graph. So the domain is $\{x | x \ge -2\}$. The relation is continuous.

You can see from the table and the vertical line test that there are two y values for each x value except x = -2. Therefore, the equation $x = y^2 - 2$ does not represent a function.

CHECK Your Progress

- **3A.** Graph the relation represented by $y = x^2 + 1$.
- **3B.** Find the domain and range. Determine if the relation is *discrete* or continuous.
- **3C.** Determine whether the relation is a function.



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Reading Math

Functions Suppose you have a job that pays by the hour. Since your pay depends on the number of hours you work, you might say that your pay is a *function* of the number of hours you work.

When an equation represents a function, the variable, usually x, whose values make up the domain is called the **independent variable**. The other variable, usually y, is called the **dependent variable** because its values depend on x.

Equations that represent functions are often written in function notation. The equation y = 2x + 1 can be written as f(x) = 2x + 1. The symbol f(x) replaces the *y* and is read "*f* of *x*." The *f* is just the name of the function. It is not a variable that is multiplied by x. Suppose you want to find the value in the range that corresponds to the element 4 in the domain of the function. This is written as *f*(4) and is read "*f* of 4." The value *f*(4) is found by substituting 4 for each *x* in the equation. Therefore, f(4) = 2(4) + 1 or 9. Letters other than f can be used to represent a function. For example, g(x) = 2x + 1.

EXAMPLE Evaluate a Function



Given $f(x) = x^2 + 2$, find each value.

a.
$$f(-3)$$

$$f(x) = x^2 + 2$$
 Original function

b.
$$f(3z)$$

$$f(x) = x^2 + 2$$
 Original function

$$f(-3) = (-3)^2 + 2$$
 Substitute.

$$= 9 + 2 \text{ or } 11$$
 Simplify.

$$f(3z) = (3z)^2 + 2$$
 Substitute.
= $9z^2 + 2$ $(ab)^2 = a^2b^2$

CHECK Your Progress

Given
$$g(x) = 0.5x^2 - 5x + 3.5$$
, find each value.

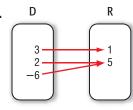
4A.
$$g(2.8)$$

4B.
$$g(4a)$$

Examples 1, 2 (pp. 59-60)

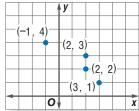
State the domain and range of each relation. Then determine whether each relation is a function. Write yes or no.

1.



х	у
5	2
10	-2
15	-2
20	-2

3.



WEATHER For Exercises 4–6, use the table that shows the record high temperatures (°F) for January and July for four states.

- 4. Identify the domain and range. Assume that the January temperatures are the domain.
- **5.** Write a relation of ordered pairs for the data.
- **6.** Graph the relation. Is this relation a function?

State Jan. July California 97 134 Illinois 78 117 North Carolina 86 109 Texas 119

Source: U.S. National Oceanic and Atmospheric Administration

Examples 2, 3 (pp. 60-61)

Example 4 (p. 61) Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether it is discrete or continuous.

7.
$$\{(7,8), (7,5), (7,2), (7,-1)\}$$

9.
$$y = -2x + 1$$

10.
$$x = y^2$$

11. Find
$$f(5)$$
 if $f(x) = x^2 - 3x$.

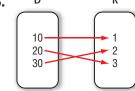
12. Find
$$h(-2)$$
 if $h(x) = x^3 + 1$.

Exercises

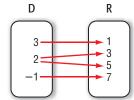
HOMEWORK HELF				
For Exercises	See Examples			
13-28	1, 2			
29-34	3			
35–42	4			

State the domain and range of each relation. Then determine whether each relation is a function. Write yes or no.

13.



14.



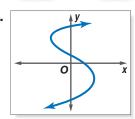
15.

X	y
0.5	-3
2	0.8
0.5	8

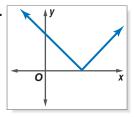
16.

X	у
2000	\$4000
2001	\$4300
2002	\$4600
2003	\$4500

17.

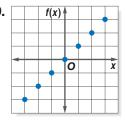


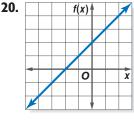
18.



Determine whether each function is discrete or continuous.

19.





21.
$$\{(-3, 0), (-1, 1), (1, 3)\}$$

22.
$$y = -x + 4$$

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether it is discrete or continuous.

25.
$$\{(-2,5), (3,7), (-2,8)\}$$

29.
$$y = -5x$$

30.
$$y = 3x$$

31.
$$y = 3x - 4$$

32.
$$y = 7x - 6$$

33.
$$y = x^2$$

34.
$$x = 2y^2 - 3$$

Find each value if f(x) = 3x - 5 and $g(x) = x^2 - x$.

35.
$$f(-3)$$

37.
$$g(\frac{1}{3})$$

38.
$$f(\frac{2}{3})$$

40.
$$g(5n)$$

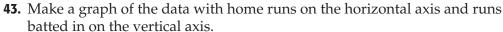
41. Find the value of
$$f(x) = -3x + 2$$
 when $x = 2$.

42. What is g(4) if $g(x) = x^2 - 5$?

SPORTS For Exercises 43–45, use the table that shows the leading home run and runs batted in totals in the National League for 2000-2004.

Year	2000	2001	2002	2003	2004
HR	50	73	49	47	48
RBI	147	160	128	141	131

Source: The World Almanac



44. Identify the domain and range.

45. Does the graph represent a function? Explain your reasoning.

STOCKS For Exercises 46–49, use the table that shows a company's stock price in recent years.

46. Write a relation to represent the data.

47. Graph the relation.

48. Identify the domain and range.

49. Is the relation a function? Explain your reasoning.

Year	Price
2002	\$39
2003	\$43
2004	\$48
2005	\$55
2006	\$61
2007	\$52

GOVERNMENT For Exercises 50–53, use the table below that shows the number of members of the U.S. House of Representatives with 30 or more consecutive years of service in Congress from 1991 to 2003.

Year	1991	1993	1995	1997	1999	2001	2003
Representatives	11	12	9	6	3	7	9

Source: Congressional Directory

50. Write a relation to represent the data.

51. Graph the relation.

52. Identify the domain and range. Determine whether the relation is *discrete* or continuous.

53. Is the relation a function? Explain your reasoning.

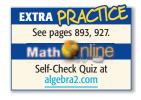
54. AUDIO BOOK DOWNLOADS Chaz has a collection of 15 audio books. After he gets a part-time job, he decides to download 3 more audio books each month. The function A(t) = 15 + 3t counts the number of audio books A(t)he has after t months. How many audio books will he have after 8 months?





The major league record for runs batted in (RBIs) is 191 by Hack Wilson.

Source: www.baseballalmanac.com



- **55. OPEN ENDED** Write a relation of four ordered pairs that is *not* a function. Explain why it is not a function.
- **56. FIND THE ERROR** Teisha and Molly are finding g(2a) for the function $g(x) = x^2 + x - 1$. Who is correct? Explain your reasoning.

Teisha

$$g(2a) = 2(a^2 + a - 1)$$

 $= 2a^2 + 2a - 2$

Molly
$$g(2a) = (2a)^{2} + 2a - 1$$

$$= 4a^{2} + 2a - 1$$

- **57. CHALLENGE** If f(3a-1)=12a-7, find one possible expression for f(x).
- **58.** Writing in Math Use the information about animal lifetimes on page 58 to explain how relations and functions apply to biology. Include an explanation of how a relation can be used to represent data and a sentence that includes the words average lifetime, maximum lifetime, and function.

STANDARDIZED TEST PRACTICE

- **59. ACT/SAT** If $g(x) = x^2$, which expression is equal to g(x + 1)?
 - **A** 1
 - **B** $x^2 + 1$
 - C $x^2 + 2x + 1$
 - **D** $x^2 x$

60. REVIEW Which set of dimensions represent a triangle similar to the triangle shown below?



- F 7 units, 11 units, 12 units
- G 10 units, 23 units, 24 units
- H 20 units, 48 units, 52 units
- 1 unit, 2 units, 3 units

Spiral Review

Solve each inequality. (Lessons 1-5 and 1-6)

61.
$$|y+1| < 7$$

62.
$$|5 - m| < 1$$

63.
$$x - 5 < 0.1$$

64. SHOPPING Javier had \$25.04 when he went to the mall. His friend Sally had \$32.67. Javier wanted to buy a shirt for \$27.89. How much money did Javier borrow from Sally? How much money did that leave Sally? (Lesson 1-3)

Simplify each expression. (Lessons 1-1 and 1-2)

65.
$$32(22 - 12) + 42$$

66.
$$3(5a+6b)+8(2a-b)$$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solution. (Lesson 1-3)

67.
$$x + 3 = 2$$

68.
$$-4 + 2y = 0$$

68.
$$-4 + 2y = 0$$
 69. $0 = \frac{1}{2}x - 3$ **70.** $\frac{1}{3}x - 4 = 1$

70.
$$\frac{1}{3}x - 4 = 1$$

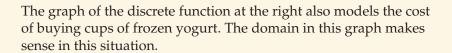
READING MATH



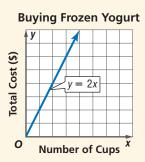
Discrete and Continuous Functions in the Real World

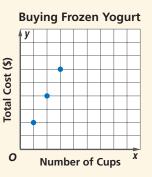
A cup of frozen yogurt costs \$2 at the Yogurt Shack. We might describe the cost of *x* cups of yogurt using the *continuous* function y = 2x, where y is the total cost in dollars. The graph of that function is shown at the right.

From the graph, you can see that 2 cups of yogurt cost \$4, 3 cups cost \$6, and so on. The graph also shows that 1.5 cups of yogurt cost 2(1.5) or \$3. However, the Yogurt Shack probably will not sell partial cups of yogurt. This function is more accurately modeled with a discrete function.



When choosing a discrete function or a continuous function to model a real-world situation, be sure to consider whether all real numbers are reasonable as part of the domain.

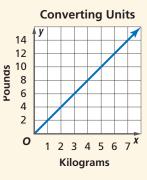




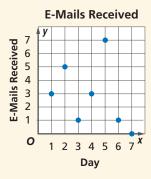
Reading to Learn

Determine whether each function is better modeled using a discrete or continuous function. Explain your reasoning.

1.



2.



- **3.** *y* represents the distance a car travels in *x* hours.
- **4.** *y* represents the total number of riders who have ridden a roller coaster after x rides.
- **5.** Give an example of a real-world function that is discrete and a real-world function that is continuous. Explain your reasoning.



Linear Equations

Main Ideas

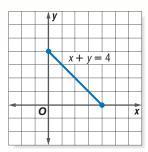
- · Identify linear equations and functions.
- Write linear equations in standard form and graph them.

New Vocabulary

linear equation linear function standard form y-intercept x-intercept

GET READY for the Lesson

Lolita has 4 hours after dinner to study and do homework. She has brought home math and chemistry. If she spends x hours on math and y hours on chemistry, a portion of the graph of the equation x + y = 4 can be used to relate how much time she spends on each.



Identify Linear Equations and Functions An equation such as x + y = 4is called a linear equation. A linear equation has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

Linear equations	Not linear equations
5x - 3y = 7	$7a + 4b^2 = -8$
x = 9	$y = \sqrt{x+5}$
6s = -3t - 15	x + xy = 1
$y = \frac{1}{2}x$	$y = \frac{1}{x}$

A **linear function** is a function whose ordered pairs satisfy a linear equation. Any linear function can be written in the form f(x) = mx + b, where m and b are real numbers.

EXAMPLE Identify Linear Functions



State whether each function is a linear function. Explain.

- **a.** f(x) = 10 5x This is a linear function because it can be written as f(x) = -5x + 10. m = -5, b = 10
- **b.** $g(x) = x^4 5$ This is not a linear function because *x* has an exponent other than 1.
- c. h(x, y) = 2xyThis is not a linear function because the two variables are multiplied together.

CHECK Your Progress

1A.
$$f(x) = \frac{5}{x+6}$$

1B.
$$g(x) = -\frac{3}{2}x + \frac{1}{3}$$









Real-World Link ...

To avoid decompression sickness, it is recommended that divers ascend no faster than 30 feet per minute.

Source: www.emedicine.com



WATER PRESSURE The linear function P(d) = 62.5d + 2117 can be used to find the pressure (lb/ft^2) d feet below the surface of the water.

a. Find the pressure at a depth of 350 feet.

$$P(d) = 62.5d + 2117$$

Original function

$$P(350) = 62.5(350) + 2117$$
 Substitute.

$$= 23.992$$

Simplify.

The pressure at a depth of 350 feet is about 24,000 lb/ft².

b. The term 2117 in the function represents the atmospheric pressure at the surface of the water. How many times as great is the pressure at a depth of 350 feet as the pressure at the surface?

Divide the pressure 350 feet down by the pressure at the surface.

$$\frac{23,992}{2117} \approx 11.33$$
 Use a calculator.

The pressure at that depth is more than 11 times that at the surface.

CHECK Your Progress

- **2.** At what depth is the pressure $33,367 \text{ lb/ft}^2$?
- Milional Tutor at algebra2.com

Standard Form Many linear equations can be written in standard form, Ax + By = C, where A, B, and C are integers whose greatest common factor is 1.

Standard Form of a Linear Equation

The standard form of a linear equation is Ax + By = C, where A, B, and C are integers whose greatest common factor is 1, $A \ge 0$, and A and B are not both zero.

EXAMPLE Standard Form



a.
$$y = -2x + 3$$

$$y = -2x + 3$$
 Original equation

$$y + 11 - 3$$

2x + y = 3 Add 2x to each side.

So,
$$A = 2$$
, $B = 1$, and $C = 3$.

b.
$$-\frac{3}{5}x = 3y - 2$$

$$-\frac{3}{5}x = 3y - 2$$
 Original equation

$$-\frac{3}{5}x - 3y = -2$$
 Subtract 3y from each side.

$$3x + 15y = 10$$
 Multiply each side by -5 so that the coefficients are integers and $A \ge 0$.

So,
$$A = 3$$
, $B = 15$, and $C = 10$.

CHECK Your Progress

3A.
$$2y = 4x + 5$$

3B.
$$3x - 6y - 9 = 0$$





Study Tip

Vertical and **Horizontal Lines**

An equation of the form x = C represents a vertical line, which has only an x-intercept. y = C represents a horizontal line, which has only a y-intercept.

Since two points determine a line, one way to graph a linear equation or function is to find the points at which the graph intersects each axis and connect them with a line. The y-coordinate of the point at which a graph crosses the y-axis is called the y-intercept. Likewise, the x-coordinate of the point at which it crosses the *x*-axis is the *x*-intercept.

EXAMPLEUse Intercepts to Graph a Line



Find the x-intercept and the y-intercept of the graph of 3x - 4y + 12 = 0. Then graph the equation.

The *x*-intercept is the value of *x* when y = 0.

$$3x - 4y + 12 = 0$$
 Original equation

$$3x - 4(0) + 12 = 0$$
 Substitute 0 for *y*.

$$3x = -12$$
 Subtract 12 from each side.

$$x = -4$$
 Divide each side by 3.

The x-intercept is -4. The graph crosses the x-axis at (-4, 0).

Likewise, the *y*-intercept is the value of *y* when x = 0.

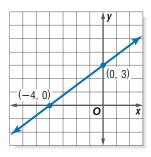
$$3x - 4y + 12 = 0$$
 Original equation

$$3(0) - 4y + 12 = 0$$
 Substitute 0 for x.

$$-4y = -12$$
 Subtract 12 from each side.

$$y = 3$$
 Divide each side by -4 .

The *y*-intercept is 3. The graph crosses the *y*-axis at (0, 3). Use these ordered pairs to graph the equation.



CHECK Your Progress

4. Find the *x*-intercept and the *y*-intercept of the graph of 2x + 5y - 10 = 0. Then graph the equation.

Your Understanding

Example 1 (p. 66)

State whether each equation or function is linear. Write yes or no. If no, explain your reasoning.

1.
$$x^2 + y^2 = 4$$

2.
$$h(x) = 1.1 - 2x$$

Example 2

ECONOMICS For Exercises 3 and 4, use the following information.

(p. 67)

On January 1, 1999, the euro became legal tender in 11 participating countries in Europe. Based on the exchange rate on one particular day, the linear function d(x) = 0.8881x could be used to convert x euros to U.S. dollars.

- **3.** On that day, what was the value in U.S. dollars of 200 euros?
- **4.** On that day, what was the value in euros of 500 U.S. dollars?

Example 3

Write each equation in standard form. Identify A, B, and C.

5.
$$y = 3x - 5$$

6.
$$4x = 10y + 6$$

7.
$$y = \frac{2}{3}x + 1$$

Example 4 (p. 68)

Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation.

8.
$$y = -3x - 5$$

9.
$$x - y - 2 = 0$$

Exercises

HOMEWO	RK HELP
For Exercises	See Examples
10-17	1
18-21	2
22-27	3
28-33	4

Real-World Link.....

The troposphere is the lowest layer of the atmosphere. All

weather events take place in the troposphere.

State whether each equation or function is linear. Write yes or no. If no, explain your reasoning.

10.
$$x + y = 5$$

12.
$$f(x) = 7x^5 + x - 1$$

14.
$$g(x) = 10 + \frac{2}{x^2}$$

16.
$$x + \sqrt{y} = 4$$

11.
$$f(x) = 6x - 19$$

13.
$$h(x) = 2x^3 - 4x^2 + 5$$

15.
$$\frac{1}{x} + 3y = -5$$

17.
$$y = \sqrt{2x - 5}$$

PHYSICS For Exercises 18 and 19, use the following information.

When a sound travels through water, the distance *y* in meters that the sound travels in x seconds is given by the equation y = 1440x.

- **18.** How far does a sound travel underwater in 5 seconds?
- **19.** In air, the equation is y = 343x. Does sound travel faster in air or water? Explain.

ATMOSPHERE For Exercises 20 and 21, use the following information.

Suppose the temperature *T* in °F above the Earth's surface is given by T(h) = -3.6h + 68, where h is the height (in thousands of feet).

- **20.** Find the temperature at a height of 10,000 feet.
- **21.** Find the height if the temperature is -58° F.

Write each equation in standard form. Identify A, B, and C.

22.
$$y = -3x + 4$$

23.
$$y = 12x$$

24.
$$x = 4y - 5$$

25.
$$x = 7y + 2$$

26.
$$5y = 10x - 25$$

27.
$$4x = 8y - 12$$

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

28.
$$5x + 3y = 15$$

29.
$$2x - 6y = 12$$

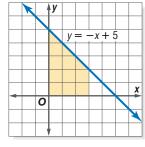
30.
$$3x - 4y - 10 = 0$$

31.
$$2x + 5y - 10 = 0$$

32.
$$y = x$$

33.
$$y = 4x - 2$$

34. GEOMETRY Find the area of the shaded region in the graph. (Hint: The area of a trapezoid is given by $A = \frac{1}{2}h(b_1 + b_2).$



Write each equation in standard form. Identify A, B, and C.

35.
$$\frac{1}{2}x + \frac{1}{2}y = 6$$

36.
$$\frac{1}{3}x - \frac{1}{3}y = -2$$

37.
$$0.5x = 3$$

38.
$$0.25y = 10$$

39.
$$\frac{5}{6}x + \frac{1}{15}y = \frac{3}{10}$$

40.
$$0.25x = 0.1 + 0.2y$$

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

41.
$$y = -2$$

42.
$$y = 4$$

43.
$$x = 8$$

44.
$$3x + 2y = 6$$

45.
$$x = 1$$

46.
$$f(x) = 4x - 1$$

46.
$$f(x) = 4x - 1$$
 47. $g(x) = 0.5x - 3$ **48.** $4x + 8y = 12$

48.
$$4x + 8y = 12$$

49. ATMOSPHERE Graph the linear function in Exercises 20 and 21.



See pages 893, 927. Math Maline Self-Check Quiz at

COMMISSION For Exercises 50–52, use the following information.

Latonya earns a commission of \$1.75 for each magazine subscription she sells and \$1.50 for each newspaper subscription she sells. Her goal is to earn a total of \$525 in commissions in the next two weeks.

- **50.** Write an equation that is a model for the different numbers of magazine and newspaper subscriptions that can be sold to meet the goal.
- **51.** Graph the equation. Does this equation represent a function? Explain.
- **52.** If Latonya sells 100 magazine subscriptions and 200 newspaper subscriptions, will she meet her goal? Explain.

H.O.T. Problems

- **53. OPEN ENDED** Write an equation of a line with an *x*-intercept of 2.
- **54. REASONING** Explain why $f(x) = \frac{x+2}{2}$ is a linear function.

CHALLENGE For Exercises 55 and 56, use x + y = 0, x + y = 5, and x + y = -5.

- **55.** Graph the equations. Then compare and contrast the graphs.
- **56.** Write a linear equation whose graph is between the graphs of x + y = 0and x + y = 5.
- **57. REASONING** Explain why the graph of x + 3y = 0 has only one intercept.
- **58.** Writing in Math Use the information about study time on page 66 to explain how linear equations relate to time spent studying. Explain why only the part of the graph in the first quadrant is shown and an interpretation of the graph's intercepts in terms of the situation.

STANDARDIZED TEST PRACTICE

59. ACT/SAT Which function is linear?

$$\mathbf{A} \ f(x) = x^2$$

B
$$g(x) = 2.7$$

C
$$f(x) = \sqrt{9 - x^2}$$

D
$$g(x) = \sqrt{x - 1}$$

60. REVIEW What is the complete solution to the equation?

$$|9 - 3x| = 18$$

F
$$x = -9$$
; $x = 3$ **H** $x = -3$; $x = 9$

G
$$x = -9$$
; $x = -3$ **J** $x = 3$; $x = 9$

Spiral Review

State the domain and range of each relation. Then graph the relation and determine whether it is a function. (Lesson 2-1)

61.
$$\{(-1, 5), (1, 3), (2, -4), (4, 3)\}$$

Solve each inequality. (Lesson 1-6)

63.
$$-2 < 3x + 1 < 7$$

64.
$$|x+4| > 2$$

65. TAX Including a 6% sales tax, a paperback book costs \$8.43. What is the price before tax? (Lesson 1-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Find the reciprocal of each number.

67.
$$\frac{1}{2}$$

68.
$$3\frac{3}{4}$$



Slope

Main Ideas

- · Find and use the slope of a line.
- · Graph parallel and perpendicular lines.

New Vocabulary

rate of change slope family of graphs parent graph oblique

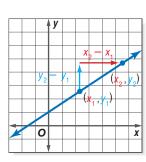
GET READY for the Lesson

The grade of a road is a percent that measures the steepness of the road. It is found by dividing the amount the road rises by the corresponding horizontal distance.



horizontal distance

Slope A rate of change measures how much a quantity changes, on average, relative to the change in another quantity, often time. The idea of rate of change can be applied to points in the coordinate plane to determine the steepness of the line between the points. The **slope** of a line is the ratio of the change in y-coordinates to the corresponding change in x-coordinates. Suppose a line passes through points at (x_1, y_1) and (x_2, y_2) .



slope =
$$\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

= $\frac{y_2 - y_1}{x_2 - x_1}$

KEY CONCEPT

Slope of a Line

Words The slope of a line is the ratio of the change in y-coordinates to the change in x-coordinates.

The slope m of the line passing through (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$. **Symbols**

Study Tip

Slope

The formula for slope is often remembered as rise over run, where the rise is the difference in y-coordinates and the run is the difference in x-coordinates.

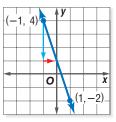
EXAMPLE Find Slope and Use Slope to Graph

Find the slope of the line that passes through (-1, 4) and (1, -2). Then graph the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 Slope formula
= $\frac{-2 - 4}{1 - (-1)}$ $(x_1, y_1) = (-1, 4), (x_2, y_2) = (1, -2)$
= $\frac{-6}{2}$ or -3 The slope is -3 .

(continued on the next page)

Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any point on the line. Then go down 3 units and right 1 unit or go up 3 units and left 1 unit. This point should also be on the line.



CHECK Your Progress

1. Find the slope of the line that passes through (1, -3) and (3, 5). Then graph the line.

The slope of a line tells the direction in which it rises or falls.



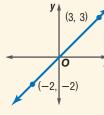
Study Tip

Slope is Constant

The slope of a line is the same, no matter what two points on the line are used.

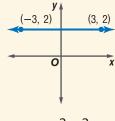
CONCEPT **SUMMARY**

If the line rises to the right, then the slope is positive.



$$m = \frac{3 - (-2)}{3 - (-2)}$$
$$= 1$$

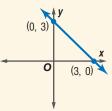
If the line is horizontal, then the slope is zero.



$$m = \frac{2-2}{3-(-3)} = 0$$

If the line falls to the right, then the slope

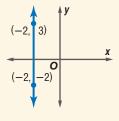
is negative.



$$m = \frac{0-3}{3-0}$$
$$= -1$$

Slope





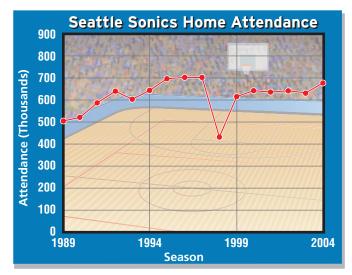
$$x_1 = x_2$$
, so m is undefined.

Real-World EXAMPLE

BASKETBALL Refer to the graph at the right. Find the rate of change of the number of people attending Seattle Sonics home games from 1993 to 1996.

$$m=rac{y_2-y_1}{x_2-x_1}$$
 Slope formula
$$=rac{700-601}{1996-1993}$$
 Substitute.
$$\approx 33$$
 Simplify.

Between 1993 and 1996, the number of people attending Seattle Sonics home games



Source: Kenn.com

increased at an average rate of about 33(1000) or 33,000 people per year.

CHECK Your Progress

2. In 1999, 45,616 students applied for admission to UCLA. In 2004, 56,878 students applied. Find the rate of change in the number of students applying for admission from 1999 to 2004.



Parallel and Perpendicular Lines A family of graphs is a group of graphs that displays one or more similar characteristics. The parent graph is the simplest of the graphs in a family.

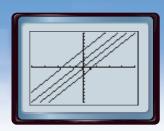
GRAPHING CALCULATOR LAB

Lines with the Same Slope

The calculator screen shows the graphs of y = 3x, y = 3x + 2, y = 3x - 2, and y = 3x + 5.

THINK AND DISCUSS

1. What is similar about the graphs? What is different about the graphs?



[-4, 4] scl: 1 by [-10, 10] scl: 1

2. Write another function that has the same characteristics as these graphs. Check by graphing.

In the Lab, you saw that lines that have the same slope are parallel.



Study Tip

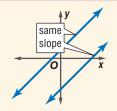
Horizontal Lines

All horizontal lines are parallel because they all have a slope of 0.

Words

In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.

Model



EXAMPLE Parallel Lines

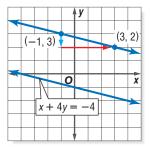


Graph the line through (-1, 3) that is parallel to the line with equation x + 4y = -4.

The x-intercept is -4, and the y-intercept is -1. Use the intercepts to graph x + 4y = -4.

The line falls 1 unit for every 4 units it moves to the right, so the slope is $-\frac{1}{4}$.

Now use the slope and the point at (-1, 3) to graph the line parallel to the graph of x + 4y = -4.



Parallel Lines

CHECK Your Progress

3. Graph the line through (-2, 4) that is parallel to the line with equation x - 3y = 3.



Personal Tutor at algebra2.com

The graphs of \overrightarrow{AB} and \overrightarrow{CD} are perpendicular.



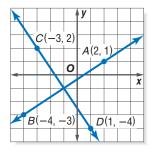
slope of line *AB*

slope of line CD

$$\frac{-3-1}{-4-2} = \frac{-4}{-6} \text{ or } \frac{2}{3}$$

$$\frac{-3-1}{-4-2} = \frac{-4}{-6} \text{ or } \frac{2}{3} \qquad \frac{-4-2}{1-(-3)} = \frac{-6}{4} \text{ or } -\frac{3}{2}$$

The slopes are opposite reciprocals of each other. The product of the slopes of two perpendicular lines is always -1.





Reading Math

Oblique

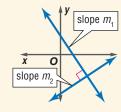
An oblique line is a line that is neither horizontal nor vertical. CONCEPT

Perpendicular Lines

Words

In a plane, two oblique lines are perpendicular if and only if the product of their slopes is -1.

Symbols Suppose m_1 and m_2 are the slopes of two oblique lines. Then the lines are perpendicular if and only if $m_1 m_2 = -1$, or $m_1 = -\frac{1}{m_2}$ Model



Any vertical line is perpendicular to any horizontal line.

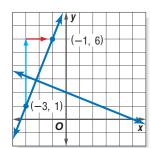
EXAMPLE Perpendicular Lines



Graph the line through (-3, 1) that is perpendicular to the line with equation 2x + 5y = 10.

The *x*-intercept is 5, and the *y*-intercept is 2. Use the intercepts to graph 2x + 5y = 10.

The line falls 2 units for every 5 units it moves to the right, so the slope is $-\frac{2}{5}$. The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{5}$, or $\frac{5}{2}$.



Start at (-3, 1) and go up 5 units and right 2 units. Use this point and (-3, 1) to graph the line.

CHECK Your Progress

4. Graph the line through (-6, 2) that is perpendicular to the line with equation 3x - 2y = 6.

Your Understanding

Example 1

Find the slope of the line that passes through each pair of points.

(pp. 71-72)

1. (-2, -1), (2, -3)

2. (2, 2), (4, 2)

3. (4, 5), (-1, 0)

Graph the line passing through the given point with the given slope.

4. (2, -1), -3

5. $(-3, -4), \frac{3}{2}$

Example 2 (p. 72) **WEATHER** For Exercises 6–8, use the table that shows the temperatures at different times on the same day.

Time	8:00 A.M.	10:00 A.M.	12:00 P.M.	2:00 P.M.	4:00 P.M.
Temp (°F)	36	47	55	58	60

- **6.** What was the average rate of change of the temperature from 8:00 A.M. to 10:00 A.M.?
- 7. What was the average rate of change of the temperature from 12:00 P.M. to 4:00 P.M.?
- **8.** During what 2-hour period was the average rate of change of the temperature the least?

Example 3

Graph the line that satisfies each set of conditions.

(p. 73)

9. passes through (0, 3), parallel to graph of 6y - 10x = 30

10. passes through (1, 1) parallel to graph of x + y = 5

Example 4 (p. 74)

11. passes through (4, -2), perpendicular to graph of 3x - 2y = 6

12. passes through (-1, 5), perpendicular to graph of 5x - 3y - 3 = 0

HOMEWO	rk HELP
For Exercises	See Examples
13-24	1
25-29	2
30–37	3, 4

Find the slope of the line that passes through each pair of points.

13.
$$(4, -1), (6, -6)$$

14.
$$(-8, -3), (2, 3)$$

15.
$$(8, 7), (7, -6)$$

16.
$$(-2, -3), (0, -5)$$

18.
$$(4, -1.5), (4, 4.5)$$

Graph the line passing through the given point with the given slope.

19.
$$(-1, 4), m = \frac{2}{3}$$

20.
$$(-3, -1), m = -\frac{1}{5}$$

21.
$$(3, -4), m = 2$$

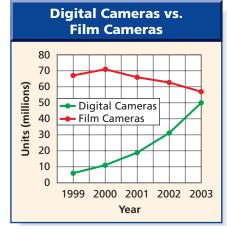
22.
$$(1, 2), m = -3$$

23.
$$(6, 2), m = 0$$

24.
$$(-2, -3)$$
, undefined

CAMERAS For Exercises 25 and 26, refer to the graph that shows the number of digital still cameras and film cameras sold in recent years.

- **25.** Find the average rate of change of the number of digital cameras sold from 1999 to 2003.
- **26.** Find the average rate of change of the number of film cameras sold from 1999 to 2003. What does the sign of the rate mean?



Source: Digital Photography Review

TRAVEL For Exercises 27–29, use the following information. Mr. and Mrs. Wellman are taking their daughter to college. The table shows their distance from home after various amounts of time.

- **27.** Find the average rate of change of their distance from home between 1 and 3 hours after leaving home.
- **28.** Find the average rate of change of their distance from home between 0 and 5 hours after leaving home.
- **29.** What is another word for *rate of change* in this situation?

Time (h)	Distance (mi)
0	0
1	55
2	110
3	165
4	165

Graph the line that satisfies each set of conditions.

- **30.** passes through (-2, 2), parallel to a line whose slope is -1
- **31.** passes through (2, -5), parallel to graph of x = 4
- **32.** passes through origin, parallel to graph of x + y = 10
- **33.** passes through (2, -1), parallel to graph of 2x + 3y = 6
- **34.** passes through (2, -1), perpendicular to graph of 2x + 3y = 6**35.** passes through (-4, 1), perpendicular to a line whose slope is $-\frac{3}{2}$
- **36.** passes through (3, 3), perpendicular to graph of y = 3
- **37.** passes through (0, 0), perpendicular to graph of y = -x



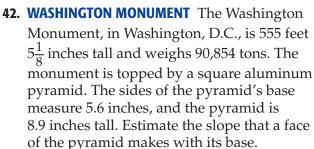
Find the slope of the line that passes through each pair of points.

38.
$$\left(\frac{1}{2}, -\frac{1}{3}\right), \left(\frac{1}{4}, \frac{2}{3}\right)$$

39. $\left(\frac{1}{2}, \frac{2}{3}\right), \left(\frac{5}{6}, \frac{1}{4}\right)$

40.
$$(c, 5), (c, -2)$$

41. (3, d), (-5, d)





- **43.** Determine the value of *r* so that the line through (5, r) and (2, 3) has slope 2.
- **44.** Determine the value of *r* so that the line through (6, r) and (9, 2) has slope $\frac{1}{2}$.



Graph the line that satisfies each set of conditions.

- **45.** perpendicular to graph of 3x 2y = 24, intersects that graph at its *x*-intercept
- **46.** perpendicular to graph of 2x + 5y = 10, intersects that graph at its *y*-intercept
- **47. GEOMETRY** Determine whether quadrilateral *ABCD* with vertices A(-2, -1), B(1, 1), C(3, -2), and D(0, -4) is a rectangle. Explain.



For Exercises 48 and 49, use a graphing calculator to investigate the graphs of each set of equations. Explain how changing the slope affects the graph of the line.

48.
$$y = 2x + 3$$
, $y = 4x + 3$, $y = 8x + 3$, $y = x + 3$

49.
$$y = -3x + 1$$
, $y = -x + 1$, $y = -5x + 1$, $y = -7x + 1$

H.O.T. Problems.....

- **50. OPEN ENDED** Write an equation of a line with slope 0. Describe the graph of the equation.
- **51. CHALLENGE** If the graph of the equation ax + 3y = 9 is perpendicular to the graph of the equation 3x + y = -4, find the value of a.
- **52. FIND THE ERROR** Gabriel and Luisa are finding the slope of the line through (2, 4) and (-1, 5). Who is correct? Explain your reasoning.

$$m = \frac{\text{Gabriel}}{5-4} \text{ or } \frac{1}{3}$$

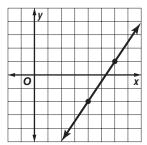
$$m = \frac{\text{Gabriel}}{\frac{5-4}{2-(-1)}} \text{ or } \frac{1}{3}$$

$$m = \frac{\frac{\text{Luisa}}{4-5}}{\frac{4-5}{2-(-1)}} \text{ or } -\frac{1}{3}$$

- **53. REASONING** Determine whether the statement *A line has a slope that is a real* number is sometimes, always, or never true. Explain your reasoning.
- **54.** Writing in Math Use the information about the grade of a road on page 71 to explain how slope applies to the steepness of roads. Include a graph of y = 0.08x, which corresponds to a grade of 8%.

STANDARDIZED TEST PRACTICE

55. ACT/SAT What is the slope of the line shown in the graph?



- $D_{\frac{3}{2}}$

56. REVIEW The table below shows the cost of bananas depending on the amount purchased. Which conclusion can be made based on information in the table?

Cost of Bananas					
Number of					
Pounds	Cost (\$)				
5	1.45				
20	4.60				
50	10.50				
100	19.00				

- **F** The cost of 10 pounds of bananas would be more than \$4.00.
- **G** The cost of 200 pounds of bananas would be at most \$38.00.
- H The cost of bananas is always more than \$0.20 per pound.
- The cost of bananas is always less than \$0.28 per pound.

Spiral Review

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation. (Lesson 2-2)

57.
$$-2x + 5y = 20$$

58.
$$4x - 3y + 8 = 0$$

59.
$$y = 7x$$

Find each value if f(x) = 3x - 4. (Lesson 2-1)

60.
$$f(-1)$$

62.
$$f(\frac{1}{2})$$

Solve each inequality. (Lessons 1-5 and 1-6)

64.
$$5 < 2x + 7 < 13$$

65.
$$2z + 5 \ge 1475$$

66. SCHOOL A test has multiple-choice questions worth 4 points each and truefalse questions worth 3 points each. Marco answers 14 multiple-choice questions correctly. How many true-false questions must be answer correctly to get at least 80 points total? (Lesson 1-5)

Simplify. (Lessons 1-1 and 1-2)

67.
$$\frac{1}{3}(15a + 9b) - \frac{1}{7}(28b - 84a)$$

68.
$$3 + (21 \div 7) \times 8 \div 4$$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation for y. (Lesson 1-3)

69.
$$x + y = 9$$

70.
$$4x + y = 2$$

71.
$$-3x - y + 7 = 0$$

72.
$$5x - 2y - 1 = 0$$

73.
$$3x - 5y + 4 = 0$$

74.
$$2x + 3y - 11 = 0$$



Graphing Calculator Lab

The Family of Linear Functions

The parent function of the family of linear functions is f(x) = x. You can use a graphing calculator to investigate how changing the parameters *m* and *b* in f(x) = mx + b affects the graphs as compared to the parent function.

$ACTIVITY \qquad b \text{ in } f(x) = mx + b$

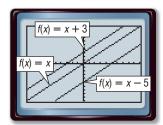
$$b \text{ in } f(x) = mx + b$$

Graph f(x) = x, f(x) = x + 3, and f(x) = x - 5 in the standard viewing window.

Enter the equations in the Y = list as Y1, Y2, and Y3. Then graph the equations.

KEYSTROKES: Y=
$$X,T,\theta,n$$
 ENTER X,T,θ,n + 3 ENTER X,T,θ,n - 5 ENTER

- **1A.** Compare and contrast the graphs.
- **1B.** How would you obtain the graphs of f(x) = x + 3 and f(x) = x - 5 from the graph of f(x) = x?



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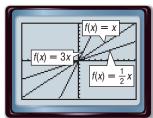
The parameter m in f(x) = mx + b affects the graphs in a different way than b.

ACTIVITY 2 m in f(x) = mx + b

Graph f(x) = x, f(x) = 3x, and $f(x) = \frac{1}{2}x$ in the standard viewing window.

Enter the equations in the Y =list and graph.

- **2A.** How do the graphs compare?
- **2B.** Which graph is steepest? Which graph is the least steep?
- **2C.** Graph f(x) = -x, f(x) = -3x, and $f(x) = -\frac{1}{2}x$ in the standard viewing window. How do these graphs compare?



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ANALYZE THE RESULTS

Graph each set of equations on the same screen. Describe the similarities or differences among the graphs.

1.
$$f(x) = 3x$$

2.
$$f(x) = x + 2$$

2.
$$f(x) = x + 2$$
 3. $f(x) = x - 3$

$$f(x) = 3x + 1$$

$$f(x) = 5x + 2$$
 $f(x) = 2x - 3$

$$f(x) = 2x - 3$$

$$f(x) = 3x - 2$$

$$f(x) = \frac{1}{2}x + 2$$

$$f(x) = 0.75x - 3$$

- **4.** What do the graphs of equations of the form f(x) = mx + b have in common?
- **5.** How do the values of b and m affect the graph of f(x) = mx + bas compared to the parent function f(x) = x?
- **6.** Summarize your results. How can knowing about the effects of *m* and *b* help you sketch the graph of a function?

Writing Linear Equations

Main Ideas

- Write an equation of a line given the slope and a point on the
- · Write an equation of a line parallel or perpendicular to a given line.

New Vocabulary

slope-intercept form point-slope form

GET READY for the Lesson

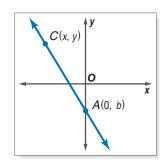
When a company manufactures a product, they must consider two types of cost. There is the *fixed cost*, which they must pay no matter how many of the product they produce, and there is variable cost, which depends on how many of the product they produce. In some cases, the total cost can be found using a linear equation such as y = 5400 + 1.37x.

Forms of Equations Consider the graph at the right. The line passes through A(0, b) and C(x, y). Notice that b is the y-intercept of \overrightarrow{AC} . You can use these two points to find the slope of AC. Substitute the coordinates of points *A* and *C* into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 Slope formula

$$m = \frac{y - b}{x - 0}$$
 $(x_1, y_1) = (0, b), (x_2, y_2) = (x, y)$

$$m = \frac{y - b}{x}$$
 Simplify.



Now solve the equation for y.

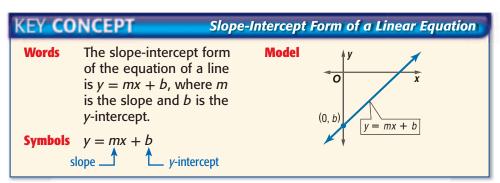
$$mx = y - b$$
 Multiply each side by x .
 $mx + b = y$ Add b to each side.
 $y = mx + b$ Symmetric Property of Equality

When an equation is written in this form, it is in slope-intercept form.

Study Tip

Slope-Intercept Form

The equation of a vertical line cannot be written in slopeintercept form because its slope is undefined.



If you are given the slope and *y*-intercept of a line, you can find an equation of the line by substituting the values of *m* and *b* into the slope-intercept form. You can also use the slope-intercept form to find an equation of a line if you know the slope and the coordinates of any point on the line.







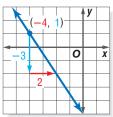
EXAMPLEWrite an Equation Given Slope and a Point

🚺 Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{2}$ and passes through (-4, 1).

$$y = mx + b$$
 Slope-intercept form
$$1 = -\frac{3}{2}(-4) + b \quad (x, y) = (-4, 1), m = -\frac{3}{2}$$

$$1 = 6 + b$$
 Simplify.
$$-5 = b$$
 Subtract 6 from each side.

The equation in slope-intercept form is $y = -\frac{3}{2}x - 5$.



CHECK Your Progress

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

1A. slope $\frac{4}{3}$, passes through (3, 2) **1B.** slope -4, passes through (-2, -2)

If you are given the coordinates of two points on a line, you can use the **point-slope form** to find an equation of the line that passes through them.

CONCEPT

Point-Slope Form of a Linear Equation

Words

The point-slope form of the equation of a line is $y - y_1 =$ $m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

Symbols

 $y - y_1 = m(x - x_1)$

STANDARDIZED TEST EXAMPLE Write an Equation Given Two Points



M What is an equation of the line through (-1, 4) and (-4, 5)?

A
$$y = -\frac{1}{3}x + \frac{11}{3}$$
 B $y = \frac{1}{3}x + \frac{13}{3}$ **C** $y = -\frac{1}{3}x + \frac{13}{3}$ **D** $y = -3x + 1$

B
$$y = \frac{1}{3}x + \frac{13}{3}$$

C
$$y = -\frac{1}{3}x + \frac{13}{3}$$

$$\mathbf{D} \ y = -3x + 1$$

Read the Test Item

You are given the coordinates of two points on the line.

Solve the Test Item

First, find the slope of the line.

Then write an equation.

CHECK Your Progress

2. What is an equation of the line through (2, 3) and (-4, -5)?

F
$$y = \frac{4}{3}x + \frac{1}{3}$$
 G $y =$
Personal Tutor at algebra2.com

G
$$y = \frac{4}{3x} + 8$$

F
$$y = \frac{4}{3}x + \frac{1}{3}$$
 G $y = \frac{4}{3x} + 8$ **H** $y = \frac{1}{3}x + \frac{17}{3}$ **J** $y = \frac{1}{3}x - 8$

$$\mathbf{J} \ \ y = \frac{1}{3}x - 8$$



Test-Taking Tip

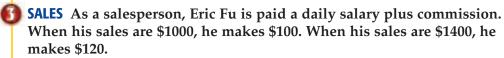
To check your answer,

substitute each ordered pair into your answer. Each should satisfy the

equation.

When changes in real-world situations occur at a linear rate, a linear equation can be used as a model for describing the situation.

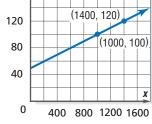
Real-World EXAMPLE



a. Write a linear equation to model this situation.

Let *x* be his sales and let *y* be the amount of money he makes. Use the points (1000, 100) and (1400, 120) to make a graph to represent the situation.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 Slope formula
= $\frac{120 - 100}{1400 - 1000}$ $(x_1, y_1) = (1000, 100), (x_2, y_2) = (1400, 120)$
= 0.05 Simplify.



Now use the slope and either of the given points with the point-slope form to write the equation.

$$y - y_1 = m(x - x_1)$$
 Point-slope form
 $y - 100 = 0.05(x - 1000)$ $m = 0.05, (x_1, y_1) = (1000, 100)$
 $y - 100 = 0.05x - 50$ Distributive Property
 $y = 0.05x + 50$ Add 100 to each side.

The slope-intercept form of the equation is y = 0.05x + 50.

b. What are Mr. Fu's daily salary and commission rate?

The *y*-intercept of the line is 50. The *y*-intercept represents the money Eric would make if he had no sales. In other words, \$50 is his daily salary. The slope of the line is 0.05. Since the slope is the coefficient of x, which is his sales, he makes 5% commission.

c. How much would Mr. Fu make in a day if his sales were \$2000?

Find the value of y when x = 2000.

$$y = 0.05x + 50$$
 Use the equation you found in part **a**.
 $= 0.05(2000) + 50$ Replace x with 2000.
 $= 100 + 50$ or 150 Simplify.

Mr. Fu would make \$150 if his sales were \$2000.

Study Tip

Alternative Method

You could also find Mr. Fu's salary in part c by extending the graph. Then find the y-value when x is 2000.

CHECK Your Progress

SCHOOL CLUBS For each meeting of the Putnam High School book club, \$25 is taken from the activities account to buy snacks and materials. After their sixth meeting, there will be \$350 left in the activities account.

- **3A.** If no money is put back into the account, what equation can be used to show how much money is left in the activities account after having *x* number of meetings?
- **3B.** How much money was originally in the account?
- **3C.** After how many meetings will there be no money left in the activities account?







Parallel and Perpendicular Lines The slope-intercept and point-slope forms can be used to find equations of lines that are parallel or perpendicular to given lines.

EXAMPLEWrite an Equation of a Perpendicular Line



I Write an equation for the line that passes through (−4, 3) and is perpendicular to the line whose equation is y = -4x - 1.

The slope of the given line is -4. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\frac{1}{4}$.

Use the point-slope form and the ordered pair (-4, 3).

$$y - y_1 = m(x - x_1)$$
 Point-slope form

$$y-3=\frac{1}{4}[x-(-4)]$$
 $(x_1,y_1)=(-4,3), m=\frac{1}{4}$

$$(x_1, y_1) = (-4, 3), m = \frac{1}{4}$$

$$y - 3 = \frac{1}{4}x + 1$$
 Distributive Property

$$y = \frac{1}{4}x + 4$$
 Add 3 to each side.

CHECK Your Progress

4. Write an equation for the line that passes through (3, 7) and is perpendicular to the line whose equation is $y = \frac{3}{4}x - 5$.

Your Understanding

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

Example 1

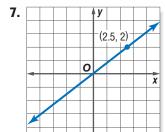
- **1.** slope 0.5, passes through (6, 4)
- **2.** slope $-\frac{3}{4}$, passes through $\left(2, \frac{1}{2}\right)$

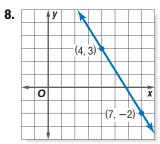
0

- (p. 80)
- **3.** slope 3, passes through (0, -6)
- **4.** slope 0.25, passes through (0, 4) **5.** passes through (6, 1) and (8, -4) **6.** passes through (-3, 5) and (2, 2)

Example 2 (p. 80)

Write an equation in slope-intercept form for each graph.





9. STANDARDIZED TEST PRACTICE What is an equation of the line through (2, -4) and (-3, -1)?

A
$$y = -\frac{3}{5}x + \frac{26}{5}$$

C
$$y = \frac{3}{5}x - \frac{26}{5}$$

B
$$y = -\frac{3}{5}x - \frac{14}{5}$$

D
$$y = \frac{3}{5}x + \frac{14}{5}$$

Example 3 (p. 81) 10. PART-TIME JOB Each week Carmen earns \$15 plus \$0.17 for every pamphlet that she delivers. Write an equation that can be used to find how much Carmen earns each week. How much will she earn the week she delivers 300 pamphlets?

Example 4 (p. 82)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

- 11. perpendicular to $y = \frac{3}{4}x 2$, passes through (2, 0) 12. perpendicular to $y = \frac{1}{2}x + 6$, passes through (-5, 7)

Exercises

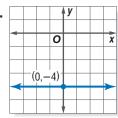
HOMEWORK HELP						
For Exercises	See Examples					
13-16	1					
17, 18,	2					
21, 22						
19, 20	4					
23, 24	3					

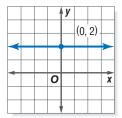
Write an equation in slope-intercept form for the line that satisfies each set of conditions.

- **13.** slope 3, passes through (0, -6) **14.** slope 0.25, passes through (0, 4) **15.** slope $-\frac{1}{2}$, passes through (1, 3) **16.** slope $\frac{3}{2}$ passes through (-5, 1)
- **17.** passes through (-2, 5) and (3, 1) **18.** passes through (7, 1) and (7, 8)
- **19.** passes through (4, 6), parallel to the graph of $y = \frac{2}{3}x + 5$
- **20.** passes through (2, -5), perpendicular to the graph of $y = \frac{1}{4}x + 7$

Write an equation in slope-intercept form for each graph.







- **23. ECOLOGY** A park ranger at Creekside Woods estimates there are 6000 deer in the park. She also estimates that the population will increase by 75 deer each year to come. Write an equation that represents how many deer will be in the park in x years.
- **24. BUSINESS** For what distance do the two stores charge the same amount for a balloon arrangement?







Real-World Link..

The number of whitetail deer in the United States increased from about half a million in the early 1900s to 25 to 30 million in 2005.

Source: espn.com

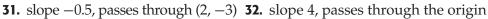
- **GEOMETRY** For Exercises 25–27, use the equation d = 180(c 2) that gives the total number of degrees d in any convex polygon with c sides.
- **25.** Write this equation in slope-intercept form.
- **26.** Identify the slope and *d*-intercept.
- **27.** Find the number of degrees in a pentagon.

SCIENCE For Exercises 28–30, use the following information.

Ice forms at a temperature of 0°C, which corresponds to a temperature of 32°F. A temperature of 100°C corresponds to a temperature of 212°F.

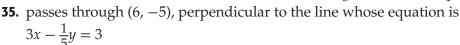
- **28.** Write and graph the linear equation that gives the number *y* of degrees Fahrenheit in terms of the number *x* of degrees Celsius.
- **29.** What temperature corresponds to 20°C?
- **30.** What temperature is the same on both scales?

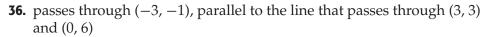
Write an equation in slope-intercept form for the line that satisfies each set of conditions.

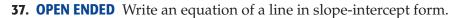


33.
$$x$$
-intercept -4 , y -intercept 4

33. *x*-intercept
$$-4$$
, *y*-intercept 4 **34.** *x*-intercept $\frac{1}{3}$, *y*-intercept $-\frac{1}{4}$







38. REASONING What are the slope and *y*-intercept of the equation cx + y = d?

39. CHALLENGE Given $\triangle ABC$ with vertices A(-6, -8), B(6, 4), and C(-6, 10), write an equation of the line containing the altitude from A. (Hint: The altitude from *A* is a segment that is perpendicular to *BC*.)

40. Writing in Math Use the information on page 79 to explain how linear equations apply to business. Relate the terms fixed cost and variable cost to the equation y = 5400 + 1.37x, where y is the cost to produce x units of a product. Give the cost to produce 1000 units of the product.



Math

See pages 894, 927.

Self-Check Quiz at algebra2.com

STANDARDIZED TEST PRACTICE

41. ACT/SAT What is an equation of the line through $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$?

A
$$y = -2x - \frac{1}{2}$$
 C $y = 2x - \frac{5}{2}$

C
$$y = 2x - \frac{5}{2}$$

$$\mathbf{B} \ \ y = -3x$$

B
$$y = -3x$$
 D $y = \frac{1}{2}x + 1$

42. REVIEW The total cost *c* in dollars to go to a fair and ride *n* roller coasters is given by the equation

$$c = 15 + 3n.$$

If the total cost was \$33, how many roller coasters were ridden?



Find the slope of the line that passes through each pair of points. (Lesson 2-3)

46. INTERNET A Webmaster estimates that the time (seconds) to connect to the server when *n* people are connecting is given by t(n) = 0.005n + 0.3. Estimate the time to connect when 50 people are connecting. (Lesson 2-2)

Solve each inequality. (Lessons 1-5 and 1-6)

47.
$$|x-2| \le -99$$

48.
$$-4x + 7 \le 31$$

49.
$$2(r-4)+5 \ge 9$$

GET READY for the Next Lesson

PREREQUISITE SKILL Find the median of each set of numbers. (Page 760)

- 1. State the domain and range of the relation $\{(2,5), (-3,2), (2,1), (-7,4), (0,-2)\}$. Is the relation a function? Write yes or no. (Lesson 2-1)
- **2.** Find f(15) if $f(x) = 100x 5x^2$. (Lesson 2-1)

For Exercises 3–5, use the table that shows a teacher's class size in recent years. (Lesson 2-1)

Year	Class Size
2002	27
2003	30
2004	29
2005	33

- **3.** Graph the relation.
- **4.** Identify the domain and range.
- **5.** Is the relation a function? Explain your reasoning.
- **6.** Write y = -6x + 4 in standard form. Identify A, B, and C. (Lesson 2-2)
- **7.** Find the *x*-intercept and the *y*-intercept of the graph of 3x + 5y = 30. Then graph the equation. (Lesson 2-2)
- **8. MULTIPLE CHOICE** What is the y-intercept of the graph of 10 - x = 2y? (Lesson 2-2)

A 2

B 5

C 6

D 10

9. What is the slope of the line containing the points shown in the table? (Lesson 2-3)

X	у
1	-1
8	7
15	15

10. Graph the line that passes through (4, -3)and is parallel to the line with equation 2x + 5y = 10. (Lesson 2-3)

Find the slope of the line that passes through each pair of points. (Lesson 2-3)

11. (7, 3), (8, 5)

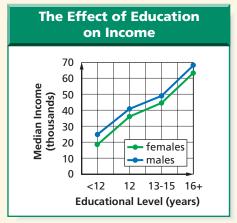
12. (12, 9), (9, 1)

13. (4, -4), (3, -7)

14. (0, 9), (4, 6)

SCHOOL For Exercises 15 and 16, use the following information.

The graph shows the effect that education levels have on income. (Lesson 2-3)



Source: healthypeople.gov

- **15.** Find the average rate of change of income for females that have 12 years of education to females that have 16+ years of education.
- **16.** Find the average rate of change of income for males that have 12 years of education to males that have 16+ years of education.
- **17.** Write an equation in slope-intercept form of the line with slope $-\frac{2}{3}$ that passes through the point (-3, 5). (Lesson 2-4)
- **18. MULTIPLE CHOICE** Find the equation of the line that passes through (0, -3) and (4, 1). (Lesson 2-4)

F
$$y = -x + 3$$

G
$$y = -x - 3$$

H
$$y = x - 3$$

$$y = x + 3$$

PART-TIME JOB Jesse is a pizza delivery driver. Each day his employer gives him \$20 plus \$0.50 for every pizza that he delivers. (Lesson 2-4)

- **19.** Write an equation that can be used to determine how much Jesse earns each day if he delivers x pizzas.
- **20.** How much will he earn the day he delivers 20 pizzas?





Statistics: Using Scatter Plots

Main Ideas

- Draw scatter plots.
- Find and use prediction equations.

New Vocabulary

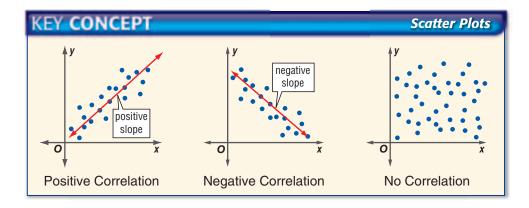
bivariate data scatter plot positive correlation negative correlation no correlation line of fit prediction equation

GET READY for the Lesson

The table shows the number of Calories burned per hour by a 140-pound person running at various speeds. A linear function can be used to model these data.

Speed (mph)	Calories
5	508
6	636
7	731
8	858

Scatter Plots Data with two variables, such as speed and Calories, is called **bivariate data**. A set of bivariate data graphed as ordered pairs in a coordinate plane is called a scatter plot. A scatter plot can show whether there is a **positive**, **negative**, or **no correlation** between the data.



The more closely data can be approximated by a line, the stronger the correlation. Correlations are usually described as strong or weak.

Prediction Equations When you find a line that closely approximates a set of data, you are finding a line of fit for the data. An equation of such a line is often called a prediction equation because it can be used to predict one of the variables given the other variable.

To find a line of fit and a prediction equation for a set of data, select two points that appear to represent the data well. This is a matter of personal judgment, so your line and prediction equation may be different from someone else's.





Study Tip 🏈

Choosing the Independent Variable

Letting x be the number of years since the first year in the data set sometimes simplifies the calculations involved in finding a function to model the data.

Reading Math

Predictions

When you are predicting for an *x*-value greater than or less than any in the data set, the process is known as extrapolation.

When you are predicting for an *x*-value between the least and greatest in the data set, the process is known as interpolation.

Real-World EXAMPLE Find and Use a Prediction Equation

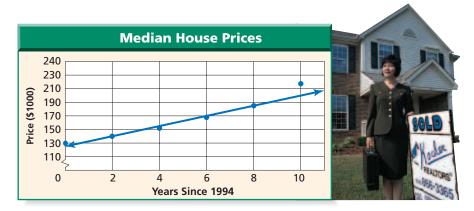
HOUSING The table below shows the median selling price of new, privately-owned, one-family houses for some recent years.

Year	1994	1996	1998	2000	2002	2004
Price (\$1000)	130.0	140.0	152.5	169.0	187.6	219.6

Source: U.S. Census Bureau and U.S. Department of Housing and Urban Development

a. Draw a scatter plot and a line of fit for the data. How well does the line fit the data?

Graph the data as ordered pairs, with the number of years since 1994 on the horizontal axis and the price on the vertical axis. The points (2, 140.0) and (8, 187.6) appear to represent the data well. Draw a line through these two points. Except for (10, 219.6), this line fits the data very well.



b. Find a prediction equation. What do the slope and y-intercept indicate?

Find an equation of the line through (2, 140.0) and (8, 187.6).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 Slope formula $y - y_1 = m(x - x_1)$ Point-slope form $= \frac{187.6 - 140.0}{8 - 2}$ Substitute. $y - 140.0 = 7.93(x - 2)$ Substitute. $y = 7.93x + 124.14$ Simplify. $y = 7.93x + 124.14$ Simplify.

One prediction equation is y = 7.93x + 124.14. The slope indicates that the median price is increasing at a rate of about \$7930 per year. The y-intercept indicates that, according to the trend of the rest of the data, the median price in 1994 should have been about \$124,140.

c. Predict the median price in 2014.

The year 2014 is 20 years after 1994, so use the prediction equation to find the value of y when x = 20.

$$y = 7.93x + 124.14$$
 Prediction equation
= $7.93(20) + 124.14$ $x = 20$
= 282.74 Simplify.

The model predicts that the median price in 2014 will be about \$282,740. (continued on the next page)

d. How accurate does the prediction appear to be?

Except for the outlier, the line fits the data very well, so the predicted value should be fairly accurate.

CHECK Your Progress

1. The table shows the mean selling price of new, privately owned one-family homes for some recent years. Draw a scatter plot and a line of fit for the data. Then find a prediction equation and predict the mean price in 2014.

Year	1994	1996	1998	2000	2002	2004
Price (\$1000)	154.5	166.4	181.9	207.0	228.7	273.5

Source: U.S. Census Bureau and U.S. Department of Housing and Urban Development



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ALGEBRA LAB

Head versus Height

COLLECT AND ORGANIZE THE DATA

Collect data from several of your classmates. Measure the circumference of each person's head and his or her height. Record the data as ordered pairs of the form (height, circumference).

ANALYZE THE DATA

- **1.** Graph the data in a scatter plot and write a prediction equation.
- **2.** Explain the meaning of the slope in the prediction equation.
- **3.** Predict the head circumference of a person who is 66 inches tall.
- **4.** Predict the height of an individual whose head circumference is 18 inches.

Your Understanding

Example (p. 87)

Study Tip

Outliers

If your scatter plot

includes points that are far from the others on the graph, check

your data before

deciding that the point is an outlier. You may

have made a graphing

or recording mistake.

Complete parts a-c for each set of data in Exercises 1 and 2.

- **a.** Draw a scatter plot and a line of fit, and describe the correlation.
- **b.** Use two ordered pairs to write a prediction equation.
- c. Use your prediction equation to predict the missing value.
- **1. SCIENCE** The table shows the temperature in the atmosphere at various altitudes.

Altitude (ft)	0	1000	2000	3000	4000	5000
Temp (°C)	15.0	13.0	11.0	9.1	7.1	?

Source: NASA

2. TELEVISION The table shows the percentage of U.S. households with televisions that also had cable service in some recent years.

Year	1995	1997	1999	2001	2003	2015
Percent	65.7	67.3	68.0	69.2	68.0	?

Source: Nielsen Media Research



Complete parts a-c for each set of data in Exercises 3–6.

- **a.** Draw a scatter plot and a line of fit, and describe the correlation.
- **b.** Use two ordered pairs to write a prediction equation.
- c. Use your prediction equation to predict the missing value.
 - **3. SAFETY** All states and the District of Columbia have enacted laws setting 21 as the minimum drinking age. The table shows the estimated cumulative number of lives these laws have saved by reducing traffic fatalities.

Year	1999	2000	2001	2002	2003	2015
Lives (1000s)	19.1	20.0	21.0	21.9	22.8	?

Source: National Highway Traffic Safety Administration

Cross-Curricular Project A scatter plot of loan payments can help you analyze home loans. Visit algebra2.com to continue work on your project.

4. HOCKEY The table shows the number of goals and assists for some of the members of the Detroit Red Wings in a recent NHL season.

Goals	30	25	18	14	15	14	10	6	4	30	?
Assists	49	43	33	32	28	29	12	9	15	38	20

Source: www.detroitredwings.com

5. HEALTH The table shows the number of gallons of bottled water consumed per person in some recent years.

Year	1998	1999	2000	2001	2002	2003	2015
Gallons	15.0	16.4	17.4	18.8	20.7	22.0	?

Source: U.S. Department of Agriculture

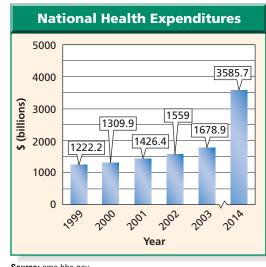
6. THEATER The table shows the total revenue of all Broadway plays for recent seasons.

Coocon	1999-	2000-	2001-	2002-	2003-	2013-
Season	2000	2001	2002	2003	2004	2014
Revenue (\$ millions)	603	666	643	721	771	?

Source: The League of American Theatres and Producers, Inc.

MEDICINE For Exercises 7–9, use the graph that shows how much Americans spent on health care in some recent years and a prediction for how much they will spend in 2014.

- **7.** Write a prediction equation from the data for 1999 to 2003.
- **8.** Use your equation to predict the amount for 2014.
- **9.** Compare your prediction to the one given in the graph.



Source: cms.hhs.gov

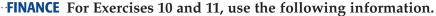




A financial analyst can advise people about how to invest their money and plan for retirement.



For more information, go to algebra2.com.



Della has \$1000 that she wants to invest in the stock market. She is considering buying stock in either Company 1 or Company 2. The values of the stocks at the end of each of the last 4 months are shown in the tables below.

- **10.** Based only on these data, which stock should Della buy? Explain.
- **11.** Do you think investment decisions should be based on this type of reasoning? If not, what other factors should be considered?

Company 1				
Month	Share Price (\$)			
Aug.	25.13			
Sept.	22.94			
Oct.	24.19			
Nov.	22.56			

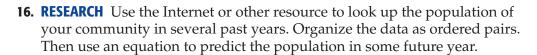
Company 2				
Month	Share Price (\$)			
Aug.	31.25			
Sept.	32.38			
Oct.	32.06			
Nov.	32.44			

PLANETS For Exercises 12–15, use the table below that shows the average distance from the Sun and average temperature for eight of the planets.

Planet	Average Distance from the Sun (million miles)	Average Temperature (°F)
Mercury	36	333
Venus	67.2	867
Earth	93	59
Mars	141.6	-85
Jupiter	483.8	—166
Saturn	890.8	-200
Uranus	1784.8	-320
Pluto	3647.2	-375

Source: World Meteorological Association

- **12.** Draw a scatter plot with average distance as the independent variable.
- **13.** Write a prediction equation.
- **14.** Predict the average temperature for Neptune, which has an average distance from the Sun of 2793.1 million miles.
- **15.** Compare your prediction to the actual value of -330° F.





H.O.T. Problems.....

CHALLENGE For Exercises 17 and 18, use the table that shows the percent of people ages 25 and over with a high school diploma over the last few decades.

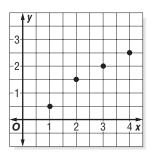
- **17.** Use a prediction equation to predict the percent in 2015.
- **18.** Do you think your prediction is accurate? Explain.



- **19. OPEN ENDED** Write a different prediction equation for the data in the example on page 87.
- **20.** Writing in Math Use the information on page 86 to explain how a linear equation can model the number of Calories you burn while exercising. Include a scatter plot, a description of the correlation, and a prediction equation for the data. Then predict the number of Calories burned in an hour by a 140-pound person running at 9 miles per hour and compare your predicted value with the actual value of 953.

STANDARDIZED TEST PRACTICE

21. ACT/SAT Which line best fits the data in the graph?



$$\mathbf{A} \ y = x$$

C
$$y = -0.5x - 4$$

B
$$y = -0.5x + 4$$

D
$$y = 0.5 + 0.5x$$

22. REVIEW Anna took brownies to a club meeting. She gave half of her brownies to Sarah. Sarah gave a third of her brownies to Rob. Rob gave a fourth of his brownies to Trina. If Trina has 3 brownies, how many brownies did Anna have in the beginning?

F 12

G 36

H 72

I 144

Spiral Review

Write an equation in slope-intercept form that satisfies each set of conditions. (Lesson 2-4)

23. slope 4, passes through (0, 6)

24. passes through (5, -3) and (-2, 0)

TELEPHONES For Exercises 25 and 26, use the following information. (Lesson 2-4)

Namid is examining the calling card portion of his phone bill. A 4-minute call at the night rate cost \$2.65. A 10-minute call at the night rate cost \$4.75.

- **25.** Write a linear equation to model this situation.
- **26.** How much would it cost to talk for half an hour at the night rate?

Find the slope of the line that passes through each pair of points. (Lesson 2-3)

28.
$$(-1, -2), (4, -2)$$

30. PROFIT Kara is planning to set up a booth at a local festival to sell her paintings. She determines that the amount of profit she will make is determined by the function P(x) = 11x - 100, where x is the number of paintings she sells. How much profit will Kara make if she sells 35 of her paintings? (Lesson 2-1)

GET READY for the Next Lesson

PREREQUISITE SKILL Find each absolute value. (Lesson 1-4)

34.
$$\left| -\frac{2}{3} \right|$$



Graphing Calculator Lab Lines of Regression

You can use a TI-83/84 Plus graphing calculator to find a function that best fits a set of data. The graph of a linear function that models a set of data is called a regression line or line of best fit. You can also use the calculator to draw scatter plots and make predictions.



Interactive Lab algebra2.com



ACTIVITY

INCOME The table shows the median income of U.S. families for the period 1970-2002.

Year	1970	1980	1985	1990	1995	1998	2000	2002
Income (\$)	9867	21,023	27,735	35,353	40,611	46,737	50,732	51,680

Source: U.S. Census Bureau

Make a scatter plot of the data. Find a function and graph a regression line. Then use the function to predict the median income in 2015.

STEP 1 Make a scatter plot.

• Enter the years in L1 and the income in L2.

KEYSTROKES: STAT ENTER 1970 ENTER 1980 ENTER ...

• Set the viewing window to fit the data.

KEYSTROKES: WINDOW 1965 ENTER 2015 ENTER 5 ENTER 0 ENTER 55000 ENTER 10000 ENTER

• Use STAT PLOT to graph a scatter plot. KEYSTROKES: 2nd [STAT PLOT] ENTER ENTER



[1965, 2015] scl: 5 by [0, 55,000] scl: 10,000

STEP 2 Find the equation of a regression

• Find the regression equation by selecting LinReg(ax + b) on the STAT CALC menu.

KEYSTROKES: STAT ▶ 4 ENTER



The regression equation is about y =1349.87x - 2,650,768.34. The slope indicates that family incomes were increasing at a rate of about \$1350 per year.

The number r is called the **linear** correlation coefficient. The closer the value of r is to 1 or -1, the closer the data points are to the line. In this case, *r* is very close to 1 so the line fits the data well. If the values of r^2 and r are not displayed, use DiagnosticOn from the CATALOG menu.



STEP 3 Graph the regression equation.

 Copy the equation to the Y= list and graph.

KEYSTROKES: Y= VARS 5 ▶ 1 GRAPH



[1965, 2015] scl: 5 by [0, 55,000] scl: 10,000

The graph of the line will be displayed with the scatter plot. Notice that the regression line seems to pass through only one of the data points, but comes close to all of them. As the correlation coefficient indicated, the line fits the data very well.

STEP 4 Predict using the function.

Find y when x = 2015. Use VALUE on the CALC menu.

KEYSTROKES: 2nd [CALC] 1 2015 ENTER



According to the function, the median family income in 2015 will be about \$69,214. Because the function is a very good fit to the data, the prediction should be quite accurate.

EXERCISES

BASEBALL For Exercises 1–3, use the table at the right that shows the total attendance for minor league baseball in some recent years.

- **1.** Make a scatter plot of the data.
- **2.** Find a regression equation for the data.
- **3.** Predict the attendance in 2010.

Year	Attendance (millions)
1985	18.4
1990	25.2
1995	33.1
2000	37.6

Source: National Association of Professional Baseball Leagues

GOVERNMENT For Exercises 4–6, use the table below that shows the population and the number of representatives in Congress for the most populous states.

State	CA	TX	NY	FL	IL	PA	ОН
Population (millions)	35.5	22.1	19.2	17.0	12.7	12.4	11.4
Representatives	53	32	29	25	19	19	18

Source: World Almanac

- **4.** Make a scatter plot of the data.
- **5.** Find a regression equation for the data.
- **6.** Predict the number of representatives for South Carolina, which has a population of about 4.1 million.

MUSIC For Exercises 7–11, use the table at the right that shows the percent of music sales that were made in record stores in the United States for the period 1995–2004.

- **7.** Make a scatter plot of the data. Is the correlation of the data positive or negative? Explain.
- **8.** Find a regression equation for the data.
- **9.** According to the regression equation, what was the average rate of change of record store sales during the period?
- **10.** Use the function to predict the percent of sales made in record stores in 2015.
- 11. How accurate do you think your prediction is? Explain.

Record Store Sales				
Year	Sales (percent)			
1995	52			
1996	49.9			
1997	51.8			
1998	50.8			
1999	44.5			
2000	42.4			
2001	42.5			
2002	36.8			
2003	33.2			
2004	32.5			

Source: Recording Industry Association of

RECREATION For Exercises 12–16, use the table at the right that shows the amount of money spent on sporting footwear in some recent years.

- **12.** Find a regression equation for the data.
- **13.** Use the regression equation to predict the sales in 2010.
- **14.** Delete the outlier (1999, 12,546) from the data set and find a new regression equation for the data.
- **15.** Use the new regression equation to predict the sales in 2010.
- **16.** Compare the correlation coefficients for the two regression equations. Which function fits the data better? Which prediction would you expect to be more accurate?

Sporting Footwear Sales				
Year	Sales (\$ millions)			
1998	13,068			
1999	12,546			
2000	13,026			
2001	13,814			
2002	14,144			
2003	14,446			
2004	14,752			

Source: National Sporting Goods Association

EXTENSION

For Exercises 17–20, design and complete your own data analysis.

- **17.** Write a question that could be answered by examining data. For example, you might estimate the number of students who will attend your school 5 years from now or predict the future cost of a piece of electronic equipment.
- **18.** Collect and organize the data you need to answer the question you wrote. You may need to research your topic on the Internet or conduct a survey to collect the data you need.
- **19.** Make a scatter plot and find a regression equation for your data. Then use the regression equation to answer the question.
- **20.** Analyze your results. How accurate do you think your model is? Explain your reasoning.





Special Functions

Main Ideas

- Identify and graph step, constant, and identity functions.
- · Identify and graph absolute value and piecewise functions.

New Vocabulary

step function greatest integer function constant function identity function absolute value function piecewise function

GET READY for the Lesson

The cost of the postage to mail a letter is a function of the weight of the letter. But the function is not linear. It is a special function called a step function.

For letters with weights between whole numbers, the cost "steps up" to the next higher cost. So the cost to mail a 1.5-ounce letter is the same as the cost to mail a 2-ounce letter, \$0.63.



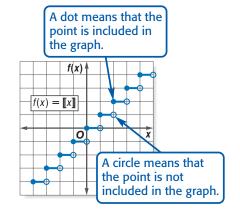
Study Tip

Greatest Integer Function

Notice that the domain of this step function is all real numbers and the range is all integers.

Step Functions, Constant Functions, and the Identity Function The graph of a step function is not linear. It consists of line segments or rays. The **greatest integer function**, written f(x) = [x], is an example of a step function. The symbol [x] means the greatest integer less than or equal to x. For example, [7.3] = 7 and [-1.5] = -2 because -1 > -1.5.

f(x) = [x]				
X	f(x)			
$-3 \le x < -2$	-3			
$-2 \le x < -1$	-2			
$-1 \le x < 0$	-1			
$0 \le x < 1$	0			
1 ≤ <i>x</i> < 2	1			
2 ≤ <i>x</i> < 3	2			
3 ≤ <i>x</i> < 4	3			





Real-World EXAMPLE Step Function

\$120, and so on.

BUSINESS The No Leak Plumbing Repair Company charges \$60 per hour or any fraction thereof for labor. Draw a graph that represents this situation.

Explore The total labor charge must be a multiple of \$60, so the graph will be the graph of a step function.

Plan If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor cost is \$60. If the time is greater than 1 hour but less than or equal to 2 hours, then the labor cost is

(continued on the next page)



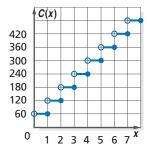


Concepts in MOtion Animation algebra2.com

Solve

Use the pattern of times and costs to make a table, where x is the number of hours of labor and C(x) is the total labor cost. Then graph.

X	C(x)
$0 < x \le 1$	\$60
1 < <i>x</i> ≤ 2	\$120
$2 < x \le 3$	\$180
$3 < x \le 4$	\$240
4 < <i>x</i> ≤ 5	\$300



Check

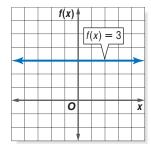
Since the company rounds any fraction of an hour up to the next whole number, each segment on the graph has a circle at the left endpoint and a dot at the right endpoint.



1. RECYCLING A recycling company pays \$5 for every full box of newspaper. They do not give any money for partial boxes. Draw a graph that shows the amount of money for the number of boxes brought to the center.

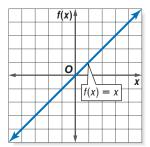
You learned in Lesson 2-4 that the slope-intercept form of a linear function is y = mx + b, or in function notation, f(x) = mx + b.

When m = 0, the value of the function is f(x) = b for every *x*-value. So, f(x) = b is called a **constant function**. The function f(x) = 0is called the zero function.



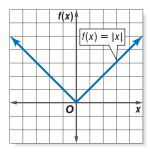
Another special case of slopeintercept form is m = 1, b = 0. This is the function f(x) = x. The graph is the line through the origin with slope 1.

Since the function does not change the input value, f(x) = x is called the identity function.



Absolute Value and Piecewise Functions Another special function is the absolute value function, f(x) = |x|.

f(x) = x				
x $f(x)$				
-2	2			
-1	1			
0	0			
1	1			
2	2			





Absolute Value Function

Notice that the domain is all real numbers and the range is all nonnegative real numbers.

The absolute value function can be written as $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$. A function that is written using two or more expressions is called a piecewise function. Recall that a family of graphs displays one or more similar characteristics. The parent graph of most absolute value functions is y = |x|.

EXAMPLE Absolute Value Functions



If f(x) = |x| + 1 and g(x) = |x| - 2 on the same coordinate plane. Determine the similarities and differences in the two graphs.

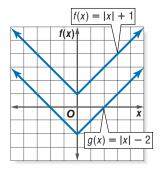
Find several ordered pairs for each function.

X	x + 1
-2	3
-1	2
0	1
1	2

х	x - 2		
-2	0		
-1	-1		
0	-2		
1	-1		

Graph the points and connect them.

- The domain of each function is all real numbers.
- The range of f(x) = |x| + 1 is $\{y | y \ge 1\}$. The range of g(x) = |x| - 2 is $\{y \mid y \ge -2\}$.
- The graphs have the same shape, but different *y*-intercepts.
- The graph of g(x) = |x| 2 is the graph of f(x) = |x| + 1 translated down 3 units.



CHECK Your Progress

2. Graph f(x) = |x + 1| and g(x) = |x - 2|.



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You can also use a graphing calculator to investigate families of absolute value graphs.

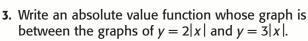
GRAPHING CALCULATOR LAB

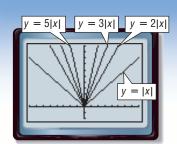
Family of Absolute Value Graphs

The calculator screen shows the graphs of y = |x|, y = 2|x|, y = 3|x|, and y = 5|x|.

THINK AND DISCUSS

- 1. What do these graphs have in common?
- **2.** Describe how the graph of y = a|x| changes as a increases. Assume a > 0.





[-8, 8] scl: 1 by [-2, 10] scl: 1

- **4.** Graph y = |x| and y = -|x| on the same screen. Then graph y = 2|x|and y = -2|x| on the same screen. What is true in each case?
- **5.** In general, what is true about the graph of y = a|x| when a < 0?





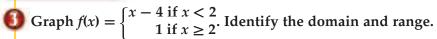
To graph other piecewise functions, examine the inequalities in the definition of the function to determine how much of each piece to include.

Study Tip

Graphs of Piecewise Functions

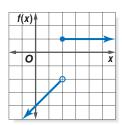
The graphs of each part of a piecewise function may or may not connect. A graph may stop at a given x value and then begin again at a different y value for the same x value.

EXAMPLE Piecewise Function



Step 1 Graph the linear function f(x) = x - 4 for x < 2. Since 2 does not satisfy this inequality, stop with an open circle at (2, -2).

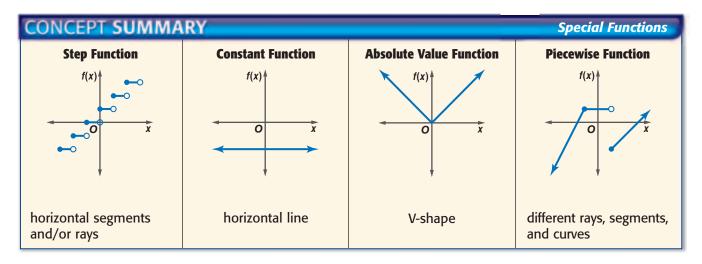
Step 2 Graph the constant function f(x) = 1 for $x \ge 2$. Since 2 does satisfy this inequality, begin with a closed circle at (2, 1) and draw a horizontal ray to the right.



The function is defined for all values of *x*, so the domain is all real numbers. The values that are y-coordinates of points on the graph are 1 and all real numbers less than -2, so the range is $\{y|y < -2 \text{ or } y = 1\}$.

CHECK Your Progress

3. Graph $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$ Identify the domain and range.

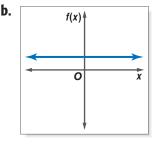


EXAMPLE Identify Functions

Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a. $f(x)^{I}$

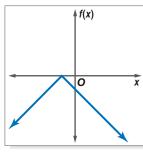
The graph has multiple horizontal segments. It represents a step function.



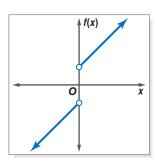
The graph is a horizontal line. It represents a constant function.



4A.



4B.



Your Understanding

Examples 1-3 (pp. 95-98)

Graph each function. Identify the domain and range.

1.
$$f(x) = - ||x||$$

2.
$$g(x) = [2x]$$

3.
$$f(x) = 4$$

4.
$$z(x) = -3$$

5.
$$h(x) = |x| - 3$$

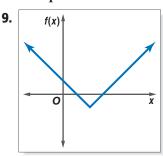
6.
$$f(x) = |3x - 2|$$

4.
$$z(x) = -3$$
5. $h(x) = |x| - 3$
6. $f(x) = |3x - 2|$
7. $g(x) =\begin{cases} -1 & \text{if } x < 0 \\ -x + 2 & \text{if } x \ge 0 \end{cases}$
8. $h(x) =\begin{cases} x + 3 & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$

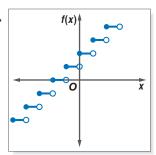
8.
$$h(x) = \begin{cases} x + 3 \text{ if } x \le -1 \\ 2x \text{ if } x > -1 \end{cases}$$

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

Example 4 (pp. 98-99)



10.



PARKING For Exercises 11–13, use the following information.

A downtown parking lot charges \$2 for the first hour and \$1 for each additional hour or part of an hour.

- **11.** What type of special function models this situation?
- **12.** Draw a graph of a function that represents this situation.
- **13.** Use the graph to find the cost of parking there for $4\frac{1}{2}$ hours.

Exercises

HOMEWO	RK HELP		
For Exercises	See Examples		
14-19	1		
20-25	2		
26–27	3		
28–33	4		

Graph each function. Identify the domain and range.

14.
$$f(x) = [x + 3]$$

15.
$$g(x) = [x-2]$$
 16. $f(x) = 2[x]$

16.
$$f(x) = 2[x]$$

17.
$$h(x) = -3[x]$$
 18. $g(x) = [x] + 3$ **19.** $f(x) = [x] - 1$

18.
$$g(x) = [\![x]\!] + 3$$

19.
$$f(x) = [x] -$$

20.
$$f(x) = |2x|$$

21.
$$h(x) = |-x|$$

21.
$$h(x) = |-x|$$
 22. $g(x) = |x| + 3$

23.
$$g(x) = |x| - 4$$

24.
$$h(x) = |x + 3|$$

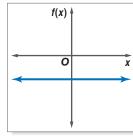
25.
$$f(x) = |x + 2|$$

26.
$$f(x) = \begin{cases} -x & \text{if } x \le 3\\ 2 & \text{if } x > 3 \end{cases}$$

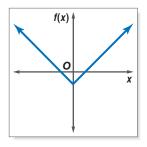
23.
$$g(x) = |x| - 4$$
 24. $h(x) = |x + 3|$ **25.** $f(x) = |x + 2|$ **26.** $f(x) = \begin{cases} -x & \text{if } x \le 3 \\ 2 & \text{if } x > 3 \end{cases}$ **27.** $h(x) = \begin{cases} -1 & \text{if } x < -2 \\ 1 & \text{if } x > 2 \end{cases}$

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

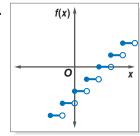
28.



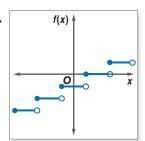
29.



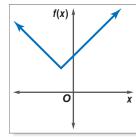
30.



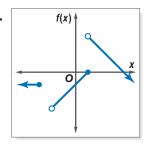
31.



32.



33.



34. THEATER Springfield High School's theater can hold 250 students. The drama club is performing a play in the theater. Draw a graph of a step function that shows the relationship between the number of tickets sold x and the minimum number of performances *y* that the drama club must do.

Graph each function. Identify the domain and range.

35.
$$f(x) = \left| x - \frac{1}{4} \right|$$

36.
$$f(x) = \left| x + \frac{1}{2} \right|$$

35.
$$f(x) = \left| x - \frac{1}{4} \right|$$
36. $f(x) = \left| x + \frac{1}{2} \right|$
37. $f(x) = \begin{cases} x \text{ if } x < -3 \\ 2 \text{ if } -3 \le x < 1 \\ -2x + 2 \text{ if } x \ge 1 \end{cases}$
38. $g(x) = \begin{cases} -1 \text{ if } x \le -2 \\ x \text{ if } -2 < x < 2 \\ -x + 1 \text{ if } x \ge 2 \end{cases}$

38.
$$g(x) = \begin{cases} -1 & \text{if } x \le -2 \\ x & \text{if } -2 < x < 2 \\ -x + 1 & \text{if } x \ge 2 \end{cases}$$

39.
$$f(x) = [|x|]$$

40.
$$g(x) = |[x]|$$

TELEPHONE RATES For Exercises 41 and 42, use the following information.

Masao has a long-distance telephone plan where she pays 10¢ for each minute or part of a minute that she talks, regardless of the time of day.

- **41.** Graph a step function that represents this situation.
- **42.** How much would a call that lasts 9 minutes and 40 seconds cost?

NUTRITION For Exercises 43–45, use the following information.

The recommended dietary allowance for vitamin C is 2 micrograms per day.

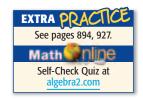
- **43.** Write an absolute value function for the difference between the number of micrograms of vitamin C you ate today *x* and the recommended amount.
- **44.** What is an appropriate domain for the function?
- **45.** Use the domain to graph the function.
- **46. INSURANCE** According to the terms of Lavon's insurance plan, he must pay the first \$300 of his annual medical expenses. The insurance company pays 80% of the rest of his medical expenses. Write a function for how much the insurance company pays if *x* represents Lavon's annual medical expenses.
- **47. OPEN ENDED** Write a function involving absolute value for which f(-2) = 3.
- **48. REASONING** Find a counterexample to the statement *To find the greatest* integer function of x when x is not an integer, round x to the nearest integer.
- **49. CHALLENGE** Graph |x| + |y| = 3.





Good sources of vitamin C include citrus fruits and juices, cantaloupe, broccoli, brussels sprouts, potatoes, sweet potatoes, tomatoes, and cabbage.

Source: The World Almanac



H.O.T. Problems.....



50. Writing in Math Use the information on page 95 to explain how step functions apply to postage rates. Explain why a step function is the best model for this situation while your gas mileage as a function of time as you drive to the post office cannot be modeled with a step function. Then graph the function that represents the cost of a first-class letter.

STANDARDIZED TEST PRACTICE

51. ACT/SAT For which function does

$$f\left(-\frac{1}{2}\right) \neq -1?$$

$$\mathbf{A} \ f(x) = 2x$$

C
$$f(x) = [x]$$

B
$$f(x) = |-2x|$$

$$\mathbf{D} f(x) = [2x]$$

52. ACT/SAT For which function is the range $\{y \mid y \le 0\}$?

$$\mathbf{F} \quad f(x) = -x$$

$$\mathbf{G} \ f(x) = [\![x]\!]$$

$$\mathbf{H} f(x) = |x|$$

$$\mathbf{J} \quad f(x) = -|x|$$

53. REVIEW Solve: 5(x + 4) = x + 4

Step 1:
$$5x + 20 = x + 4$$

Step 2:
$$4x + 20 = 4$$

Step 3:
$$4x = 24$$

Step 4:
$$x = 6$$

Which is the first incorrrect step in the solution shown above?

Spiral Review

HEALTH For Exercises 54–56, use the table that shows the life expectancy for people born in various years. (Lesson 2-5)

Year	1950	1960	1970	1980	1990	2000
Expectancy	68.2	69.7	70.8	73.7	75.4	77.0

Source: National Center for Health Statistics

- **54.** Draw a scatter plot in which x is the number of years since 1940 and describe the correlation.
- **55.** Find a prediction equation.
- **56.** Predict the life expectancy of a person born in 2010.

Write an equation in slope-intercept form that satisfies each set of conditions. (Lesson 2-4)

57. slope 3, passes through
$$(-2, 4)$$

58. passes through
$$(0, -2)$$
 and $(4, 2)$

Solve each inequality. Graph the solution set. (Lesson 1-3)

59.
$$3x - 5 \ge 4$$

60.
$$28 - 6y < 23$$

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether (0, 0) satisfies each inequality. Write yes or no. (Lesson 1-5)

61.
$$y < 2x + 3$$

62.
$$y \ge -x + 1$$

63.
$$y \le \frac{3}{4}x - 5$$

64.
$$2x + 6y + 3 > 0$$

65.
$$y > |x|$$

66.
$$|x| + y \le 3$$



Graphing Inequalities

Main Ideas

- Graph linear inequalities.
- · Graph absolute value inequalities.

New Vocabulary

boundary

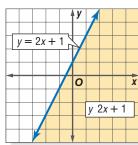
GET READY for the Lesson

Dana has Arizona Cardinals quarterback Kurt Warner as a player on his online fantasy football team. Dana gets 5 points for every yard on a completed pass and 100 points per touchdown pass that Warner makes. He considers 1000 points or more to be a good game. Dana can use a linear inequality to check whether certain combinations of yardage and touchdowns, such as those in the table, result in 1000 points or more.

CCC		66	
_	Yards	TDs	
Game 1	168	3	
Game 2	144	2	
Game 3	136	1	

Graph Linear Inequalities A linear inequality resembles a linear equation, but with an inequality symbol instead of an equals symbol. For example, $y \le 2x + 1$ is a linear inequality and y = 2x + 1 is the related linear equation.

The graph of the inequality $y \le 2x + 1$ is the shaded region. Every point in the shaded region satisfies the inequality. The graph of y = 2x + 1 is the **boundary** of the region. It is drawn as a solid line to show that points on the line satisfy the inequality. If the inequality symbol were $\langle or \rangle$, then points on the boundary would not satisfy the inequality, so the boundary would be drawn as a dashed line.



EXAMPLE Dashed Boundary



Graph 2x + 3y > 6.

The boundary is the graph of 2x + 3y = 6. Since the inequality symbol is >, the boundary will be dashed.

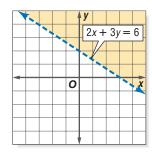
Now test the point (0, 0).

$$2x + 3y > 6$$
 Original inequality

$$2(0) + 3(0) > 6$$
 $(x, y) = (0, 0)$

$$0 > 6$$
 false

Shade the region that does *not* contain (0, 0).



CHECK Your Progress

1A. Graph
$$3x + \frac{1}{2}y < 2$$
.

1B. Graph
$$-x + 2y > 4$$
.

usually a good point to test because it results in easy calculations that you can often perform mentally.

Study Tip

Mental Math The point (0, 0) is





Look Back

To review translating verbal expressions to inequalities, see Lesson 1-5.



BUSINESS A mail-order company is hiring temporary employees to help in its packing and shipping departments during their peak season.

a. Write and graph an inequality to describe the number of employees that can be assigned to each department if the company has 20 temporary employees available.

Let p be the number of employees assigned to packing and let s be the number assigned to shipping. Since the company can assign at most 20 employees total to the two departments, use a \leq symbol.

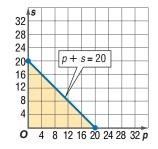
Since the inequality symbol is \leq , the graph of the related linear equation p + s = 20 is solid.

Test
$$(0, 0)$$
.

$$p + s \le 20$$
 Original inequality

$$0 + 0 \le 20$$
 $(p, s) = (0, 0)$

$$0 \le 20$$
 true

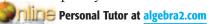


Shade the region that contains (0, 0). Since the variables cannot be negative, shade only the part in the first quadrant.

b. Can the company assign 8 employees to packing and 10 to shipping? The point (8, 10) is in the shaded region, so it satisfies the inequality. The company can assign 8 employees to packing and 10 to shipping.

CHECK Your Progress

2. Manuel has \$15 to spend at the fair. It costs \$5 for admission, \$0.75 for each ride ticket, and \$0.25 for each game ticket. Write and graph an inequality for the number of ride and game tickets that he can buy.



Graph Absolute Value Inequalities Graphing absolute value inequalities is similar to graphing linear inequalities.

EXAMPLE Absolute Value Inequality



Since the inequality symbol is <, the boundary is dashed. Graph the equation. Then test (0, 0).

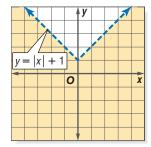
$$y < |x| + 1$$
 Original inequality

$$0 < |0| + 1$$
 $(x, y) = (0, 0)$

$$0 < 0 + 1$$
 $|0| = 0$

$$0 < 1$$
 true

Shade the region that includes (0, 0).



CHECK Your Progress

3. Graph
$$y > 2|x| - 3$$
.



Examples 1-3 (pp. 102-103)

Graph each inequality.

2.
$$y > 2x - 3$$

3.
$$x - y \ge 0$$

4.
$$x - 2y \le 5$$

5.
$$y > |2x|$$

6.
$$y \le 3|x| - 1$$

Example 2 (p. 103)

SHOPPING For Exercises 7–9, use the following information.

Gwen wants to buy some used CDs that cost \$10 each and some used DVDs that cost \$13 each. She has \$40 to spend.

- **7.** Write an inequality to represent the situation, where *c* is the number of CDs she buys and *d* is the number of DVDs.
- **8.** Graph the inequality.
- 9. Can she buy 2 CDs and 3 DVDs? Explain.

Exercises

HOMEWO	RK HELP
For Exercises	See Examples
10-15	1
16–19, 22–26	2
20-21	3

Graph each inequality.

10.
$$x + y > -5$$

11.
$$y > 6x - 2$$

12.
$$y + 1 < 4$$

13.
$$y - 2 < 3x$$

14.
$$x - 6y + 3 > 0$$

15.
$$y > \frac{1}{3}x + 5$$

16.
$$y \ge 1$$

17.
$$3 \ge x - 3y$$

18.
$$x - 5 \le y$$

19.
$$y \ge -4x + 3$$

20.
$$y \le |x|$$

21.
$$y > |4x|$$

COLLEGE For Exercises 22 and 23, use the following information.

Rosa's professor says that the midterm exam will count for 40% of each student's grade and the final exam will count for 60%. A score of at least 90 is required for an A.

- **22.** The inequality $0.4x + 0.6y \ge 90$ represents this situation, where x is the midterm score and y is the final exam score. Graph this inequality.
- **23.** Refer to the graph. If she scores 85 on the midterm and 95 on the final, will Rosa get an A?



FINANCE For Exercises 24–26, use the following information.

Carl Talbert estimates that he will need to earn at least \$9000 per year combined in dividend income from the two stocks he owns to supplement his retirement plan.

Company	Dividend per Share
Able Records	\$1.20
Best Bakes	\$1.30

- **24.** Write an inequality to represent this situation.
- **25.** Graph the inequality.
- **26.** Will he make enough from 3000 shares of each company?
- **27.** Graph all the points on the coordinate plane to the left of the graph of x = -2. Write an inequality to describe these points.
- **28.** Graph all the points on the coordinate plane below the graph of y = 3x 5. Write an inequality to describe these points.

Real-World Link

A dividend is a payment from a company to an investor. It is a way to make money on a stock without selling it.

Graph each inequality. **29.** $4x - 5y - 10 \le 0$

30.
$$y \ge \frac{1}{2}x - 5$$

31.
$$y + |x| < 3$$

32.
$$y \ge |x-1|-2$$

33.
$$|x+y| > 1$$

34.
$$|x| \le |y|$$



SHADE(COMMAND You can graph inequalities by using the SHADE(command located in the DRAW menu. Enter two functions.

- The first function defines the lower boundary of the shaded region. If the inequality is " $y \le$," use the Ymin window value as the lower boundary.
- The second function defines the upper boundary of the region. If the inequality is " $y \ge$," use the Ymax window value as the upper boundary.

Graph each inequality.

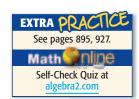
35.
$$y \ge 3$$

36.
$$y \le x + 2$$

36.
$$y \le x + 2$$
 37. $y \le -2x - 4$ **38.** $x - 7 \le y$

38.
$$x - 7 < y$$

H.O.T. Problems....



39. REASONING Explain how to determine which region to shade when graphing an inequality.

40. CHALLENGE Graph |y| < x.

41. Writing in Math Use the information on page 102 to write an inequality that defines a good game for Kurt Warner in Dana's fantasy football league, and explain how you obtained it.

AD STANDARDIZED TEST PRACTICE

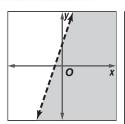
42. ACT/SAT Which could be the inequality for the graph?

A
$$y < 3x + 2$$

$$\mathbf{B} \ y \le 3x + 2$$

C
$$y > 3x + 2$$

$$\mathbf{D} \ y \ge 3x + 2$$



43. REVIEW What is the solution set of the inequality?

$$|6 - |x + 7| \le -2$$

$$F -15 \le x + \le 1$$

$$\mathbf{G} - 1 \le x \le 3$$

H
$$x \le -1$$
 or $x \ge 3$

$$J \quad x \le -15 \text{ or } x \ge 1$$

Spiral Review

Graph each function. Identify the domain and range. (Lesson 2-6)

44.
$$f(x) = [x] - 4$$

45.
$$g(x) = |x| - 1$$

46.
$$h(x) = |x - 3|$$

SALARY For Exercises 47–49, use the table which shows the years of experience for eight computer programmers and their yearly salary. (Lesson 2-5)

Years	6	5	3	1	4	3	6	2
Salary (\$)	55,000	53,000	45,000	42,000	48,500	46,500	53,000	43,000

47. Draw a scatter plot and describe the correlation.

48. Find a prediction equation.

49. Predict the salary for a representative with 9 years of experience.

Solve each equation. Check your solution. (Lesson 1-3)

50.
$$4x - 9 = 23$$

51.
$$11 - 2y = 5$$

52.
$$2z - 3 = -6z + 1$$

CHAPTER Study Guide and Review





OLDABLES GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Relations and Functions (Lesson 2-1)

- A relation is a set of ordered pairs. The domain is the set of all x-coordinates, and the range is the set of all y-coordinates.
- A function is a relation where each member of the domain is paired with exactly one member of the range.

Linear Equations and Slope (Lessons 2-2 to 2-4)

- A linear equation is an equation whose graph is a line.
- Slope is the ratio of the change in *y*-coordinates to the corresponding change in *x*-coordinates.
- Lines with the same slope are parallel. Lines with slopes that are opposite reciprocals are perpendicular.
- Standard Form: Ax + By = C, where A, B, and C are integers whose greatest common factor is 1, $A \ge 0$, and A and B are not both zero
- Slope-Intercept Form: y = mx + b
- Point-Slope Form: $y y_1 = m(x x_1)$

Using Scatter Plots (Lesson 2-5)

• A prediction equation can be used to predict the value of one of the variables given the value of the other variable.

Graphing Inequalities (Lesson 2-7)

- You can graph an inequality by following these steps.
 - **Step 1** Determine whether the boundary is solid or dashed. Graph the boundary.
 - **Step 2** Choose a point not on the boundary and test it in the inequality.
 - **Step 3** If a true inequality results, shade the region containing your test point. If a false inequality results, shade the other region.

Key Vocabulary

absolute value function (p. 96)boundary (p. 102) constant function (p. 96) continuous function (p. 65) coordinate plane (p. 58) dependent variable (p. 61) discrete function (p. 65) domain (p. 58) family of graphs (p. 73) function (p. 58) function notation (p. 61) greatest integer function (p. 95)

identity function (p. 96) independent variable (p. 61)

linear equation (p. 66) linear function (p. 66) line of fit (p. 86) mapping (p. 58)

negative correlation (p. 86) no correlation (p. 86) one-to-one function (p. 58) ordered pair (p. 58) parent graph (p. 73) piecewise function (p. 97) point-slope form (p. 80) positive correlation (p. 86) prediction equation (p. 86) quadrant (p. 58) range (p. 58) rate of change (p. 71) relation (p. 58) scatter plot (p. 86) slope (p. 71) slope-intercept form (p. 79) standard form (p. 67) step function (p. 95) vertical line test (p. 59)

x-intercept (p. 68)

Vocabulary Check

Choose the correct term to complete each

- 1. The (constant, identity) function is a linear function described by f(x) = x.
- **2.** The graph of the (absolute value, greatest integer) function forms a V-shape.
- **3.** The (slope-intercept, standard) form of the equation of a line is y = mx + b.
- **4.** Two lines in the same plane having the same slope are (parallel, perpendicular).
- **5.** The (line of fit, vertical line test) can be used to determine if a relation is a function.
- **6.** The (domain, range) of a relation is the set of all first coordinates from the ordered pairs which determine the relation.



Lesson-by-Lesson Review

2-1 Relations and Functions (pp. 58-64)

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function. Is the relation discrete or continuous?

8.
$$\{(-5, 2), (2, 4), (1, 1), (-5, -2)\}$$

9.
$$y = 0.5x$$

10.
$$y = 2x + 1$$

Find each value if f(x) = 5x - 9.

12.
$$f(-2)$$

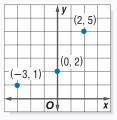
14.
$$f(-2v)$$

15. TAXI RIDE A taxi company charges \$2.80 for the first mile and \$1.60 for each additional mile. The amount a passenger will be charged can be expressed as f(x) = 1.20 + 1.60x, when $x \ge 1$. Graph this equation and find the domain and range. Then determine whether the equation is a function. Is the equation discrete or continuous?

Example 1 Graph the relation $\{(-3, 1),$ (0, 2), (2, 5)} and find the domain and range. Then determine whether the relation is a function. Is the relation discrete or continuous?

The domain is $\{-3, 0, 2\}$, and the range is {1, 2, 5}.

Since each *x*-value is paired with exactly one y-value, the relation is a function. The relation is discrete because the points are not connected.



Linear Equations (pp. 66-70)

State whether each equation or function is linear. Write yes or no. If no, explain your reasoning.

16.
$$2x + y = 11$$

17.
$$h(x) = \sqrt{2x+1}$$

Write each equation in standard form. Identify A, B, and C.

18.
$$\frac{2}{3}x - \frac{3}{4}y = 6$$

19.
$$0.5x = -0.2y - 0.4$$

Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation.

20.
$$-\frac{1}{5}y = x + 4$$

21.
$$6x = -12y + 48$$

22. CUBES Julián thinks that the equation for the volume of a cube,
$$V = s^3$$
, is a linear equation. Is he correct? Explain.

Example 2 Write 2x - 6 = y + 8 in standard form. Identify A, B, and C.

$$2x - 6 = y + 8$$
 Original equation

$$2x - y - 6 = 8$$

Subtract y from each side.

$$2x - y = 14$$

Add 6 to each side.

The standard form is
$$2x - y = 14$$
. So, $A = 2$, $B = -1$, and $C = 14$.

Study Guide and Review

Slope (pp. 71-77)

Find the slope of the line that passes through each pair of points.

Graph the line passing through the given point with the given slope.

25.
$$(0, 1), m = 2$$

26.
$$(-5, 2), m = -\frac{1}{4}$$

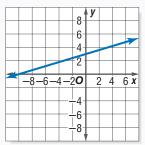
Graph the line that satisfies each set of conditions.

- **27.** passes through (-1, -2), perpendicular to a line whose slope is $\frac{1}{2}$
- **28.** passes through (-1, 2), parallel to the graph of x - 3y = 14
- **29. RAMPS** Jack measures his bicycle ramp and finds that it is 5 feet long and 3 feet high. What is the slope of his ramp?

Example 3 Graph the line passing through (3, 4) with slope $m = \frac{1}{2}$.

Graph the ordered pair (3, 4). Then, according to the slope, go up 1 unit and right 3 units. Plot the new point at (6, 5). You can also go right 3 units and then up 1 unit to plot the new point.

Draw the line containing the points.



Writing Linear Equations (pp. 79-84)

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

- **30.** slope $\frac{3}{4}$, passes through (-6, 9)
- **31.** passes through (-1, 2), parallel to the graph of x - 3y = 14
- **32.** passes through (3, -8) and (-3, 2)
- **33.** passes through (3, 2), perpendicular to the graph of 4x - 3y = 12
- **34. LANDSCAPING** Mr. Ryan is planning to plant rows of roses in a garden he is designing for a client. Before planting, he sketches out his plans on a coordinate grid. A row of white roses will be planted along the line with equation y = 2x + 1. A row of red roses will be parallel to the white roses and pass through the point (3, 5). What equation would represent the line for the row of red roses?

Example 4 Write an equation in slopeintercept form for the line through (4, 5) that is parallel to the line through (-1, -3)and (2, -1).

First, find the slope of the given line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1 - (-3)}{2 - (-1)}$$
$$= \frac{2}{-1}$$

The parallel line will also have a slope of $\frac{2}{3}$.

$$y - y_1 = m(x - x_1)$$

$$= \frac{-1 - (-3)}{2 - (-1)}$$

$$= \frac{2}{3}$$

$$y - 5 = \frac{2}{3}(x - 4)$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

Statistics: Using Scatter Plots (pp. 86-91)

HEALTH INSURANCE For Exercises 35 and 36 use the table that shows the number of people covered by private or government health insurance in the United States.

Year	People (millions)
1988	211
1992	218
1996	225
2000	240
2004	245

Source: U.S. Census

- **35.** Draw a scatter plot and describe the correlation.
- **36.** Use two ordered pairs to write a prediction equation. Then use your prediction equation to predict the number of people with health insurance in 2010.

GOLD PRODUCTION For Exercises 37 and 38, use the table that shows the number of ounces of gold produced in the United States for several years.

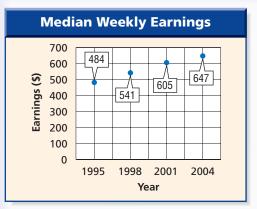
Year	Troy ounces (millions)
1998	11.8
1999	11.0
2000	11.3
2001	10.8
2002	9.6
2003	8.9

Source: World Almanac

- **37.** Draw a scatter plot and describe the correlation.
- **38.** Use two ordered pairs to write a prediction equation. Then use your prediction equation to predict the number of ounces of gold that will be produced in 2010.

Example 5 WEEKLY PAY The table below shows the median weekly earnings for American workers for the period 1985–1999. Predict the median weekly earnings for 2010.

Year	Year 1995		2001	2004	2010	
Earnings (\$)	484	541	605	647	?	



Source: U.S. Bureau of Labor Statistics

Use (1995, 484) and (2004, 647) to find a prediction equation.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 Slope formula

$$= \frac{647 - 484}{2004 - 1995} \quad (x_1, y_1) = (1995, 484),$$

$$= \frac{163}{9} \text{ or about } 18.1 \text{ Simplify.}$$

$$y - y_1 = m(x - x_1)$$
 Point-slope form
$$y - 484 = 18.1(x - 1995)$$
 Substitute.
$$y - 484 = 18.1x - 36,109.5$$
 Multiply.

To predict earnings for 2010, substitute 2010 for *x*.

y = 18.1x - 35,625.5 Add 484 to each

side.

$$y = 18.1$$
(2010) $-35,625.5$ $x = 2010$
= 755.5 Simplify.

The model predicts median weekly earnings of \$755.50 in 2010.

Study Guide and Review

Special Functions (pp. 95–101)

Graph each function. Identify the domain and range.

39.
$$f(x) = [x] - 2$$

40.
$$h(x) = [2x - 1]$$

41.
$$g(x) = |x| + 4$$

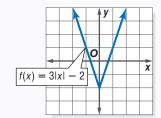
41.
$$g(x) = |x| + 4$$
 42. $h(x) = |x - 1| - 7$

43.
$$f(x) = \begin{cases} 2 \text{ if } x < -1 \\ -x - 1 \text{ if } x \ge -1 \end{cases}$$

44.
$$g(x) = \begin{cases} -2x - 3 & \text{if } x < 1 \\ x - 4 & \text{if } x > 1 \end{cases}$$

45. WIRELESS INTERNET A wireless Internet provider charges \$40 a month plus an additional 30 cents a minute or any fraction thereof. Draw a graph that represents this situation.

Example 6 Graph the function f(x) = 3|x| - 2. Identify the domain and range.



The domain is all real numbers. The range is all real numbers greater than or equal to -2.

Graphing Inequalities (pp. 102–105)

Graph each inequality.

46.
$$y \le 3x - 5$$

47.
$$x > y - 1$$

48.
$$y + 0.5x < 4$$
 49. $2x + y \ge 3$

49.
$$2x + y \ge 3$$

50.
$$y \ge |x| + 2$$

51.
$$y > |x - 3|$$

52. BASEBALL The Cincinnati Reds must score more runs than their opponent to win a game. Write an inequality to represent this situation. Graph the inequality.

Example 7 Graph $x + 4y \le 4$.

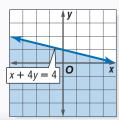
Since the inequality symbol is \leq , the graph of the boundary should be solid. Graph the equation.

Test (0, 0).

$$x + 4y \le 4$$
 Original inequality

$$0 + 4(0) \le 4 \quad (x, y) = (0, 0)$$

 $0 \le 4$ Shade the region that contains (0, 0).



Graph each relation and find the domain and range. Then determine whether the relation is a function.

1.
$$\{(-4, -8), (-2, 2), (0, 5), (2, 3), (4, -9)\}$$

2.
$$y = 3x - 3$$

Find each value.

3.
$$f(3)$$
 if $f(x) = 7 - x^2$

4.
$$f(0)$$
 if $f(x) = x - 3x^2$

Graph each equation or inequality.

5.
$$y = \frac{3}{5}x - 4$$

7. $x = -4$

6.
$$4x - y = 2$$

7.
$$x = -4$$

8.
$$y = 2x - 5$$

9.
$$f(x) = 3x - 1$$

10.
$$f(x) = [3x] + 3$$

11.
$$g(x) = |x + 2|$$

12.
$$y \le 10$$

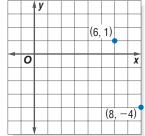
13.
$$-2x + 5 \le 3y$$

14.
$$y < 4|x - 1|$$

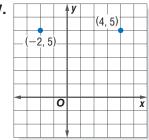
15.
$$h(x) = \begin{cases} x + 2 & \text{if } x < -2 \\ 2x - 1 & \text{if } x \ge -2 \end{cases}$$

Find the slope of the line that passes through each pair of points.

16.



17.



Graph the line passing through the given point with the given slope.

20.
$$(1, -3), 2$$

21.
$$(-2, 2), -\frac{1}{3}$$
 22. $(3, -2)$, undefined

22.
$$(3, -2)$$
, undefined

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

24.
$$x$$
-intercept 9, y -intercept -4

25. passes through
$$(-6, 15)$$
, parallel to the graph of $2x + 3y = 1$

26. passes through (5, 2), perpendicular to the graph of
$$x + 3y = 7$$

RECREATION For Exercises 27–29, use the table that shows the amount Americans spent on admission to spectator amusements in some recent years.

Year	Amount (billion \$)
2000	30.4
2001	32.2
2002	34.6
2003	35.6

Source: Bureau of Economic Analysis, U.S. Dept. of Commerce

- **27.** Draw a scatter plot. Let x represents the number of years since 2000.
- **28.** Write a prediction equation.
- **29.** Predict the amount that will be spent on recreation in 2015.
- **30. MULTIPLE CHOICE** What is the slope of a line parallel to y - 2 = 4(x + 1)?

$$\mathbf{A}$$
 -4

B
$$-\frac{1}{4}$$
 C $\frac{1}{4}$

$$\mathbf{C} \frac{1}{4}$$

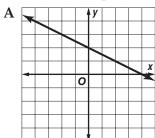
$$D_4$$

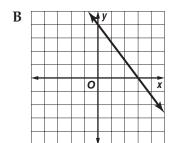
Standardized Test Practice

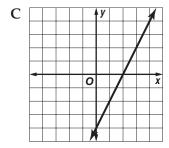
Cumulative, Chapters 1-2

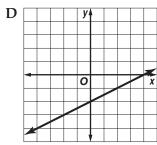
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which graph best represents a line parallel to the line with equation $y = -\frac{4}{3}x + 1$?









2. GRIDDABLE Miranda traveled half of her trip by train. She then traveled one fourth of the rest of the distance by bus. She rented a car and drove the remaining 120 miles. How many miles away was her destination?

3. Rich's Pet Store sells cat food. The cost of two 5-pound bags is \$7.99. The total cost *c* of purchasing *n* bags can be found by—

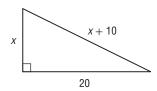
F multiplying n by c.

G multiplying *n* by 5.

H multiplying n by the cost of 1 bag.

J dividing n by c.

4. GRIDDABLE What is the value of *x* in the drawing below?



5. Peyton works as a nanny. She charges at least \$10 to drive to a home and \$10.50 an hour. Which best represents the relationship between the number of hours working *n* and the total charge *c*?

A
$$c \ge 10 + 10.50n$$

B
$$c \ge 10.50 + 10n$$

C
$$c \le 10.50 + 10$$

D
$$c \le 10n + 10.50n$$

TEST-TAKING TIP

Question 5 Watch for the phrases "at least" or "at most." Think logically about the conditions that make a value less than or greater than another variable. Notice what types of numbers are used—positive, even, prime, or integers.

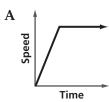
- **6.** Given the function y = 2.24x + 16.45, which statement best describes the effect of decreasing the *y*-intercept by 20.25?
 - **F** The *x*-intercept increases.
 - **G** The *y*-intercept increases.
 - H The new line has a greater rate of change.
 - J The new line is perpendicular to the original.

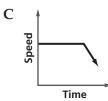


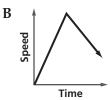
Preparing for Standardized Tests

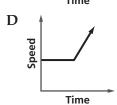
For test-taking strategies and more practice, see pages 941-955.

7. Stephen walks at a steady pace from his house. He then walks up a hill at a slower pace. Which graph best represents this situation?









8. Use the table to determine the expression that best represents the sum of the degree measures of the interior angles of a polygon with n sides.

Polygon	Number of Sides	Sum of Measures		
Triangle	3	180		
Quadrilateral	4	360		
Pentagon	5	540		
Hexagon	6	720		
Heptagon	7	900		
Octagon	8	1080		

F
$$180 + n$$

H
$$180(n-2)$$

9. What are the coordinates of the *x*-intercept of the equation 2y = 4x + 3?

$$\mathbf{A}\left(-\frac{1}{4},0\right)$$

C
$$(0, \frac{3}{2})$$

B
$$\left(-\frac{3}{4}, 0\right)$$

$$\mathbf{D}\left(0,\frac{7}{2}\right)$$

- **10.** Which two 3-dimensional figures have the same number of vertices?
 - F pentagonal prism and a rectangular pyramid.
 - **G** triangular prism and a pentagonal pyramid
 - H rectangular prism and a square pyramid
 - triangular prism and a rectangular prism

Pre-AP

Record your answers on a sheet of paper. Show your work.

11. The amount that a certain online retailer charges for shipping an electronics purchase is determined by the weight of the package. The charges for several different weights

Electronics Shipping Charges								
Weight (lb)	Shipping (\$)							
1	5.58							
3	6.76							
4	7.35							
7	9.12							
10	10.89							
13	12.66							
15	13.84							

- are given in the table.
- **a.** Write a relation to represent the data. Use weight as the independent variable and the shipping charges as the dependent variable.
- **b.** Graph the relation on a coordinate plane.
- **c.** Find the rate of change of the shipping charge per pound.
- **d.** Write an equation that could be used to find the shipping charge y for a package that weights *x* pounds.
- **e.** Find the shipping charge for a package that weighs 19 pounds.

NEED EXTRA HELP?											
If You Missed Question	1	3	4	5	6	7	8	9	2	10	11
Go to Lesson or Page	2-4	2-4	1-4	2-4	2-2	2-1	2-4	2-4	2-4	879	2-4