



CHAPTER 3

Systems of Equations and Inequalities

BIG Ideas

- Solve systems of linear equations in two or three variables.
- Solve systems of inequalities.
- Use linear programming to find minimum and maximum values of functions.

Key Vocabulary

- elimination method (p. 125)
- linear programming (p. 140)
- ordered triple (p. 146)
- system of equations (p. 116)

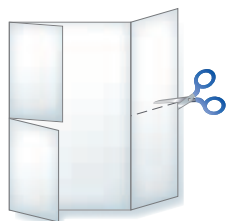
Real-World Link

Attendance Figures Nearly three hundred thousand people attend the annual Missouri State Fair in Sedalia. A system of equations can be used to determine how many children and how many adults attend if the total number of tickets sold and the income from the ticket sales are known.

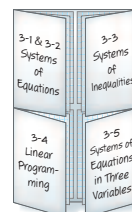
FOLDABLES Study Organizer

Systems of Equations and Inequalities Make this Foldable to record information about systems of linear equations and inequalities. Begin with one sheet of $11'' \times 17''$ paper and four sheets of grid paper.

- 1** Fold the short sides of the $11'' \times 17''$ paper to meet in the middle. Cut each tab in half as shown.



- 2** Cut 4 sheets of grid paper in half and fold the half-sheets in half. Insert two folded half-sheets under each of the four tabs and staple along the fold. Label each tab as shown.



GET READY for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2



Take the Online Readiness Quiz at algebra2.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICK Check

Graph each equation. (Lesson 2-1)

- | | |
|--------------------|-------------------|
| 1. $2y = x$ | 2. $y = x - 4$ |
| 3. $y = 2x - 3$ | 4. $x + 3y = 6$ |
| 5. $2x + 3y = -12$ | 6. $4y - 5x = 10$ |

FUND-RAISING For Exercises 7–10, use the following information.

The Jackson Band Boosters sell beverages for \$1.75 and candy for \$1.50 at home games. Their goal is to have total sales of \$525 for each game. (Lesson 2-3)

- Write an equation that is a model for the different numbers of beverages and candy that can be sold to meet the goal.
- Graph the equation.
- Does this equation represent a function? Explain.
- If they sell 100 beverages and 200 pieces of candy, will the Band Boosters meet their goal?

Graph each inequality. (Lesson 2-7)

- | | |
|---------------------|--------------------|
| 11. $y \geq -2$ | 12. $x + y \leq 0$ |
| 13. $y < 2x - 2$ | 14. $x + 4y < 3$ |
| 15. $2x - y \geq 6$ | 16. $3x - 4y < 10$ |
17. **DRAMA** Tickets for the spring play cost \$4 for adults and \$3 for students. The club must make \$2000 to cover expenses. Write and graph an inequality that describes this situation. (Lesson 2-7)

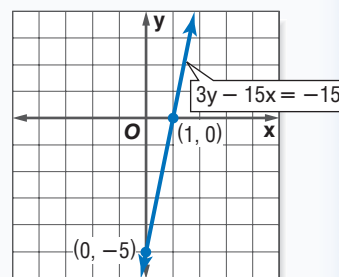
QUICK Review

EXAMPLE 1 Graph $3y - 15x = -15$.

Find the x - and y -intercepts.

$3(0) - 15x = -15$	$3y - 15(0) = -15$
$-15x = -15$	$3y = -15$
$x = 1$	$y = -5$

The graph crosses the x -axis at $(1, 0)$ and the y -axis at $(0, -5)$. Use these ordered pairs to graph the equation.



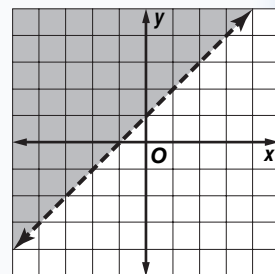
EXAMPLE 2 Graph $y > x + 1$.

The boundary is the graph of $y = x + 1$. Since the inequality symbol is $>$, the boundary will be dashed. Test the point $(0, 0)$.

$$0 > 0 + 1 \quad (x, y) = (0, 0)$$

$$0 > 1 \quad \text{false}$$

Shade the region that does *not* contain $(0, 0)$.



3-1

Solving Systems of Equations by Graphing

Main Ideas

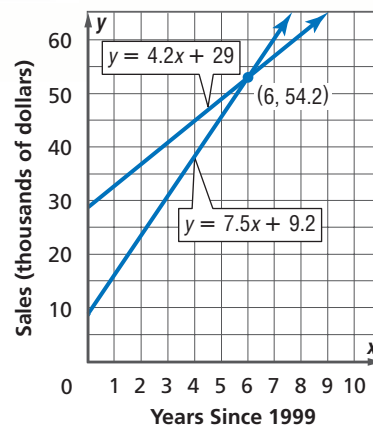
- Solve systems of linear equations by graphing.
- Determine whether a system of linear equations is consistent and independent, consistent and dependent, or inconsistent.

New Vocabulary

system of equations
consistent
inconsistent
independent
dependent

GET READY for the Lesson

Since 1999, the growth of in-store sales for Custom Creations can be modeled by $y = 4.2x + 29$. The growth of their online sales can be modeled by $y = 7.5x + 9.2$. In these equations, x represents the number of years since 1999, and y represents the amount of sales in thousands of dollars.



The equations $y = 4.2x + 29$ and $y = 7.5x + 9.2$ are called a system of equations.

Solve Systems Using Tables and Graphs A **system of equations** is two or more equations with the same variables. To solve a system of equations, find the ordered pair that satisfies all of the equations.

EXAMPLE Solve the System of Equations by Completing a Table

1 Solve the system of equations by completing a table.

$$-2x + 2y = 4$$

$$-4x + y = -1$$

Write each equation in slope-intercept form.

$$-2x + 2y = 4 \rightarrow y = x + 2$$

$$-4x + y = -1 \rightarrow y = 4x - 1$$

Use a table to find the solution that satisfies both equations.

x	$y_1 = x + 2$	y_1	$y_2 = 4x - 1$	y_2	(x, y_1)	(x, y_2)
-1	$y_1 = (-1) + 2$	1	$y_2 = 4(-1) - 1$	-5	$(-1, 1)$	$(-1, -5)$
0	$y_1 = 0 + 2$	2	$y_2 = 4(0) - 1$	-1	$(0, 2)$	$(0, -1)$
1	$y_1 = (1) + 2$	3	$y_2 = 4(1) - 1$	3	$(1, 3)$	$(1, 3)$

The solution of the system is $(1, 3)$.

The solution of the system of equations is the ordered pair that satisfies both equations.

CHECK Your Progress

1A. $-3x + y = 4$
 $2x + y = -6$

1B. $2x + 3y = 4$
 $5x + 6y = 5$



Another way to solve a system of equations is to graph the equations on the same coordinate plane. The point of intersection represents the solution.

EXAMPLE Solve by Graphing

2 Solve the system of equations by graphing.

$$2x + y = 5$$

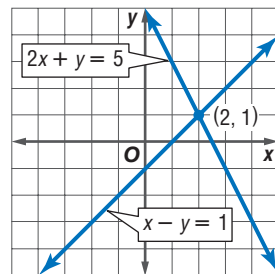
$$x - y = 1$$

Write each equation in slope-intercept form.

$$2x + y = 5 \rightarrow y = -2x + 5$$

$$x - y = 1 \rightarrow y = x - 1$$

The graphs appear to intersect at (2, 1).



CHECK Substitute the coordinates into each equation.

$$2x + y = 5 \quad x - y = 1 \quad \text{Original equations}$$

$$2(2) + 1 \stackrel{?}{=} 5 \quad 2 - 1 \stackrel{?}{=} 1 \quad \text{Replace } x \text{ with 2 and } y \text{ with 1.}$$

$$5 = 5 \checkmark \quad 1 = 1 \checkmark \quad \text{Simplify.}$$

The solution of the system is (2, 1).

Study Tip

Checking Solutions

When using a graph to find a solution, always check the ordered pair in *both* original equations.

CHECK Your Progress

2A. $4x + \frac{1}{3}y = 8$
 $3x + y = 6$

2B. $5x + 4y = 7$
 $-x - 4y = -3$



Personal Tutor at algebra2.com

Systems of equations are used in businesses to determine the *break-even point*. The break-even point is the point at which the income equals the cost.



Real-World EXAMPLE Break-Even Point Analysis

3 MUSIC The initial cost for Travis and his band to record their first CD was \$1500. Each CD will cost \$4 to produce. If they sell their CDs for \$10 each, how many must they sell before they make a profit?

Let x = the number of CDs and let y = the number of dollars.

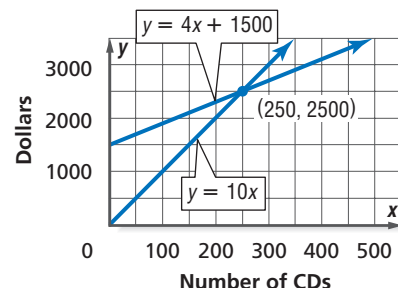
Costs of CDs is cost per CD plus start-up cost.

$$y = 4x + 1500$$

Income for CDs is price per CD times number sold.

$$y = 10 \cdot x$$

The graphs intersect at (250, 2500). This is the break-even point. If the band sells fewer than 250 CDs, they will lose money. If the band sells more than 250 CDs, they will make a profit.



Real-World Link

Compact discs (CDs) store music digitally. The recorded sound is converted to a series of 1s and 0s. This coded pattern can then be read by an infrared laser in a CD player.



Study Tip

Graphs of Linear Systems

Graphs of systems of linear equations may be intersecting lines, parallel lines, or the same line.

CHECK Your Progress

- 3A. RUNNING** Curtis will run 4 miles the first week of training and increase the mileage by one mile each week. With another schedule, Curtis will run 1 mile the first week and increase his total mileage by 2 miles each week. During what week do the two schedules break even? How many miles will Curtis run during this week?

Classify Systems of Equations A system of equations is **consistent** if it has at least one solution and **inconsistent** if it has no solutions. A consistent system is **independent** if it has exactly one solution or **dependent** if it has an infinite number of solutions.

EXAMPLE Intersecting Lines

- 4** Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$$x + \frac{1}{2}y = 5$$

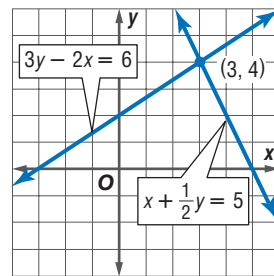
$$3y - 2x = 6$$

Write each equation in slope-intercept form.

$$x + \frac{1}{2}y = 5 \rightarrow y = -2x + 10$$

$$3y - 2x = 6 \rightarrow y = \frac{2}{3}x + 2$$

The graphs intersect at (3, 4). Since there is one solution, this system is *consistent and independent*.



CHECK Your Progress

4A. $2x - y = 5$

$$x + 3y = 6$$

4B. $2x - y = 5$

$$y + \frac{1}{2}x = 5$$

The graph of a system of linear equations that is consistent and dependent is one line.

EXAMPLE Same Line

- 5** Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$$9x - 6y = 24$$

$$6x - 4y = 16$$

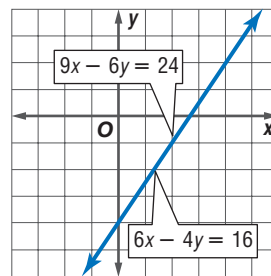
Write each equation in slope-intercept form.

$$9x - 6y = 24 \rightarrow y = \frac{3}{2}x - 4$$

$$6x - 4y = 16 \rightarrow y = \frac{3}{2}x - 4$$

Since the equations are equivalent, their graphs are the same line. Any ordered pair representing a point on that line will satisfy both equations.

So, there are infinitely many solutions to this system. It is *consistent and dependent*.



CHECK Your Progress

5A. $5x - 3y = -2$
 $4x + 2y = 5$

5B. $4x + 2y = 5$
 $2x + y = \frac{5}{2}$

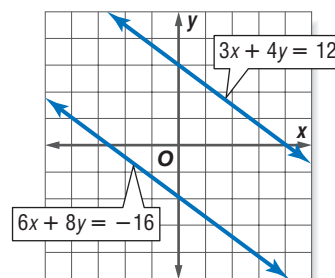
EXAMPLE Parallel Lines

6 Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$3x + 4y = 12$
 $6x + 8y = -16$

$3x + 4y = 12 \rightarrow y = -\frac{3}{4}x + 3$

$6x + 8y = -16 \rightarrow y = -\frac{3}{4}x - 2$



The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system is *inconsistent*.

CHECK Your Progress

6A. $y - \frac{4}{3}x = -2$
 $y + \frac{3}{4}x = -2$

6B. $y - \frac{4}{3}x = -2$
 $y - \frac{4}{3}x = 3$

Study Tip

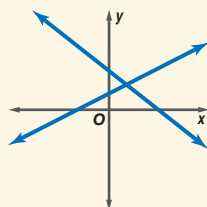
Parallel Lines

Notice from their equations that the lines have the same slope and different y-intercepts.

The relationship between the graph of a system of equations and the number of its solutions is summarized below.

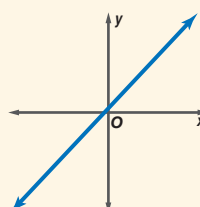
CONCEPT SUMMARY

consistent and independent



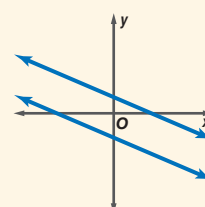
intersecting lines;
one solution

consistent and dependent



same line; infinitely
many solutions

inconsistent



parallel lines;
no solution

Example 1
(p. 116)

Solve each system of equations by completing a table.

$$\begin{aligned} 1. \quad y &= 2x + 9 \\ y &= -x + 3 \end{aligned}$$

$$\begin{aligned} 2. \quad 3x + 2y &= 10 \\ 2x + 3y &= 10 \end{aligned}$$

Example 2
(p. 117)

Solve each system of equations by graphing.

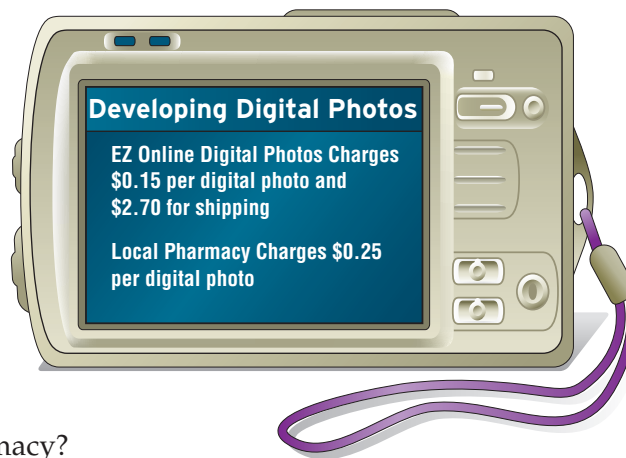
$$\begin{aligned} 3. \quad 4x - 2y &= 22 \\ 6x + 9y &= -3 \end{aligned}$$

$$\begin{aligned} 4. \quad y &= 2x - 4 \\ y &= -3x + 1 \end{aligned}$$

Example 3
(pp. 117–118)

DIGITAL PHOTOS For Exercises 5–7, use the information in the graphic.

5. Write equations that represent the cost of printing digital photos at each lab.
6. Under what conditions is the cost to print digital photos the same for either store?
7. When is it best to use EZ Online Digital Photos and when is it best to use the local pharmacy?



Examples 4–6
(pp. 118–119)

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$$\begin{aligned} 8. \quad y &= 6 - x \\ y &= x + 4 \end{aligned}$$

$$\begin{aligned} 9. \quad x + 2y &= 2 \\ 2x + 4y &= 8 \end{aligned}$$

$$\begin{aligned} 10. \quad x - 2y &= 8 \\ \frac{1}{2}x - y &= 4 \end{aligned}$$

Exercises

HOMEWORK	HELP
For Exercises	See Examples
11, 12	1
13–18	2
19–26	4–6
27–32	3

Solve each system of linear equations by completing a table.

$$\begin{aligned} 11. \quad y &= 3x - 8 \\ y &= x - 8 \end{aligned}$$

$$\begin{aligned} 12. \quad x + 2y &= 6 \\ 2x + y &= 9 \end{aligned}$$

Solve each system of linear equations by graphing.

$$\begin{aligned} 13. \quad 2x + 3y &= 12 \\ 2x - y &= 4 \end{aligned}$$

$$\begin{aligned} 14. \quad 3x - 7y &= -6 \\ x + 2y &= 11 \end{aligned}$$

$$\begin{aligned} 15. \quad 5x - 11 &= 4y \\ 7x - 1 &= 8y \end{aligned}$$

$$\begin{aligned} 16. \quad 2x + 3y &= 7 \\ 2x - 3y &= 7 \end{aligned}$$

$$\begin{aligned} 17. \quad 8x - 3y &= -3 \\ 4x - 2y &= -4 \end{aligned}$$

$$\begin{aligned} 18. \quad \frac{1}{4}x + 2y &= 5 \\ 2x - y &= 6 \end{aligned}$$

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$$\begin{aligned} 19. \quad y &= x + 4 \\ y &= x - 4 \end{aligned}$$

$$\begin{aligned} 20. \quad y &= x + 3 \\ y &= 2x + 6 \end{aligned}$$

$$\begin{aligned} 21. \quad x + y &= 4 \\ -4x + y &= 9 \end{aligned}$$

$$\begin{aligned} 22. \quad 3x + y &= 3 \\ 6x + 2y &= 6 \end{aligned}$$

$$\begin{aligned} 23. \quad y - x &= 5 \\ 2y - 2x &= 8 \end{aligned}$$

$$\begin{aligned} 24. \quad 4x - 2y &= 6 \\ 6x - 3y &= 9 \end{aligned}$$

25. **GEOMETRY** The sides of an angle are parts of two lines whose equations are $2y + 3x = -7$ and $3y - 2x = 9$. The angle's vertex is the point where the two sides meet. Find the coordinates of the vertex of the angle.

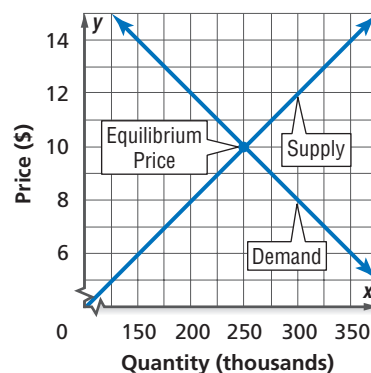


EXTRA PRACTICE
See pages 895, 928.
Math online
Self-Check Quiz at
algebra2.com

- 26. GEOMETRY** The graphs of $y - 2x = 1$, $4x + y = 7$, and $2y - x = -4$ contain the sides of a triangle. Find the coordinates of the vertices of the triangle.

ECONOMICS For Exercises 27–29, use the graph that shows the supply and demand curves for a new multivitamin.

In economics, the point at which the supply equals the demand is the *equilibrium price*. If the supply of a product is greater than the demand, there is a surplus and prices fall. If the supply is less than the demand, there is a shortage and prices rise.



- 27.** If the price for vitamins is \$8.00 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?
- 28.** If the price for vitamins is \$12.00 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?
- 29.** At what quantity will the prices stabilize? What is the equilibrium price for this product?

ANALYZE TABLES For Exercises 30–32, use the table showing state populations.

- 30.** Write equations that represent populations of Florida and New York x years after 2003. Assume that both states continue to gain the same number of residents every year. Let y equal the population.

Rank	State	Population 2003	Average Annual Gain (2000–2003)
1	California	25,484,000	567,000
2	Texas	22,118,000	447,000
3	New York	19,190,000	70,000
4	Florida	17,019,000	304,000
5	Illinois	12,653,000	80,000

Source: U.S. Census Bureau

- 31.** Graph both equations for the years 2003 to 2020. Estimate when the populations of both states will be equal.
- 32.** Do you think New York will overtake Texas as the second most populous state by 2010? by 2020? Explain your reasoning.



Real-World Link

In the United States there is approximately one birth every 8 seconds and one death every 14 seconds.

Source: U.S. Census Bureau

Solve each system of equations by graphing.

$$\begin{aligned} 33. \quad \frac{2}{3}x + y &= -3 \\ y - \frac{1}{3}x &= 6 \end{aligned}$$

$$\begin{aligned} 34. \quad \frac{1}{2}x - y &= 0 \\ \frac{1}{4}x + \frac{1}{2}y &= -2 \end{aligned}$$

$$\begin{aligned} 35. \quad \frac{4}{3}x + \frac{1}{5}y &= 3 \\ \frac{2}{3}x - \frac{3}{5}y &= 5 \end{aligned}$$

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$$\begin{aligned} 36. \quad 1.6y &= 0.4x + 1 \\ 0.4y &= 0.1x + 0.25 \end{aligned}$$

$$\begin{aligned} 37. \quad 3y - x &= -2 \\ y - \frac{1}{3}x &= 2 \end{aligned}$$

$$\begin{aligned} 38. \quad 2y - 4x &= 3 \\ \frac{4}{3}x - y &= -2 \end{aligned}$$

To use a TI-83/84 Plus to solve a system of equations, graph the equations. Then, select INTERSECT, which is option 5 under the CALC menu, to find the coordinates of the point of intersection to the nearest hundredth.

$$\begin{aligned} 39. \quad y &= 0.125x - 3.005 \\ y &= -2.58 \end{aligned}$$

$$\begin{aligned} 40. \quad 3.6x - 2y &= 4 \\ -2.7x + y &= 3 \end{aligned}$$

$$\begin{aligned} 41. \quad y &= 0.18x + 2.7 \\ y &= -0.42x + 5.1 \end{aligned}$$

H.O.T. Problems

- 42. OPEN ENDED** Give an example of a system of equations that is consistent and independent.
- 43. REASONING** Explain why a system of linear equations cannot have exactly two solutions.



Graphing Calculator

44. **CHALLENGE** State the conditions for which the system below is:
 (a) consistent and dependent, (b) consistent and independent, and
 (c) inconsistent if none of the variables are equal to 0.

$$ax + by = c$$

$$dx + ey = f$$

45. **Writing in Math** Use the information about sales on page 116 to explain how a system of equations can be used to predict sales. Include an explanation of the meaning of the solution of the system of equations in the application at the beginning of the lesson. How reasonable would it be to use this system of equations to predict the company's online and in-store profits in 100 years? Explain your reasoning.

STANDARDIZED TEST PRACTICE

46. **ACT/SAT** Which of the following best describes the graph of the equations?

$$4y = 3x + 8$$

$$-6x = -8y + 24$$

- A The lines are parallel.
 B The lines have the same x -intercept.
 C The lines are perpendicular.
 D The lines have the same y -intercept.

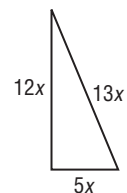
47. **REVIEW** Which set of dimensions corresponds to a triangle similar to the one shown below?

F 7 units, 11 units, 12 units

G 10 units, 23 units, 24 units

H 20 units, 48 units, 52 units

J 1 unit, 2 units, 3 units

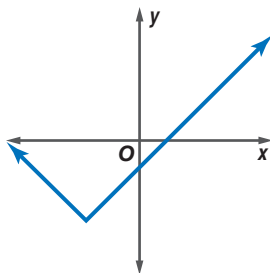


Spiral Review

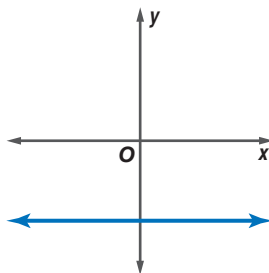
48. **CHORES** Simon is putting up fence around his yard at a rate no faster than 15 feet per hour. Draw a graph that represents the length of fence that Simon has built. (Lesson 2-7)

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. (Lesson 2-6)

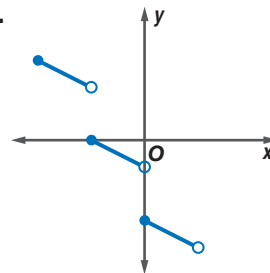
49.



50.



51.



GET READY for the Next Lesson

PREREQUISITE SKILL Simplify each expression. (Lesson 1-2)

52. $(3x + 5) - (2x + 3)$

53. $(3y - 11) + (6y + 12)$

54. $(5x - y) + (-8x + 7y)$

55. $6(2x + 3y - 1)$

56. $5(4x + 2y - x + 2)$

57. $3(x + 4y) - 2(x + 4y)$

3-2

Solving Systems of Equations Algebraically

Main Ideas

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

New Vocabulary

substitution method
elimination method

GET READY for the Lesson

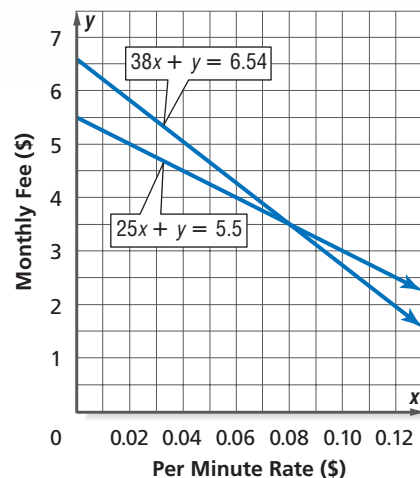
In January, Yolanda's long-distance bill was \$5.50 for 25 minutes of calls. The bill was \$6.54 in February, when Yolanda made 38 minutes of calls. What are the rate per minute and flat fee the company charges?

Let x equal the rate per minute, and let y equal the monthly fee.

January bill: $25x + y = 5.5$

February bill: $38x + y = 6.54$

Sometimes it is difficult to determine the exact coordinates of the point where the lines intersect from the graph. For systems of equations like this one, it may be easier to solve the system by using algebraic methods.



Substitution One algebraic method is the **substitution method**. Using this method, one equation is solved for one variable in terms of the other. Then, this expression is substituted for the variable in the other equation.

EXAMPLE Solve by Using Substitution

1 Use substitution to solve the system of equations.

$$x + 2y = 8$$

$$\frac{1}{2}x - y = 18$$

Solve the first equation for x in terms of y .

$$x + 2y = 8$$

First equation

$$x = 8 - 2y$$

Subtract $2y$ from each side.

Substitute $8 - 2y$ for x in the second equation and solve for y .

$$\frac{1}{2}x - y = 18$$

Second equation

$$\frac{1}{2}(8 - 2y) - y = 18$$

Substitute $8 - 2y$ for x .

$$4 - y - y = 18$$

Distributive Property

$$-2y = 14$$

Subtract 4 from each side.

$$y = -7$$

Divide each side by -2 .

Study Tip

Coefficient of 1

It is easier to solve for the variable that has a coefficient of 1.

(continued on the next page)

Now, substitute the value for y in either original equation and solve for x .

$$x + 2y = 8 \quad \text{First equation}$$

$$x + 2(-7) = 8 \quad \text{Replace } y \text{ with } -7.$$

$$x - 14 = 8 \quad \text{Simplify.}$$

$$x = 22$$

The solution of the system is $(22, -7)$.

CHECK Your Progress

1A. $2x - 3y = 2$

$$x + 2y = 15$$

1B. $7y = 26 + 11x$

$$x - 3y = 0$$

STANDARDIZED TEST EXAMPLE

Solve by Substitution

- 2** Matthew stopped for gasoline twice on a long car trip. The price of gasoline at the first station where he stopped was \$2.56 per gallon. At the second station, the price was \$2.65 per gallon. Matthew bought a total of 36.1 gallons of gasoline and spent \$94.00. How many gallons of gasoline did Matthew buy at the first gas station?

A 17.6

B 18.5

C 19.2

D 20.1

Read the Item

You are asked to find the number of gallons of gasoline that Matthew bought at the first gas station.

Solve the Item

- Step 1** Define variables and write the system of equations. Let x represent the number of gallons bought at the first station and y represent the number of gallons bought at the second station.

$$x + y = 36.1 \quad \text{The total number of gallons was 36.1.}$$

$$2.56x + 2.65y = 94 \quad \text{The total price was \$94.}$$

- Step 2** Solve one of the equations for one of the variables in terms of the other. Since the coefficient of y is 1 and you are asked to find the value of x , it makes sense to solve the first equation for y in terms of x .

$$x + y = 36.1 \quad \text{First equation}$$

$$y = 36.1 - x \quad \text{Subtract } x \text{ from each side.}$$

- Step 3** Substitute $36.1 - x$ for y in the second equation.

$$2.56x + 2.65y = 94 \quad \text{Second equation}$$

$$2.56x + 2.65(36.1 - x) = 94 \quad \text{Substitute } 36.1 - x \text{ for } y.$$

$$2.56x + 95.665 - 2.65x = 94 \quad \text{Distributive Property}$$

$$-0.09x = -1.665 \quad \text{Simplify.}$$

$$x = 18.5 \quad \text{Divide each side by } -0.09.$$

Test-Taking Tip

Even if the question does not ask you for both variables, it is still a good idea to find both so that you can check your answer.



Step 4 Matthew bought 18.5 gallons of gasoline at the first gas station. The answer is B.

CHECK Your Progress

2. **COMIC BOOKS** Dante spent \$11.25 on 3 new and 4 old comic books, and Samantha spent \$15.75 on 10 old and 3 new ones. If comics of one type are sold at the same price, what is the price in dollars of a new comic book?

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Elimination Another algebraic method is the **elimination method**. Using this method, you eliminate one of the variables by adding or subtracting the equations. When you add two true equations, the result is a new equation that is also true.

EXAMPLE Solve by Using Elimination

Use the elimination method to solve the system of equations.

$$4a + 2b = 15$$

$$2a + 2b = 7$$

In each equation, the coefficient of b is 2. If one equation is subtracted from the other, the variable b will be eliminated.

$$4a + 2b = 15$$

$$(-) 2a + 2b = 7$$

$$2a = 8 \quad \text{Subtract the equations.}$$

$$a = 4 \quad \text{Divide each side by 2.}$$

Now find b by substituting 4 for a in either original equation.

$$2a + 2b = 7 \quad \text{Second equation}$$

$$2(4) + 2b = 7 \quad \text{Replace } a \text{ with 4.}$$

$$8 + 2b = 7 \quad \text{Multiply.}$$

$$2b = -1 \quad \text{Subtract 8 from each side.}$$

$$b = -\frac{1}{2} \quad \text{Divide each side by 2.}$$

The solution is $\left(4, -\frac{1}{2}\right)$.

CHECK Your Progress

3A. $2x + y = 4$

$$3x + y = 8$$

3B. $5b = 20 + 2a$

$$2a + 4b = 7$$

Study Tip

Alternative Method

You may find it confusing to subtract equations. It may be helpful to multiply the second equation by -1 and then add the equations.

Sometimes, adding or subtracting the two equations will not eliminate either variable. You may use multiplication to write an equivalent equation so that one of the variables has the same or opposite coefficient in both equations. When you multiply an equation by a nonzero number, the new equation is equivalent to the original equation.



EXAMPLE Multiply, Then Use Elimination

- 4 Use the elimination method to solve the system of equations.

$$3x - 7y = -14$$

$$5x + 2y = 45$$

Multiply the first equation by 2 and the second equation by 7. Then add the equations to eliminate the y variable.

$$3x - 7y = -14$$

Multiply by 2.

$$6x - 14y = -28$$

$$5x + 2y = 45$$

Multiply by 7.

$$(+)\ 35x + 14y = 315$$

$$\begin{array}{r} 41x \\ = 287 \end{array}$$

Add the equations.

$$x = 7$$

Divide each side by 41.

Replace x with 7 and solve for y .

$$3x - 7y = -14 \quad \text{First equation}$$

$$3(7) - 7y = -14 \quad \text{Replace } x \text{ with 7.}$$

$$21 - 7y = -14 \quad \text{Multiply.}$$

$$-7y = -35 \quad \text{Subtract 21 from each side.}$$

$$y = 5 \quad \text{Divide each side by } -7.$$

The solution is $(7, 5)$.

CHECK Your Progress

4A. $3x + 4y = 14$

$$4x + 5y = 17$$

4B. $2x - 4y = 28$

$$4x = 17 - 5y$$

If you add or subtract two equations in a system and the result is an equation that is never true, then the system is inconsistent. If the result when you add or subtract two equations in a system is an equation that is always true, then the system is dependent.

EXAMPLE Inconsistent System

- 5 Use the elimination method to solve the system of equations.

$$8x + 2y = 17$$

$$-4x - y = 9$$

Use multiplication to eliminate x .

$$8x + 2y = 17$$

$$8x + 2y = 17$$

$$-4x - y = 9$$

Multiply by 2.

$$-8x - 2y = 18$$

$$0 = 35 \quad \text{Add the equations.}$$

Since there are no values of x and y that will make the equation $0 = 35$ true, there are no solutions for this system of equations.

CHECK Your Progress

5A. $8y = 2x + 48$

$$y - \frac{1}{4}x = 6$$

5B. $x - 0.5y = -3$

$$2x - y = 6$$

Study Tip**Alternative Method**

You could also multiply the first equation by 5 and the second equation by 3. Then subtract to eliminate the x variable.

Example 1
(pp. 123–124)

Solve each system of equations by using substitution.

1. $y = 3x - 4$

$y = 4 + x$

3. $a - b = 2$

$-2a + 3b = 3$

2. $4c + 2d = 10$

$c + 3d = 10$

4. $3g - 2h = -1$

$4g + h = 17$

Example 2
(pp. 124–125)

5. STANDARDIZED TEST PRACTICE Campus Rentals rents 2- and 3-bedroom apartments for \$700 and \$900 per month, respectively. Last month they had six vacant apartments and reported \$4600 in lost rent. How many 2-bedroom apartments were vacant?

A 2

B 3

C 4

D 5

Examples 3–5
(pp. 125–126)

Solve each system of equations by using elimination.

6. $2r - 3s = 11$

$2r + 2s = 6$

9. $\frac{1}{4}x + y = \frac{11}{4}$
 $x - \frac{1}{2}y = 2$

7. $5m + n = 10$

$4m + n = 4$

10. $\frac{1}{6}y - 2 = \frac{1}{9}$
 $12 = 18y$

8. $2p + 4q = 18$

$3p - 6q = 3$

11. $1.25x - y = -7$
 $4y = 5x + 28$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–17	1, 2
18–21	3, 4
22, 23	5

Solve each system of equations by using substitution.

12. $2j - 3k = 3$

$j + k = 14$

15. $-w - z = -2$

$4w + 5z = 16$

13. $2r + s = 11$

$6r - 2s = -2$

16. $3s + 2t = -3$

$s + \frac{1}{3}t = -4$

14. $5a - b = 17$

$3a + 2b = 5$

17. $2x + 4y = 6$

$7x = 4 + 3y$

Solve each system of equations by using elimination.

18. $u + v = 7$

$2u + v = 11$

21. $4x - 5y = 17$

$3x + 4y = 5$

19. $m - n = 9$

$7m + n = 7$

22. $2c + 6d = 14$

$-\frac{7}{3} + \frac{1}{3}c = -d$

20. $r + 4s = -8$

$3r + 2s = 6$

23. $6d + 3f = 12$

$2d = 8 - f$

SKIING For Exercises 24 and 25, use the following information.

All 28 members in Crestview High School's Ski Club went on a one-day ski trip. Members can rent skis for \$16 per day or snowboards for \$19 per day. The club paid a total of \$478 for rental equipment.

24. Write a system of equations that represents the number of members who rented the two types of equipment.

25. How many members rented skis and how many rented snowboards?

INVENTORY For Exercises 26 and 27, use the following information.

Beatriz is checking a shipment of technology equipment that contains laser printers that cost \$700 each and color monitors that cost \$200 each. She counts 30 boxes on the loading dock. The invoice states that the order totals \$15,000.

26. Write a system of two equations that represents the number of each item.

27. How many laser printers and how many color monitors were delivered?

Cross-Curricular Project



A system of equations can be used to compare home loan options. Visit algebra2.com to continue work on your project.



Real-World Career

Teacher

Besides the time they spend in a classroom, teachers spend additional time preparing lessons, grading papers, and assessing students' progress.



For more information, go to algebra2.com.

EXTRA PRACTICE

See pages 895, 928.



Self-Check Quiz at algebra2.com

H.O.T. Problems

Solve each system of equations by using either substitution or elimination.

28. $3p - 6q = 6$

$2p - 4q = 4$

31. $6g - 8h = 50$

$6h = 22 - 4g$

34. $3u + 5v = 6$

$2u - 4v = -7$

37. $8 = 0.4m + 1.8n$

$1.2m + 3.4n = 16$

29. $10m - 9n = 15$

$5m - 4n = 10$

32. $2p = 7 + q$

$6p - 3q = 24$

35. $3a = -3 + 2b$

$3a + b = 3$

38. $s + 3t = 27$

$2t = 19 - \frac{1}{2}s$

30. $3c - 7d = -3$

$2c + 6d = -34$

33. $3x = -31 + 2y$

$5x + 6y = 23$

36. $0.25x + 1.75y = 1.25$

$-0.5x + 2 = 2.5y$

39. $2f + 2g = 18$

$\frac{1}{6}f + \frac{1}{3}g = 1$

TEACHING For Exercises 40–42, use the following information.

Mr. Talbot is writing a science test. It will have true/false questions worth 2 points each and multiple-choice questions worth 4 points each for a total of 100 points. He wants to have twice as many multiple-choice questions as true/false.

40. Write a system of equations that represents the number of each type of question.

41. How many of each type of question will be on the test?

42. If most of his students can answer true/false questions within 1 minute and multiple-choice questions within $1\frac{1}{2}$ minutes, will they have enough time to finish the test in 45 minutes?

EXERCISE For Exercises 43 and 44, use the following information.

Megan exercises every morning for 40 minutes. She does a combination of step aerobics, which burns about 11 Calories per minute, and stretching, which burns about 4 Calories per minute. Her goal is to burn 335 Calories during her routine.

43. Write a system of equations that represents Megan's morning workout.

44. How long should she do each activity in order to burn 335 Calories?

45. **OPEN ENDED** Give a system of equations that is more easily solved by substitution and a system of equations that is more easily solved by elimination.

46. **REASONING** Make a conjecture about the solution of a system of equations if the result of subtracting one equation from the other is $0 = 0$.

47. **FIND THE ERROR** Juanita and Jamal are solving the system $2x - y = 6$ and $2x + y = 10$. Who is correct? Explain your reasoning.

Juanita

$$\begin{array}{r} 2x - y = 6 \\ (-)2x + y = 10 \\ \hline 0 = -4 \end{array}$$

The statement $0 = -4$ is never true,
so there is no solution.

Jamal

$$\begin{array}{r} 2x - y = 6 \\ (+)2x + y = 10 \\ \hline 4x = 16 \\ x = 4 \end{array} \quad \begin{array}{r} 2x - y = 6 \\ 2(4) - y = 6 \\ 8 - y = 6 \\ y = 2 \end{array}$$

The solution is $(4, 2)$.

48. **CHALLENGE** Solve the system of equations.

$$\frac{1}{x} + \frac{3}{y} = \frac{3}{4} \quad \left(\text{Hint: Let } m = \frac{1}{x} \text{ and } n = \frac{1}{y}.\right)$$

$$\frac{3}{x} - \frac{2}{y} = \frac{5}{12}$$



49. **Writing in Math** Use the information on page 123 to explain how a system of equations can be used to find a flat fee and a per-unit rate. Include a solution of the system of equations in the application at the beginning of the lesson.

STANDARDIZED TEST PRACTICE

50. **ACT/SAT** In order to practice at home, Tadeo purchased a basketball and a volleyball that cost a total of \$67, not including tax. If the price of the basketball b is \$4 more than twice the cost of the volleyball v which system of linear equations could be used to determine the price of each ball?

A $b + v = 67$ $b = 2v - 4$	C $b + v = 4$ $b = 2v - 67$
B $b + v = 67$ $b = 2v + 4$	D $b + v = 4$ $b = 2v + 67$

51. **REVIEW** The caterer at a brunch bought several pounds of chicken salad and several pounds of tuna salad. The chicken salad cost \$9 per pound, and the tuna salad cost \$6 per pound. He bought a total of 14 pounds of salad and paid a total of \$111. How much chicken salad did he buy?

- F** 6 pounds
G 7 pounds
H 8 pounds
J 9 pounds

Spiral Review

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*. (Lesson 3-1)

52. $y = x + 2$
 $y = x - 1$

53. $4y - 2x = 4$
 $y - \frac{1}{2}x = 1$

54. $3x + y = 1$
 $y = 2x - 4$

Graph each inequality. (Lesson 2-7)

55. $x + y \leq 3$

56. $5y - 4x < -20$

57. $3x + 9y \geq -15$

Write each equation in standard form. Identify A , B , and C . (Lesson 2-2)

58. $y = 7x + 4$

59. $x = y$

60. $3x = 2 - 5y$

61. $6x = 3y - 9$

62. $y = \frac{1}{2}x - 3$

63. $\frac{2}{3}y - 6 = 1 - x$

64. **ELECTRICITY** Find the amount of current I (in amperes) produced if the electromotive force E is 1.5 volts, the circuit resistance R is 2.35 ohms, and the resistance r within a battery is 0.15 ohms, using the formula

$$I = \frac{E}{R + r} \quad (\text{Lesson 1-1})$$

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether the given point satisfies each inequality. (Lesson 2-7)

65. $3x + 2y \leq 10$; $(2, -1)$

66. $4x - 2y > 6$; $(3, 3)$

67. $7x + 4y \geq -15$; $(-4, 2)$

3-3

Solving Systems of Inequalities by Graphing

Main Ideas

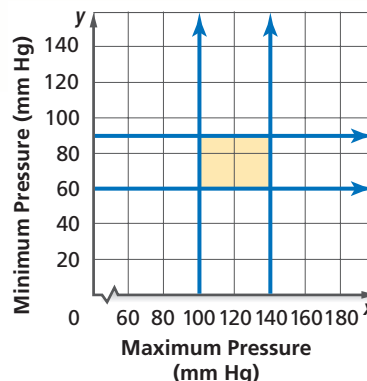
- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

New Vocabulary

system of inequalities

GET READY for the Lesson

During one heartbeat, blood pressure reaches a maximum pressure and a minimum pressure, which are measured in millimeters of mercury (mm-Hg). It is expressed as the maximum over the minimum—for example, 120/80. Normal blood pressure for people under 40 ranges from 100 to 140 mm Hg for the maximum and from 60 to 90 mm Hg for the minimum. This can be represented by a system of inequalities.



Graph Systems of Inequalities To solve a **system of inequalities**, we need to find the ordered pairs that satisfy all of the inequalities in the system. The solution set is represented by the intersection of the graphs of the inequalities.

EXAMPLE Intersecting Regions

1 Solve each system of inequalities.

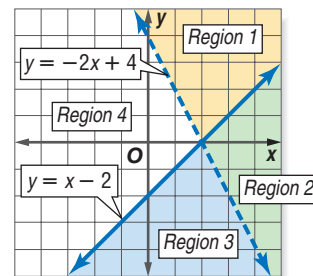
a. $y > -2x + 4$

$y \leq x - 2$

Solution of $y > -2x + 4 \rightarrow$ Regions 1 and 2

Solution of $y \leq x - 2 \rightarrow$ Regions 2 and 3

The region that provides a solution of both inequalities is the solution of the system. Region 2 is the solution of the system.

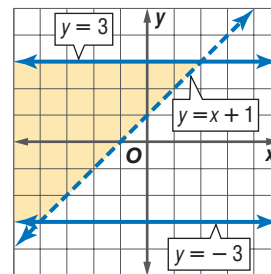


b. $y > x + 1$

$|y| \leq 3$

The inequality $|y| \leq 3$ can be written as $y \leq 3$ and $y \geq -3$.

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.



Study Tip

Look Back

To review **graphing inequalities**, see Lesson 2-7.

Concepts in Motion

Animation algebra2.com

CHECK Your Progress

1A. $y \leq 3x - 4$

$y > -2x + 3$

1B. $|y| < 3$

$y \geq x - 1$



Reading Math

Empty Set The empty set is also called the *null set*. It can be represented as \emptyset or $\{\}$.

It is possible that two regions do *not* intersect. In such cases, we say the solution set is the empty set (\emptyset) and no solution exists.

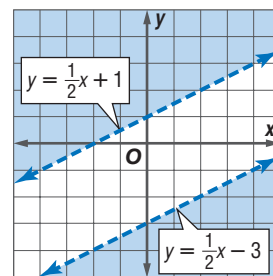
EXAMPLE Separate Regions

- 2** Solve the system of inequalities by graphing.

$$y > \frac{1}{2}x + 1$$

$$y < \frac{1}{2}x - 3$$

Graph both inequalities. The graphs do not overlap, so the solution sets have no points in common. The solution set of the system is \emptyset .



CHECK Your Progress

2. $y > \frac{1}{4}x + 4$
 $y < \frac{1}{4}x - 2$

Real-World EXAMPLE Write and Use a System of Inequalities

- BASKETBALL** The 2005–06 Denver Nuggets roster included players of varying weights and heights. Francisco Elson was the largest at 7'0" and 235 pounds. The smallest player on the team was Earl Boykins at 5'5" and 133 pounds. Write and graph a system of inequalities that represents the range of heights and weights for the members of the team.

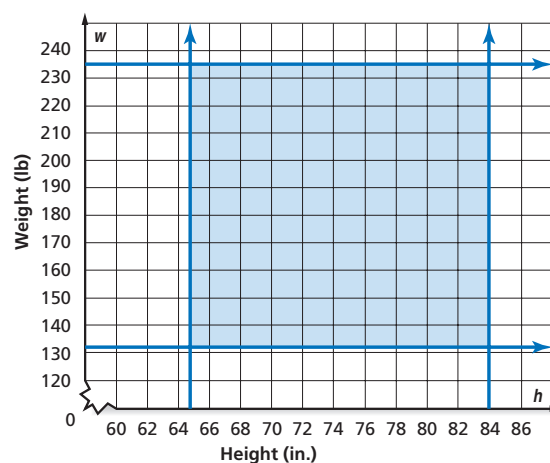
Let h represent the height of a member of the Denver Nuggets. The possible heights for a member of the team are at least 65 inches, but no more than 84 inches. We can write two inequalities.

$$h \geq 65 \text{ and } h \leq 84$$

Let w represent the weights of a player on the Denver Nuggets. The weights can be written as two inequalities.

$$w \geq 133 \text{ and } w \leq 235$$

Graph all of the inequalities. Any ordered pair in the intersection of the graphs is a solution of the system. In this case, a solution of the system of inequalities is a potential height and weight combination for a member of the Denver Nuggets.



CHECK Your Progress

- 3. CATERING** Classy Catering needs at least 15 food servers and 5 bussers to cater a large party. But in order to make a profit, they can have no more than 34 food servers and 7 bussers working at an event. Write and graph a system of inequalities that represents this information.





Find Vertices of a Polygonal Region Sometimes, the graph of a system of inequalities forms a polygonal region. To find the vertices of the region, determine the coordinates of the points at which the boundaries intersect.

EXAMPLE Find Vertices

- 4 GEOMETRY** Find the coordinates of the vertices of the figure formed by $x + y \geq -1$, $x - y \leq 6$, and $12y + x \leq 32$.

Graph each inequality. The intersection of the graphs forms a triangle.

The coordinates $(-4, 3)$ and $(8, 2)$ can be determined from the graph. To find the third vertex, solve the system of equations $x + y = -1$ and $x - y = 6$.

Add the equations to eliminate y .

$$\begin{array}{rcl} x + y & = & -1 \\ (+) x - y & = & 6 \\ \hline 2x & = & 5 \quad \text{Add the equations.} \\ x & = & \frac{5}{2} \quad \text{Divide each side by 2.} \end{array}$$

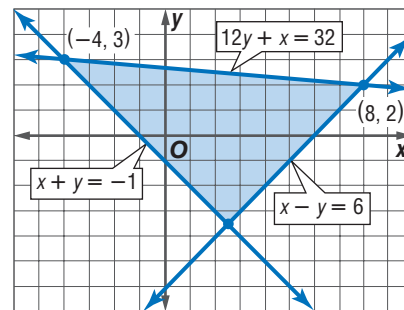
Now find y by substituting $\frac{5}{2}$ for x in the first equation.

$$\begin{array}{rcl} x + y & = & -1 \quad \text{First equation} \\ \frac{5}{2} + y & = & -1 \quad \text{Replace } x \text{ with } \frac{5}{2}. \\ y & = & -\frac{7}{2} \quad \text{Subtract } \frac{5}{2} \text{ from each side.} \end{array}$$

CHECK Compare the coordinates to the coordinates on the graph.

The x -coordinate of the third vertex is between 2 and 3, so $\frac{5}{2}$ is reasonable. The y -coordinate of the third vertex is between -3 and -4 , so $-\frac{7}{2}$ is reasonable.

The vertices of the triangle are at $(-4, 3)$, $(8, 2)$, and $(\frac{5}{2}, -\frac{7}{2})$.



CHECK Your Progress

4. Find the coordinates of the vertices of the figure formed by $x + y \leq 2$, $x - 2y \leq 8$, and $x + (-\frac{1}{3}y) \geq -\frac{2}{3}$.

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CHECK Your Understanding

Examples 1, 2
(pp. 130–131)

Solve each system of inequalities by graphing.

1. $x \leq 4$
 $y > 2$

2. $y \leq -4x - 3$
 $y > -4x + 1$

3. $|x - 1| \leq 2$
 $x + y > 2$

4. $y \geq 3x + 3$
 $y < 3x - 2$



Example 3
(p. 131)

SHOPPING For Exercises 5 and 6, use the following information.

The most Jack can spend on bagels and muffins for the cross country team is \$28. A package of 6 bagels costs \$2.50. A package of muffins costs \$3.50 and contains 8 muffins. He needs to buy at least 12 bagels and 24 muffins.

5. Graph the region that shows how many packages of each item he can purchase.
6. Give an example of three different purchases he can make.

Example 4
(p. 132)

Find the coordinates of the vertices of the figure formed by each system of inequalities.

- | | |
|-------------------|-------------------|
| 7. $y \leq x$ | 8. $y \geq x - 3$ |
| $y \geq -3$ | $y \leq x + 7$ |
| $3y + 5x \leq 16$ | $x + y \leq 11$ |
| | $x + y \geq -1$ |

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–17	1, 2
18, 19	3
20–23	4

Solve each system of inequalities by graphing.

- | | | |
|---------------------------|----------------------|-------------------------|
| 9. $x \geq 2$ | 10. $x \leq -1$ | 11. $y < 2 - x$ |
| $y > 3$ | $y \geq -4$ | $y > x + 4$ |
| 12. $x > 1$ | 13. $3x + 2y \geq 6$ | 14. $4x - 3y < 7$ |
| $x \leq -1$ | $4x - y \geq 2$ | $2y - x < -6$ |
| 15. $3y \leq 2x - 8$ | 16. $y > x - 3$ | 17. $2x + 5y \leq -15$ |
| $y \geq \frac{2}{3}x - 1$ | $ y \leq 2$ | $y > -\frac{2}{5}x + 2$ |

18. **PART-TIME JOBS** Rondell makes \$10 an hour cutting grass and \$12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Rondell can use to determine how many hours he needs to work at each job if he wants to earn at least \$120 per week.
19. **RECORDING** Jane's band wants to spend no more than \$575 recording their first CD. The studio charges at least \$35 an hour to record. Graph a system of inequalities to represent this situation.

Find the coordinates of the vertices of the figure formed by each system of inequalities.

- | | |
|-------------------|--------------------|
| 20. $y \geq 0$ | 21. $y \geq -4$ |
| $x \geq 0$ | $y \leq 2x + 2$ |
| $x + 2y \leq 8$ | $2x + y \leq 6$ |
| 22. $x \leq 3$ | 23. $x + y \leq 9$ |
| $-x + 3y \leq 12$ | $x - 2y \leq 12$ |
| $4x + 3y \geq 12$ | $y \leq 2x + 3$ |

24. **GEOMETRY** Find the area of the region defined by the system of inequalities $y + x \leq 3$, $y - x \leq 3$, and $y \geq -1$.
25. **GEOMETRY** Find the area of the region defined by the system of inequalities $x \geq -3$, $y + x \leq 8$, and $y - x \geq -2$.



Real-World Career

Atmospheric Scientist

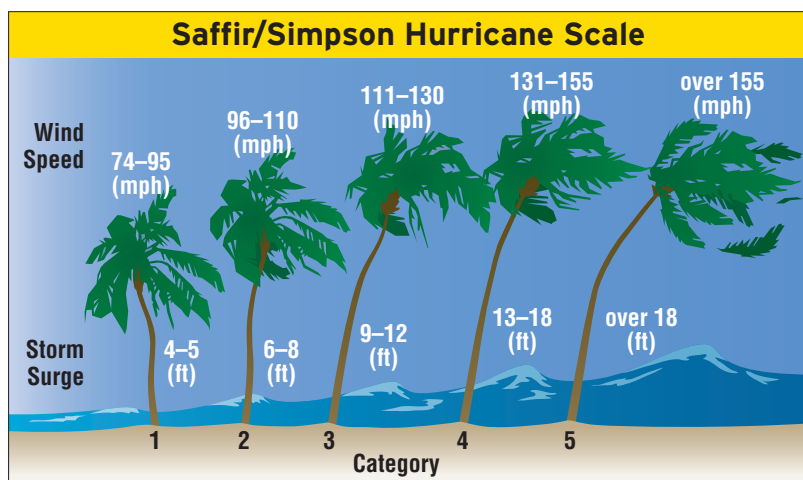
The best known use of atmospheric science is for weather forecasting. However, weather information is also studied for air-pollution control, agriculture, and transportation.



For more information, go to algebra2.com.

HURRICANES For Exercises 26 and 27, use the following information.

Hurricanes are divided into five categories according to their wind speed and storm surge. Category 5 is the most destructive type of hurricane.



Source: National Oceanic and Atmospheric Administration

26. Write and graph the system of inequalities that represents the range of wind speeds s and storm surges h for a Category 3 hurricane.
27. On August 29, 2005, Hurricane Katrina hit the Gulf coasts of Louisiana and Mississippi. At its peak, Katrina had maximum sustained winds of 145 mph. Classify the strength of Hurricane Katrina and state the expected heights of its storm surges.

BAKING For Exercises 28–30, use the recipes at the right.

The Merry Bakers are baking pumpkin bread and Swedish soda bread for this week's specials. They have at most 24 cups of flour and at most 26 teaspoons of baking powder on hand.

28. Graph the inequalities that represent how many loaves of each type of bread the bakers can make.
29. List three different combinations of breads they can make.
30. Which combination uses all of the available flour and baking soda?

Pumpkin Bread

2 c. of flour
1 tsp. baking powder

Swedish Soda Bread

$1\frac{1}{2}$ c. of flour
 $2\frac{1}{2}$ tsp. baking powder

Solve each system of inequalities by graphing.

$$\begin{aligned} 31. \quad & y < 2x - 3 \\ & y \leq \frac{1}{2}x + 1 \end{aligned}$$

$$\begin{aligned} 32. \quad & |x| \leq 3 \\ & |y| > 1 \end{aligned}$$

$$\begin{aligned} 33. \quad & |x + 1| \leq 3 \\ & x + 3y \geq 6 \end{aligned}$$

$$\begin{aligned} 34. \quad & y \geq 2x + 1 \\ & y \leq 2x - 2 \\ & 3x + y \leq 9 \end{aligned}$$

$$\begin{aligned} 35. \quad & x - 3y > 2 \\ & 2x - y < 4 \\ & 2x + 4y \geq -7 \end{aligned}$$

$$\begin{aligned} 36. \quad & x \leq 1 \\ & y < 2x + 1 \\ & x + 2y \geq -3 \end{aligned}$$

EXTRA PRACTICE

See pages 896, 928.

MathOnline

Self-Check Quiz at algebra2.com

H.O.T. Problems

37. **OPEN ENDED** Write a system of inequalities that has no solution.
38. **REASONING** Determine whether the following statement is *true* or *false*. If false, give a counterexample. *A system of two linear inequalities has either no points or infinitely many points in its solution.*

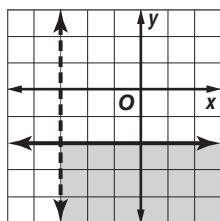


- 39. CHALLENGE** Find the area of the region defined by $|x| + |y| \leq 5$ and $|x| + |y| \geq 2$.

- 40. Writing in Math** Using the information about blood pressure on page 130, explain how you can determine whether your blood pressure is in a normal range utilizing a graph of the system of inequalities.

STANDARDIZED TEST PRACTICE

- 41. ACT/SAT** Choose the system of inequalities whose solution is represented by the graph.



- | | |
|----------------------|----------------------|
| A $y < -2$ | C $x \leq -2$ |
| $x < -3$ | $y > -3$ |
| B $y \leq -2$ | D $x < -3$ |
| $x > -3$ | $y < -3$ |

- 42. REVIEW** To be a member of the marching band, a student must have a GPA of at least 2.0 and must have attended at least five after-school practices. Choose the system of inequalities that best represents this situation.

- | | |
|---------------------|------------------|
| F $x \geq 2$ | H $x < 2$ |
| $y \geq 5$ | $y < 5$ |
| G $x \leq 2$ | J $x > 2$ |
| $y \leq 5$ | $y > 5$ |

Spiral Review

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

43. $4x - y = -20$
 $x + 2y = 13$

44. $3x - 4y = -2$
 $5x + 2y = 40$

45. $4x + 5y = 7$
 $3x - 2y = 34$

Solve each system of equations by graphing. (Lesson 3-1)

46. $y = 2x + 1$
 $y = -\frac{1}{2}x - 4$

47. $2x + y = -3$
 $6x + 3y = -9$

48. $2x - y = 6$
 $-x + 8y = 12$

- 49. RENTALS** To rent an inflatable trampoline for parties, it costs \$75 an hour plus a set-up/tear-down fee of \$200. Write an equation that represents this situation in slope-intercept form. (Lesson 2-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Find each value if $f(x) = 4x + 3$ and $g(x) = 5x - 7$. (Lesson 2-1)

- 50.** $f(-2)$ **51.** $g(-1)$ **52.** $g(3)$ **53.** $g(-0.25)$

Graphing Calculator Lab

Systems of Linear Inequalities

You can graph systems of linear inequalities with a TI-83/84 Plus graphing calculator using the $Y=$ menu. You can choose different graphing styles to shade above or below a line.

EXAMPLE

Graph the system of inequalities in the standard viewing window.

$$y \geq -2x + 3$$

$$y \leq x + 5$$

Step 1

- Enter $-2x + 3$ as $Y1$. Since y is greater than or equal to $-2x + 3$, shade above the line.

KEYSTROKES: -2 $[X,T,\theta,n]$ $+$ 3

- Use the left arrow key to move your cursor as far left as possible. Highlight the graph style icon. Press $[ENTER]$ until the shade above icon, ∇ , appears.

Step 2

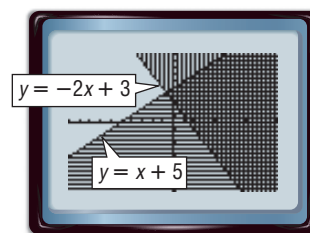
- Enter $x + 5$ as $Y2$. Since y is less than or equal to $x + 5$, shade below the line.

KEYSTROKES: $[X,T,\theta,n]$ $+$ 5

- Use the arrow and $[ENTER]$ keys to choose the shade below icon, ∇ .

Step 3

- Display the graphs by pressing $[GRAPH]$. Notice the shading pattern above the line $y = -2x + 3$ and the shading pattern below the line $y = x + 5$. The intersection of the graphs is the region where the patterns overlap. This region includes all the points that satisfy the system $y \geq -2x + 3$ and $y \leq x + 5$.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

EXERCISES

Solve each system of inequalities. Sketch each graph on a sheet of paper.

1. $y \geq 4$
 $y \leq -x$

3. $y \geq 1 - x$
 $y \leq x + 5$

5. $3y \geq 6x - 15$
 $2y \leq -x + 3$

7. $6y + 4x \geq 12$
 $5y - 3x \leq -10$

2. $y \geq -2x$
 $y \leq -3$

4. $y \geq x + 2$
 $y \leq -2x - 1$

6. $y + 3x \geq 6$
 $y - 2x \leq 9$

8. $\frac{1}{4}y - x \geq -2$
 $\frac{1}{3}y + 2x \leq 4$

Mid-Chapter Quiz

Lessons 3-1 through 3-3

Solve each system of equations by graphing. (Lesson 3-1)

1. $y = 3x + 10$
 $y = -x + 6$
2. $2x + 3y = 12$
 $2x - y = 4$
3. $x = y - 1$
 $\frac{1}{3}y = x - 3$
4. $10 = -2x + y$
 $-3x = -5y + 1$

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

5. $y = x + 5$
 $x + y = 9$
6. $2x + 6y = 2$
 $3x + 2y = 10$
7. $\frac{3}{5}x + \frac{1}{12}y = 24$
 $\frac{1}{9}x - \frac{2}{9}y = 13$
8. $-x = 16.95 - 7y$
 $4x - 18.3 = -2y$

9. **TRAVEL** The busiest airport in the world is Atlanta's Hartsfield International Airport, and the second busiest airport is Chicago's O'Hare International Airport. Together they handled 160 million passengers in 2005. If Hartsfield handled 16 million more passengers than O'Hare, how many were handled by each airport? (Lesson 3-2)

10. **MULTIPLE CHOICE** Shenae spent \$42 on 2 cans of primer and 1 can of paint for her room. If the price of paint p is 150% of the price of primer r , which system of equations can be used to find the price of paint and primer? (Lesson 3-2)

- A $p = r + \frac{1}{2}r$
 $p + 2r = 42$
- C $r = p + \frac{1}{2}p$
 $p + 2p = 42$
- B $p = r + 2r$
 $p + \frac{1}{2}r = 42$
- D $r = p + 2p$
 $p + \frac{1}{2} = 42$

11. **ART** Marta can spend no more than \$225 on the art club's supply of brushes and paint. A box of brushes costs \$7.50 and contains 3 brushes. A box of paint costs \$21.45 and contains 10 tubes of paint. She needs at least 20 brushes and 56 tubes of paint. Graph the region that shows how many packages of each item can be purchased. (Lesson 3-3)

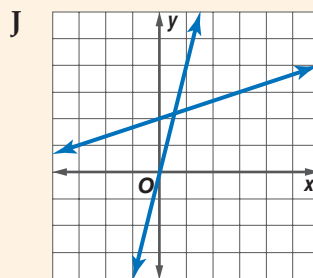
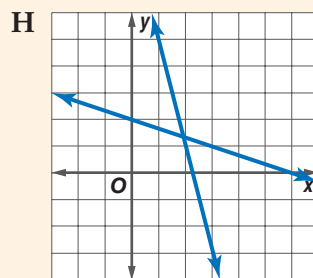
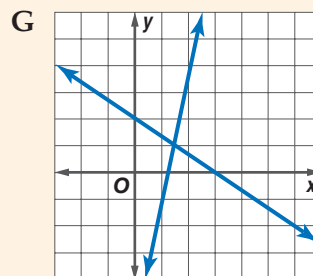
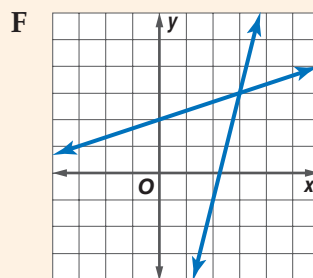
Solve each system of inequalities by graphing. (Lesson 3-3)

12. $y - x > 0$
 $y + x < 4$
13. $y \geq 3x - 4$
 $y \leq x + 3$

14. **MULTIPLE CHOICE** Which graph represents the following system of equations? (Lesson 3-3)

$$\frac{1}{3}x + 2 = y$$

$$4x - 9 = y$$



Main Ideas

- Find the maximum and minimum values of a function over a region.
- Solve real-world problems using linear programming.

New Vocabulary

constraints
feasible region
bounded
vertex
unbounded
linear programming

GET READY for the Lesson

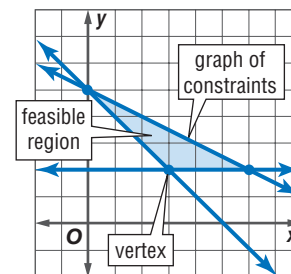
The U.S. Coast Guard maintains the buoys that ships use to navigate. The ships that service buoys are called *buoy tenders*.

Suppose a buoy tender can carry up to 8 replacement buoys. The crew can repair a buoy in 1 hour and replace a buoy in $2\frac{1}{2}$ hours.



Maximum and Minimum Values The buoy tender captain can use a system of inequalities to represent the limits of time and replacements on the ship. If these inequalities are graphed, the points in the intersection are combinations of repairs and replacements that can be scheduled.

The inequalities are called the **constraints**. The intersection of the graphs is called the **feasible region**. When the graph of a system of constraints is a polygonal region like the one graphed at the right, we say that the region is **bounded**.



Since the buoy tender captain wants to service the maximum number of buoys, he will need to find the maximum value of the function for points in the feasible region. The maximum or minimum value of a related function *always* occurs at a **vertex** of the feasible region.

EXAMPLE Bounded Region

- 1** Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

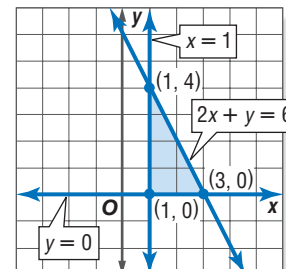
$$x \geq 1$$

$$y \geq 0$$

$$2x + y \leq 6$$

$$f(x, y) = 3x + y$$

Step 1 Graph the inequalities. The polygon formed is a triangle with vertices at $(1, 4)$, $(3, 0)$, and $(1, 0)$.



Reading Math

Function Notation

The notation $f(x, y)$ is used to represent a function with two variables x and y . It is read f of x and y .

Study Tip

Common Misconception

Do not assume that there is no minimum value if the feasible region is unbounded below the line, or that there is no maximum value if the feasible region is unbounded above the line.

Step 2 Use a table to find the maximum and minimum values of $f(x, y)$.

Substitute the coordinates of the vertices into the function.

(x, y)	$3x + y$	$f(x, y)$
(1, 4)	$3(1) + 4$	7
(3, 0)	$3(3) + 0$	9
(1, 0)	$3(1) + 0$	3

← maximum

← minimum

The maximum value is 9 at (3, 0). The minimum value is 3 at (1, 0).

CHECK Your Progress

1. $x \leq 2$

$3x - y \geq -2$

$y \geq x - 2$

$f(x, y) = 2x - 3y$

Sometimes a system of inequalities forms a region that is open. In this case, the region is said to be **unbounded**.

EXAMPLE Unbounded Region

2 Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$2x + y \geq 3$

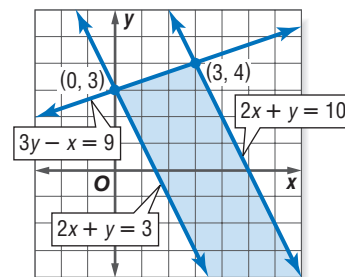
$3y - x \leq 9$

$2x + y \leq 10$

$f(x, y) = 5x + 4y$

Graph the system of inequalities. There are only two points of intersection, (0, 3) and (3, 4).

(x, y)	$5x + 4y$	$f(x, y)$
(0, 3)	$5(0) + 4(3)$	12
(3, 4)	$5(3) + 4(4)$	31



The maximum is 31 at (3, 4).

Although $f(0, 3)$ is 12, it is not the minimum value since there are other points in the solution that produce lesser values. For example, $f(3, -2) = 7$ and $f(20, -35) = -40$. It appears that because the region is unbounded, $f(x, y)$ has no minimum value.

CHECK Your Progress

2. $g \leq -3h + 4$

$g \geq -3h - 6$

$g \geq \frac{1}{3}h - 6$

$f(g, h) = 2g - 3h$

Review Vocabulary

Feasible

Everyday Use

possible or likely

Math Use the area of a graph where it is possible to find a solution to a system of inequalities





Linear Programming The process of finding maximum or minimum values of a function for a region defined by inequalities is called **linear programming**.

KEY CONCEPT

Linear Programming Procedure

- Step 1** Define the variables.
- Step 2** Write a system of inequalities.
- Step 3** Graph the system of inequalities.
- Step 4** Find the coordinates of the vertices of the feasible region.
- Step 5** Write a linear function to be maximized or minimized.
- Step 6** Substitute the coordinates of the vertices into the function.
- Step 7** Select the greatest or least result. Answer the problem.



Real-World Link

Animal surgeries are usually performed in the morning so that the animal can recover throughout the day while there is plenty of staff to monitor its progress.

Source: www.vetmedicine.miningco.com



Real-World EXAMPLE

Linear Programming

3 VETERINARY MEDICINE As a receptionist for a veterinarian, one of Dolores Alvarez's tasks is to schedule appointments. She allots 20 minutes for a routine office visit and 40 minutes for a surgery. The veterinarian cannot do more than 6 surgeries per day. The office has 7 hours available for appointments. If an office visit costs \$55 and most surgeries cost \$125, how can she maximize the income for the day?

Step 1 Define the variables.

v = the number of office visits
 s = the number of surgeries

Step 2 Write a system of inequalities.

Since the number of appointments cannot be negative, v and s must be nonnegative numbers.

$$v \geq 0 \text{ and } s \geq 0$$

An office visit is 20 minutes, and a surgery is 40 minutes. There are 7 hours available for appointments.

$$20v + 40s \leq 420 \quad 7 \text{ hours} = 420 \text{ minutes}$$

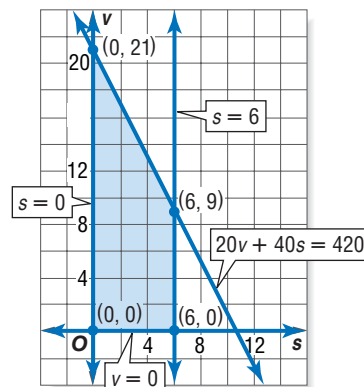
The veterinarian cannot do more than 6 surgeries per day.

$$s \leq 6$$

Step 3 Graph the system of inequalities.

Step 4 Find the coordinates of the vertices of the feasible region.

From the graph, the vertices of the feasible region are at $(0, 0)$, $(6, 0)$, $(6, 9)$, and $(0, 21)$. If the vertices could not be read from the graph easily, we could also solve a system of equations using the boundaries of the inequalities.



**Concepts
in Motion**

Animation
algebra2.com



Study Tip

Reasonableness

Check your solutions for reasonableness by thinking of the situation in context. Surgeries provide more income than office visits. So to maximize income, the veterinarian would do the most possible surgeries in a day.

Step 5 Write a function to be maximized or minimized.

The function that describes the income is $f(s, v) = 125s + 55v$. We wish to find the maximum value for this function.

Step 6 Substitute the coordinates of the vertices into the function.

(s, v)	$125s + 55v$	$f(s, v)$
$(0, 0)$	$125(0) + 55(0)$	0
$(6, 0)$	$125(6) + 55(0)$	750
$(6, 9)$	$125(6) + 55(9)$	1245
$(0, 21)$	$125(0) + 55(21)$	1155

Step 7 Select the greatest or least result. Answer the problem.

The maximum value of the function is 1245 at $(6, 9)$. This means that the maximum income is \$1245 when Dolores schedules 6 surgeries and 9 office visits.



CHECK Your Progress

3. BUSINESS A landscaper balances his daily projects between small landscape jobs and mowing lawns. He allots 30 minutes per lawn and 90 minutes per small landscape job. He works at most ten hours per day. The landscaper earns \$35 per lawn and \$125 per landscape job. He cannot do more than 3 landscape jobs per day and get all of his mowing done. Find a combination of lawns mowed and completed landscape jobs per week that will maximize income.



Personal Tutor at algebra2.com



CHECK Your Understanding

Example 1
(pp. 138–139)

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $y \geq 2$
 $x \geq 1$
 $x + 2y \leq 9$
 $f(x, y) = 2x - 3y$

3. $x \leq 5$
 $y \geq -2$
 $y \leq x - 1$
 $f(x, y) = x - 2y$

5. $y \geq -x + 2$
 $2 \leq x \leq 7$
 $y \leq \frac{1}{2}x + 5$
 $f(x, y) = 8x + 3y$

7. $x \geq -3$
 $y \leq 1$
 $3x + y \leq 6$
 $f(x, y) = 5x - 2y$

2. $y \leq 2x + 1$
 $1 \leq y \leq 3$
 $x + 2y \leq 12$
 $f(x, y) = 3x + y$

4. $y \geq -x + 3$
 $1 \leq x \leq 4$
 $y \leq x + 4$
 $f(x, y) = -x + 4y$

6. $x + 2y \leq 6$
 $2x - y \leq 7$
 $x \geq -2, y \geq -3$
 $f(x, y) = x - y$

8. $y \leq x + 2$
 $y \leq 11 - 2x$
 $2x + y \geq -7$
 $f(x, y) = 4x - 3y$

Example 2
(p. 139)



Example 3
(pp. 139–140)

MANUFACTURING For Exercises 9–14, use the following information.

The Future Homemakers Club is making canvas tote bags and leather tote bags for a fund-raiser. They will line both types of tote bags with canvas and use leather for the handles of both. For the canvas bags, they need 4 yards of canvas and 1 yard of leather. For the leather bags, they need 3 yards of leather and 2 yards of canvas. Their advisor purchased 56 yards of leather and 104 yards of canvas.

9. Let c represent the number of canvas bags and let ℓ represent the number of leather bags. Write a system of inequalities for the number of bags that can be made.
10. Draw the graph showing the feasible region.
11. List the coordinates of the vertices of the feasible region.
12. If the club plans to sell the canvas bags at a profit of \$20 each and the leather bags at a profit of \$35 each, write a function for the total profit on the bags.
13. How can the club make the maximum profit?
14. What is the maximum profit?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
15–20	1
21–27	2
28–33	3

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

- | | | |
|--|---|---|
| <p>15. $y \geq 1$
$x \leq 6$
$y \leq 2x + 1$
$f(x, y) = x + y$</p> | <p>16. $y \geq -4$
$x \leq 3$
$y \leq 3x - 4$
$f(x, y) = x - y$</p> | <p>17. $y \geq 2$
$1 \leq x \leq 5$
$y \leq x + 3$
$f(x, y) = 3x - 2y$</p> |
| <p>18. $y \geq 1$
$2 \leq x \leq 4$
$x - 2y \geq -4$
$f(x, y) = 3y + x$</p> | <p>19. $y \leq x + 6$
$y + 2x \geq 6$
$2 \leq x \leq 6$
$f(x, y) = -x + 3y$</p> | <p>20. $x - 3y \geq -7$
$5x + y \leq 13$
$x + 6y \geq -9$
$3x - 2y \geq -7$
$f(x, y) = x - y$</p> |
| <p>21. $x + y \geq 4$
$3x - 2y \leq 12$
$x - 4y \geq -16$
$f(x, y) = x - 2y$</p> | <p>22. $y \geq x - 3$
$y \leq 6 - 2x$
$2x + y \geq -3$
$f(x, y) = 3x + 4y$</p> | <p>23. $2x + 3y \geq 6$
$3x - 2y \geq -4$
$5x + y \geq 15$
$f(x, y) = x + 3y$</p> |
| <p>24. $2x + 2y \geq 4$
$2y \geq 3x - 6$
$4y \leq x + 8$
$f(x, y) = 3y + x$</p> | <p>25. $x \geq 0$
$y \geq 0$
$x + 2y \leq 6$
$2y - x \leq 2$
$x + y \leq 5$
$f(x, y) = 3x - 5y$</p> | <p>26. $x \geq 2$
$y \geq 1$
$x - 2y \geq -4$
$x + y \leq 8$
$2x - y \leq 7$
$f(x, y) = x - 4y$</p> |

27. **RESEARCH** Use the Internet or other reference to find an industry that uses linear programming. Describe the restrictions or constraints of the problem and explain how linear programming is used to help solve the problem.



PRODUCTION For Exercises 28–33, use the following information.

The total number of workers' hours per day available for production in a skateboard factory is 85 hours. There are 40 workers' hours available for finishing decks and quality control each day. The table shows the number of hours needed in each department for two different types of skateboards.

Skateboard Manufacturing Time		
Board Type	Production Time	Deck Finishing/Quality Control
Pro Boards	$1\frac{1}{2}$ hours	2 hours
Specialty Boards	1 hour	$\frac{1}{2}$ hour

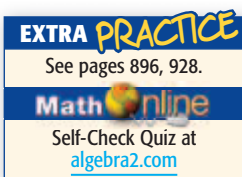
28. Let g represent the number of pro boards and let c represent the number of specialty boards. Write a system of inequalities to represent the situation.
29. Draw the graph showing the feasible region.
30. List the coordinates of the vertices of the feasible region.
31. If the profit on a pro board is \$50 and the profit on a specialty board is \$65, write a function for the total profit on the skateboards.
32. Determine the number of each type of skateboard that needs to be made to have a maximum profit.
33. What is the maximum profit?

FARMING For Exercises 34–37, use the following information.

Dean Stadler has 20 days in which to plant corn and soybeans. The corn can be planted at a rate of 250 acres per day and the soybeans at a rate of 200 acres per day. He has 4500 acres available for planting these two crops.

34. Let c represent the number of acres of corn and let s represent the number of acres of soybeans. Write a system of inequalities to represent the possible ways Mr. Stadler can plant the available acres.
35. Draw the graph showing the feasible region and list the coordinates of the vertices of the feasible region.
36. If the profit is \$26 per acre on corn and \$30 per acre on soybeans, how much of each should Mr. Stadler plant? What is the maximum profit?
37. How much of each should Mr. Stadler plant if the profit on corn is \$29 per acre and the profit on soybeans is \$24 per acre? What is the maximum profit?

38. **MANUFACTURING** The Cookie Factory wants to sell chocolate chip and peanut butter cookies in combination packages of 6–12 cookies. At least three of each type of cookie should be in each package. The cost of making a chocolate chip cookie is 19¢, and the selling price is 44¢ each. The cost of making a peanut butter cookie is 13¢, and the selling price is 39¢. How many of each type of cookie should be in each package to maximize the profit?



H.O.T. Problems

39. **OPEN ENDED** Create a system of inequalities that forms a bounded region.
40. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true.
A function defined by a feasible region has a minimum and a maximum value.



- 41. Which One Doesn't Belong?** Given the following system of inequalities, which ordered pair does not belong? Explain your reasoning.

$$y \leq \frac{1}{2}x + 5 \quad y < -3x + 7 \quad y \geq -\frac{1}{3}x - 2$$

$(0, 0)$

$(-2, 6)$

$(-3, 2)$

$(1, -1)$

- 42. CHALLENGE** The vertices of a feasible region are $A(1, 2)$, $B(5, 2)$, and $C(1, 4)$. Write a function where A is the maximum and B is the minimum.

- 43. Writing in Math** Use the information about buoy tenders on page 138 to explain how linear programming can be used in scheduling work. Include a system of inequalities that represents the constraints that are used to schedule buoy repair and replacement and an explanation of the linear function that the buoy tender captain would wish to maximize.

STANDARDIZED TEST PRACTICE

- 44. ACT/SAT** For a game she's playing, Liz must draw a card from a deck of 26 cards, one with each letter of the alphabet on it, and roll a six-sided die. What is the probability that Liz will roll an odd number and draw a letter in her name?

A $\frac{2}{3}$ B $\frac{1}{13}$ C $\frac{1}{26}$ D $\frac{3}{52}$

- 45. REVIEW** Which of the following best describes the graphs of $y = 3x - 5$ and $4y = 12x + 16$?

F The lines have the same y -intercept.
G The lines have the same x -intercept.
H The lines are perpendicular.
J The lines are parallel.

Spiral Review

Solve each system of inequalities by graphing. (Lesson 3-3)

46. $2y + x \geq 4$
 $y \geq x - 4$

47. $3x - 2y \leq -6$
 $y \leq \frac{3}{2}x - 1$

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

48. $4x + 5y = 20$
 $5x + 4y = 7$

49. $6x + y = 15$
 $x - 4y = -10$

50. $3x + 8y = 23$
 $x - y = 4$

- 51. CARD COLLECTING** Nathan has 50 baseball cards in his collection from the 1950's and 1960's. His goal is to buy 2 more cards each month. Write an equation that represents how many cards Nathan will have in his collection in x months if he meets his goal. (Lesson 2-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression if $x = -2$, $y = 6$, and $z = 5$. (Lesson 1-1)

52. $x + y + z$

53. $2x - y + 3z$

54. $-x + 4y - 2z$

55. $5x + 2y - z$

56. $3x - y + 4z$

57. $-2x - 3y + 2z$

Solving Systems of Equations in Three Variables

Main Ideas

- Solve systems of linear equations in three variables.
- Solve real-world problems using systems of linear equations in three variables.

New Vocabulary

ordered triple

GET READY for the Lesson

At the 2004 Summer Olympics in Athens, Greece, the United States won 103 medals. They won 6 more gold medals than bronze and 10 more silver medals than bronze.



You can write and solve a system of three linear equations to determine

how many of each type of medal the U.S. Olympians won. Let g represent the number of gold medals, let s represent the number of silver medals, and let b represent the number of bronze medals.

$$g + s + b = 103 \quad \text{U.S. Olympians won a total of 103 medals.}$$

$$g = b + 6 \quad \text{They won 6 more gold medals than bronze.}$$

$$s = b + 10 \quad \text{They won 10 more silver medals than bronze.}$$

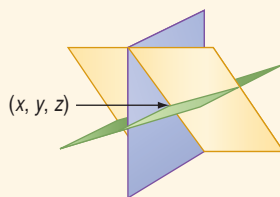
Systems in Three Variables The system of equations above has three variables. The graph of an equation in three variables, all to the first power, is a plane. The solution of a system of three equations in three variables can have one solution, infinitely many solutions, or no solution.

KEY CONCEPT

System of Equations in Three Variables

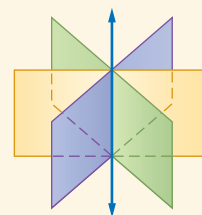
One Solution

- planes intersect in one point



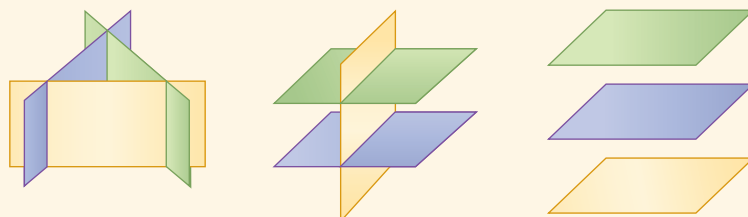
Infinitely Many Solutions

- planes intersect in a line
- planes intersect in the same plane



No Solution

- planes have no point in common





Solving systems of equations in three variables is similar to solving systems of equations in two variables. Use the strategies of substitution and elimination. The solution of a system of equations in three variables x , y , and z is called an **ordered triple** and is written as (x, y, z) .

EXAMPLE One Solution

1 Solve the system of equations.

$$x + 2y + z = 10$$

$$2x - y + 3z = -5$$

$$2x - 3y - 5z = 27$$

Step 1 Use elimination to make a system of two equations in two variables.

$$\begin{array}{rcl} x + 2y + z = 10 & \xrightarrow{\text{Multiply by 2.}} & 2x + 4y + 2z = 20 \\ 2x - y + 3z = -5 & & (-) 2x - y + 3z = -5 \\ \hline & & 5y - z = 25 \end{array} \quad \begin{array}{l} \text{Subtract to} \\ \text{eliminate } x. \end{array}$$

$$\begin{array}{rcl} 2x - y + 3z = -5 & \text{Second equation} & \\ (-) 2x - 3y - 5z = 27 & \text{Third equation} & \\ \hline & & 2y + 8z = -32 \end{array} \quad \begin{array}{l} \text{Subtract to eliminate } x. \end{array}$$

Notice that the x terms in each equation have been eliminated. The result is two equations with the same two variables y and z .

Step 2 Solve the system of two equations.

$$\begin{array}{rcl} 5y - z = 25 & \xrightarrow{\text{Multiply by 8.}} & 40y - 8z = 200 \\ 2y + 8z = -32 & & (+) 2y + 8z = -32 \\ \hline & & 42y = 168 \end{array} \quad \begin{array}{l} \text{Add to eliminate } z. \\ y = 4 \quad \text{Divide by 42.} \end{array}$$

Use one of the equations with two variables to solve for z .

$$\begin{array}{lcl} 5y - z = 25 & \text{Equation with two variables} & \\ 5(4) - z = 25 & \text{Replace } y \text{ with 4.} & \\ 20 - z = 25 & \text{Multiply.} & \\ z = -5 & \text{Simplify.} & \end{array}$$

The result is $y = 4$ and $z = -5$.

Step 3 Solve for x using one of the original equations with three variables.

$$\begin{array}{lcl} x + 2y + z = 10 & \text{Original equation with three variables} & \\ x + 2(4) + (-5) = 10 & \text{Replace } y \text{ with 4 and } z \text{ with } -5. & \\ x + 8 - 5 = 10 & \text{Multiply.} & \\ x = 7 & \text{Simplify.} & \end{array}$$

The solution is $(7, 4, -5)$. Check this solution in the other two original equations.

CHECK Your Progress

1A. $2x - y + 3z = -2$
 $x + 4y - 2z = 16$
 $5x + y - 1z = 14$

1B. $3x + y + z = 0$
 $-x + 2y - 2z = -3$
 $4x - y - 3z = 9$

Study Tip

Elimination

Remember that you can eliminate any of the three variables.

Study Tip

Common Misconception

Not every ordered triple is a solution of a system in three variables with an infinite number of solutions. The solution set contains an infinite number of ordered triples but not every ordered triple.

EXAMPLE Infinitely Many Solutions

2 Solve the system of equations.

$$4x - 6y + 4z = 12$$

$$6x - 9y + 6z = 18$$

$$5x - 8y + 10z = 20$$

Eliminate x in the first two equations.

$$4x - 6y + 4z = 12 \quad \xrightarrow{\text{Multiply by 3.}} \quad 12x - 18y + 12z = 36$$

$$6x - 9y + 6z = 18 \quad \xrightarrow{\text{Multiply by } -2.} \quad (+) -12x + 18y - 12z = -36 \quad \text{Add the equations.}$$
$$\hline 0 = 0$$

The equation $0 = 0$ is always true. This indicates that the first two equations represent the same plane. Check to see if this plane intersects the third plane.

$$4x - 6y + 4z = 12 \quad \xrightarrow{\text{Multiply by 5.}} \quad 20x - 30y + 20z = 60$$

$$5x - 8y + 10z = 20 \quad \xrightarrow{\text{Multiply by } -2.} \quad (+) -10x + 16y - 20z = -40 \quad \text{Add the equations.}$$
$$\hline 10x - 14y = 20 \quad \text{Divide by the GCF, 2.}$$
$$5x - 7y = 10$$

The planes intersect in a line. So, there are an infinite number of solutions.

CHECK Your Progress

2A. $8x + 12y - 24z = -40$

$$3x - 8y + 12z = 23$$

$$2x + 3y - 6z = -10$$

2B. $3x - 2y + 4z = 8$

$$-6x + 4y - 8z = -16$$

$$x + 2y - 4z = 4$$

EXAMPLE No Solution

3 Solve the system of equations.

$$6a + 12b - 8c = 24$$

$$9a + 18b - 12c = 30$$

$$4a + 8b - 7c = 26$$

Eliminate a in the first two equations.

$$6a + 12b - 8c = 24 \quad \xrightarrow{\text{Multiply by 3.}} \quad 18a + 36b - 24c = 72$$

$$9a + 18b - 12c = 30 \quad \xrightarrow{\text{Multiply by 2.}} \quad (-) 18a + 36b - 24c = 60 \quad \text{Subtract the equations.}$$
$$\hline 0 = 12$$

The equation $0 = 12$ is never true. So, there is no solution of this system.

CHECK Your Progress

3A. $8x + 4y - 3z = 7$

$$4x + 2y - 6z = -15$$

$$10x + 5y - 15z = -25$$

3B. $4x - 3y - 2z = 8$

$$x + 5y + 3z = 9$$

$$-8x + 6y + 4z = 2$$

Real-World Problems When solving problems involving three variables, use the four-step plan to help organize the information.



Real-World EXAMPLE

Write and Solve a System of Equations



4

INVESTMENTS Andrew Chang has \$15,000 that he wants to invest in certificates of deposit (CDs). For tax purposes, he wants his total interest per year to be \$800. He wants to put \$1000 more in a 2-year CD than in a 1-year CD and invest the rest in a 3-year CD. How much should Mr. Chang invest in each type of CD?

Number of Years	1	2	3
Rate	3.4%	5.0%	6.0%



Real-World Link

A certificate of deposit (CD) is a way to invest your money with a bank. The bank generally pays higher interest rates on CDs than savings accounts. However, you must invest your money for a specific time period, and there are penalties for early withdrawal.

Explore

Read the problem and define the variables.

a = the amount of money invested in a 1-year certificate

b = the amount of money in a 2-year certificate

c = the amount of money in a 3-year certificate

Plan

Mr. Chang has \$15,000 to invest.

$$a + b + c = 15,000$$

The interest he earns should be \$800. The interest equals the rate times the amount invested.

$$0.034a + 0.05b + 0.06c = 800$$

There is \$1000 more in the 2-year certificate than in the 1-year certificate.

$$b = a + 1000$$

Solve

Substitute $b = a + 1000$ in each of the first two equations.

$$a + (a + 1000) + c = 15,000$$

Replace b with $(a + 1000)$.

$$2a + 1000 + c = 15,000$$

Simplify.

$$2a + c = 14,000$$

Subtract 1000 from each side.

$$0.034a + 0.05(a + 1000) + 0.06c = 800$$

Replace b with $(a + 1000)$.

$$0.034a + 0.05a + 50 + 0.06c = 800$$

Distributive Property

$$0.084a + 0.06c = 750$$

Simplify.

Now solve the system of two equations in two variables.

$$\begin{array}{r} 2a + c = 14,000 \\ 0.084a + 0.06c = 750 \end{array}$$

Multiply by 0.06.

$$\begin{array}{r} 0.12a + 0.06c = 840 \\ (-) 0.084a + 0.06c = 750 \\ \hline 0.036a \qquad = 90 \\ a = 2500 \end{array}$$

Substitute 2500 for a in one of the original equations.

$$b = a + 1000$$

Third equation

$$= 2500 + 1000$$

$a = 2500$

$$= 3500$$

Add.





Substitute 2500 for a and 3500 for b in one of the original equations.

$$a + b + c = 15,000 \quad \text{First equation}$$

$$2500 + 3500 + c = 15,000 \quad a = 2500, b = 3500$$

$$6000 + c = 15,000 \quad \text{Add.}$$

$$c = 9000 \quad \text{Subtract 6000 from each side.}$$

So, Mr. Chang should invest \$2500 in a 1-year certificate, \$3500 in a 2-year certificate, and \$9000 in a 3-year certificate.

Check Is the answer reasonable? Have all the criteria been met?

The total investment is \$15,000.

$$2500 + 3500 + 9000 = 15,000 \quad \checkmark$$

The interest earned will be \$800.

$$0.034(2500) + 0.05(3500) + 0.06(9000) = 800$$

$$85 + 175 + 540 = 800 \quad \checkmark$$

There is \$1000 more in the 2-year certificate than the 1-year certificate.

$$3500 = 2500 + 1000 \quad \checkmark \quad \text{The answer is reasonable.}$$



**Concepts
in Motion**

Interactive Lab
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4. **BASKETBALL** Macario knows that he has scored a total of 70 points so far this basketball season. His coach told him that he has scored 37 times, but Macario wants to know how many free throws, field goals, and three pointers he has made. The sum of his field goals and three pointers equal twice the number of free throws minus two. How many free throws, field goals, and three pointers has Macario made?

Online Personal Tutor at algebra2.com



Examples 1–3
(pp. 146–147)

Solve each system of equations.

1. $x + 2y = 12$

$$3y - 4z = 25$$

$$x + 6y + z = 20$$

4. $2r + 3s - 4t = 20$

$$4r - s + 5t = 13$$

$$3r + 2s + 4t = 15$$

2. $9a + 7b = -30$

$$8b + 5c = 11$$

$$-3a + 10c = 73$$

5. $2x - y + z = 1$

$$x + 2y - 4z = 3$$

$$4x + 3y - 7z = -8$$

3. $r - 3s + t = 4$

$$3r - 6s + 9t = 5$$

$$4r - 9s + 10t = 9$$

6. $x + y + z = 12$

$$6x - 2y - z = 16$$

$$3x + 4y + 2z = 28$$

Example 4
(pp. 148–149)

COOKING For Exercises 7 and 8, use the following information.

Jambalaya is a Cajun dish made from chicken, sausage, and rice. Simone is making a large pot of jambalaya for a party. Chicken costs \$6 per pound, sausage costs \$3 per pound, and rice costs \$1 per pound. She spends \$42 on 13.5 pounds of food. She buys twice as much rice as sausage.

7. Write a system of three equations that represents how much food Simone purchased.
8. How much chicken, sausage, and rice will she use in her dish?



Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–19	1–3
20–23	4

Solve each system of equations.

9. $2x - y = 2$
 $3z = 21$
 $4x + z = 19$
10. $-4a = 8$
 $5a + 2c = 0$
 $7b + 3c = 22$
11. $5x + 2y = 4$
 $3x + 4y + 2z = 6$
 $7x + 3y + 4z = 29$
12. $8x - 6z = 38$
 $2x - 5y + 3z = 5$
 $x + 10y - 4z = 8$
13. $4a + 2b - 6c = 2$
 $6a + 3b - 9c = 3$
 $8a + 4b - 12c = 6$
14. $2r + s + t = 14$
 $-r - 3s + 2t = -2$
 $4r - 6s + 3t = -5$
15. $3x + y + z = 4$
 $2x + 2y + 3z = 3$
 $x + 3y + 2z = 5$
16. $4a - 2b + 8c = 30$
 $a + 2b - 7c = -12$
 $2a - b + 4c = 15$
17. $9x - 3y + 12z = 39$
 $12x - 4y + 16z = 52$
 $3x - 8y + 12z = 23$
18. The sum of three numbers is 20. The second number is 4 times the first, and the sum of the first and third is 8. Find the numbers.
19. The sum of three numbers is 12. The first number is twice the sum of the second and third. The third number is 5 less than the first. Find the numbers.

BASKETBALL For Exercises 20 and 21, use the following information.

In the 2004 season, Seattle's Lauren Jackson was ranked first in the WNBA for total points and points per game. She scored 634 points making 362 shots, including 3-point field goals, 2-point field goals, and 1-point free throws. She made 26 more 2-point field goals than free throws.

20. Write a system of equations that represents the number of goals she made.
21. Find the number of each type of goal she made.

FOOD For Exercises 22 and 23, use the following information.

Maka loves the lunch combinations at Rosita's Mexican Restaurant.

Today however, she wants a different combination than the ones listed on the menu.

22. Assume that the price of a combo meal is the same price as purchasing each item separately. Find the price for an enchilada, a taco, and a burrito.
23. If Maka wants 2 burritos and 1 enchilada, how much should she plan to spend?

Lunch Combo Meals



1. Two Tacos,
One Burrito.....\$6.55
2. One Enchilada, One Taco,
One Burrito.....\$7.10
3. Two Enchiladas,
Two Tacos.....\$8.90



Real-World Link

In 2005, Katie Smith became the first person in the WNBA to score 5000 points.

Source: www.wnba.com

24. **TRAVEL** Jonathan and members of his Spanish Club are going to Costa Rica. He purchases 10 traveler's checks in denominations of \$20, \$50, and \$100, totaling \$370. He has twice as many \$20 checks as \$50 checks. How many of each denomination of traveler's checks does he have?

Solve each system of equations.

25. $6x + 2y + 4z = 2$
 $3x + 4y - 8z = -3$
 $-3x - 6y + 12z = 5$
26. $r + s + t = 5$
 $2r - 7s - 3t = 13$
 $\frac{1}{2}r - \frac{1}{3}s + \frac{2}{3}t = -1$
27. $2a - b + 3c = -7$
 $4a + 5b + c = 29$
 $a - \frac{2b}{3} + \frac{c}{4} = -10$

EXTRA PRACTICE

See pages 896, 928.

Math online

Self-Check Quiz at
algebra2.com

- 28. OPEN ENDED** Write an example of a system of three equations in three variables that has $(-3, 5, 2)$ as a solution. Show that the ordered triple satisfies all three equations.
- 29. REASONING** Compare and contrast solving a system of two equations in two variables to solving a system of three equations in three variables.
- 30. FIND THE ERROR** Melissa is solving the system of equations $r + 2s + t = 3$, $2r + 4s + 2t = 6$, and $3r + 6s + 3t = 12$. Is she correct? Explain.

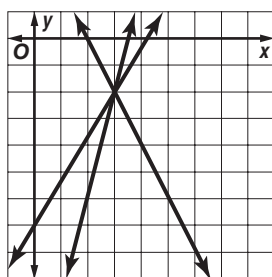
$$\begin{array}{rcl} r + 2s + t = 3 & \rightarrow & 2r + 4s + 2t = 6 \\ 2r + 4s + 2t = 6 & \rightarrow & (-)2r + 4s + 2t = 6 \\ & & 0 = 0 \end{array}$$

The second equation is a multiple of the first, so they are the same plane. There are infinitely many solutions.

- 31. CHALLENGE** The general form of an equation for a parabola is $y = ax^2 + bx + c$, where (x, y) is a point on the parabola. If three points on the parabola are $(0, 3)$, $(-1, 4)$, and $(2, 9)$, determine the values of a, b, c . Write the equation of the parabola.
- 32. Writing in Math** Use the information on page 145 to explain how you can determine the number and type of medals 2004 U.S. Olympians won in Athens. Demonstrate how to find the number of each type of medal won by the U.S. Olympians and describe another situation where you can use a system of three equations in three variables to solve a problem.

STANDARDIZED TEST PRACTICE

- 33. ACT/SAT** The graph depicts which system of equations?



- | | |
|-------------------------|-------------------------|
| A $y + 14 = 4x$ | C $y - 14 = 4x$ |
| $y = 4 - 2x$ | $y = 4 + 2x$ |
| $-7 = y - \frac{5}{3}x$ | $-7 = y + \frac{5}{3}x$ |
| B $y + 14x = 4$ | D $y - 14x = 4$ |
| $-2y = 4 + y$ | $2x = 4 + y$ |
| $-7 = y - \frac{5}{3}x$ | $7 = y - \frac{5}{3}x$ |

- 34. REVIEW** What is the solution to the system of equations shown below?

$$\begin{cases} x - y + z = 0 \\ -5x + 3y - 2z = -1 \\ 2x - y + 4z = 11 \end{cases}$$

- F** $(0, 3, 3)$
G $(2, 5, 3)$
H no solution
J infinitely many solutions

- 35. MILK** The Yoder Family Dairy produces at most 200 gallons of skim and whole milk each day for delivery to large bakeries and restaurants. Regular customers require at least 15 gallons of skim and 21 gallons of whole milk each day. If the profit on a gallon of skim milk is \$0.82 and the profit on a gallon of whole milk is \$0.75, how many gallons of each type of milk should the dairy produce each day to maximize profits? (Lesson 3-4)

Solve each system of inequalities by graphing. (Lesson 3-3)

36. $y \leq x + 2$
 $y \geq 7 - 2x$

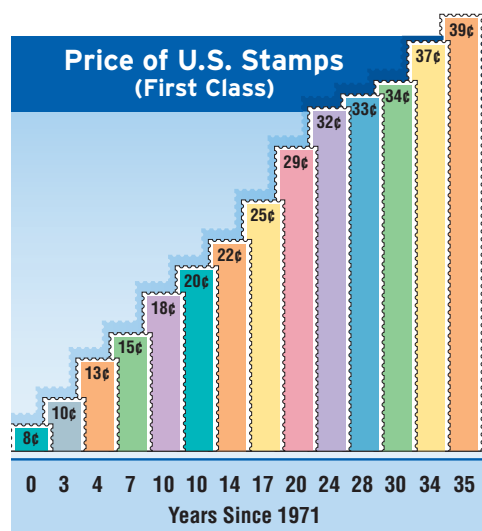
37. $4y - 2x > 4$
 $3x + y > 3$

38. $3x + y \geq 1$
 $2y - x \leq -4$

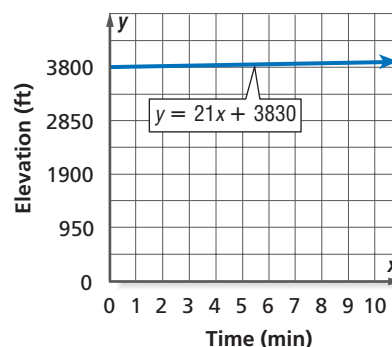
ANALYZE GRAPHS For Exercises 39 and 40, use the following information.

The table shows the price for first-class stamps since July 1, 1971. (Lesson 2-5)

- 39.** Write a prediction equation for this relationship.
- 40.** Predict the price for a first-class stamp issued in the year 2015.



- 41. HIKING** Miguel is hiking on the Alum Cave Bluff Trail in the Great Smoky Mountains. The graph represents Miguel's elevation y at each time x . At what elevation did Miguel begin his climb? How is that represented in the equation? (Lesson 2-4)



Find each value if $f(x) = 6x + 2$ and $g(x) = 3x^2 - x$. (Lesson 2-1)

42. $f(-1)$

43. $f\left(\frac{1}{2}\right)$

44. $g(1)$

45. $g(-3)$

- 46. TIDES** Ocean tides are caused by gravitational forces exerted by the Moon. Tides are also influenced by the size, boundaries, and depths of ocean basins and inlets. The highest tides on Earth occur in the Bay of Fundy in Nova Scotia, Canada. During the middle of the tidal range, the ocean shore is 30 meters from a rock bluff. The tide causes the shoreline to advance 8 meters and retreat 8 meters throughout the day. Write and solve an equation describing the maximum and minimum distances from the rock bluff to the ocean during high and low tide. (Lesson 1-4)

Study Guide and Review

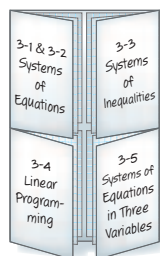


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Review from algebra2.com

FOLDABLES™
Study Organizer

GET READY to Study

Be sure the following
Key Concepts are noted
in your Foldable.



Key Concepts

Systems of Equations (Lessons 3-1 and 3-2)

- The solution of a system of equations can be found by graphing the two equations and determining at what point they intersect.
- In the substitution method, one equation is solved for a variable and substituted to find the value of another variable.
- In the elimination method, one variable is eliminated by adding or subtracting the equations.

Systems of Inequalities (Lesson 3-3)

- The solution of a system of inequalities is found by graphing the inequalities and determining the intersection of the graphs.

Linear Programming (Lesson 3-4)

- The maximum and minimum values of a function are determined by linear programming techniques.

Systems of Three Equations (Lesson 3-5)

- A system of equations in three variables can be solved algebraically by using the substitution method or the elimination method.

Key Vocabulary

bounded region (p. 138)	linear programming (p. 140)
consistent system (p. 118)	ordered triple (p. 146)
constraints (p. 138)	substitution method (p. 123)
dependent system (p. 118)	system of equations (p. 116)
elimination method (p. 125)	system of inequalities (p. 130)
feasible region (p. 138)	unbounded region (p. 139)
inconsistent system (p. 118)	vertex (p. 138)
independent system (p. 118)	

Vocabulary Check

Choose the term from the list above that best matches each phrase.

1. the inequalities of a linear programming problem
2. a system of equations that has an infinite number of solutions
3. the region of a graph where every constraint is met
4. a method of solving equations in which one equation is solved for one variable in terms of the other variable
5. a system of equations that has at least one solution
6. a system of equations that has exactly one solution
7. a method of solving equations in which one variable is eliminated when the two equations are combined
8. the solution of a system of equations in three variables (x , y , z)
9. two or more equations with the same variables
10. two or more inequalities with the same variables

Lesson-by-Lesson Review

3-1 Solving Systems of Equations by Graphing (pp. 116–122)

Solve each system of linear equations by graphing.

11. $3x + 2y = 12$

$x - 2y = 4$

12. $8x - 10y = 7$

$4x - 5y = 7$

13. $y - 2x = 8$

$y = \frac{1}{2}x - 4$

14. $20y + 13x = 10$

$0.65x + y = 0.5$

15. **PLUMBING** Two plumbers offer competitive services. The first charges a \$35 house-call fee and \$28 per hour. The second plumber charges a \$42 house-call fee and \$21 per hour. After how many hours do the two plumbers charge the same amount?

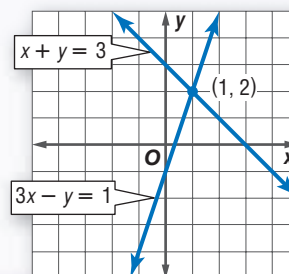
Example 1 Solve the system of equations by graphing.

$x + y = 3$

$3x - y = 1$

Graph both equations on the same coordinate plane.

The solution of the system is (1, 2).

**3-2 Solving Systems of Equations Algebraically** (pp. 123–129)

Solve each system of equations by using either substitution or elimination.

16. $x + y = 5$

$2x - y = 4$

17. $2x - 3y = 9$

$4x + 2y = -22$

18. $7y - 2x = 10$

$-3y + x = -3$

19. $x + y = 4$

$x - y = 8.5$

20. $-6y - 2x = 0$

$11y + 3x = 4$

21. $3x - 5y = -13$

$4x + 2y = 0$

22. **CLOTHING** Colleen bought 15 used and lightly used T-shirts at a thrift store. The used shirts cost \$0.70 less than the lightly used shirts. Her total, minus tax, was \$16.15. If Colleen bought 8 used shirts and paid \$0.70 less per shirt than for a lightly used shirt, how much does each type of shirt cost?

Example 2 Solve the system of equations by using either substitution or elimination.

$x = 4y + 7$

$y = -3 - x$

Substitute $-3 - x$ for y in the first equation.

$x = 4y + 7$

$x = 4(-3 - x) + 7$

$x = -12 - 4x + 7$

$5x = -5$

$x = -1$

First equation

Substitute $-3 - x$ for y .

Distributive Property

Add $4x$ to each side.

Divide each side by 5.

Now substitute the value for x in either original equation.

$y = -3 - x$

$y = -3 - (-1) \text{ or } -2$

Second equation

Replace x with -1 and simplify.

The solution of the system is $(-1, -2)$.

3-3 Solving Systems of Inequalities by Graphing (pp. 130-135)

Solve each system of inequalities by graphing. Use a table to analyze the possible solutions.

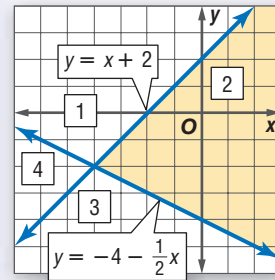
23. $y \leq 4$
 $y > -3$
24. $|y| > 3$
 $x \leq 1$
25. $y < x + 1$
 $x > 5$
26. $y \leq x + 4$
 $2y \geq x - 3$

27. **JOBS** Tamara spends no more than 5 hours working at a local manufacturing plant. It takes her 25 minutes to set up her equipment and at least 45 minutes for each unit she constructs. Draw a diagram that represents this information.

Example 3 Solve the system of inequalities by graphing.

$$y \leq x + 2$$

$$y \geq -4 - \frac{1}{2}x$$



The solution of the system is the region that satisfies both inequalities. The solution of this system is region 2.

3-4 Linear Programming (pp. 138-144)

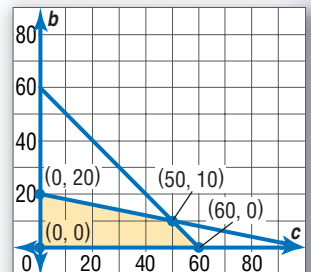
28. **MANUFACTURING** A toy manufacturer is introducing two new dolls to their customers: My First Baby, which talks, laughs, and cries, and My Real Baby, which simulates using a bottle and crawls. In one hour the company can produce 8 First Babies or 20 Real Babies. Because of the demand, the company must produce at least twice as many First Babies as Real Babies. The company spends no more than 48 hours per week making these two dolls. The profit on each First Baby is \$3.00 and the profit on each Real Baby is \$7.50. Find the number and type of dolls that should be produced to maximize the profit.

Example 4 The area of a parking lot is 600 square meters. A car requires 6 square meters of space, and a bus requires 30 square meters of space. The attendant can handle no more than 60 vehicles. If a car is charged \$3 to park and a bus is charged \$8, how many of each should the attendant accept to maximize income?

Let c = the number of cars and b = the number of buses.

$$c \geq 0, b \geq 0, 6c + 30b \leq 600, \text{ and } c + b \leq 60$$

Graph the inequalities. The vertices of the feasible region are $(0, 0)$, $(0, 20)$, $(50, 10)$, and $(60, 0)$.



The profit function is $f(c, b) = 3c + 8b$.

The maximum value of \$230 occurs at $(50, 10)$. So the attendant should accept 50 cars and 10 buses.

3-5

Solving Systems of Equations in Three Variables (pp. 145–152)

Solve each system of equations.

$$\begin{aligned} 29. \quad & x + 4y - z = 6 \\ & 3x + 2y + 3z = 16 \\ & 2x - y + z = 3 \end{aligned}$$

$$\begin{aligned} 30. \quad & 2a + b - c = 5 \\ & a - b + 3c = 9 \\ & 3a - 6c = 6 \end{aligned}$$

$$\begin{aligned} 31. \quad & e + f = 4 \\ & 2d + 4e - f = -3 \\ & 3e = -3 \end{aligned}$$

32. **SUBS** Ryan, Tyee, and Jaleel are ordering subs from a shop that lets them choose the number of meats, cheeses, and veggies that they want. Their sandwiches and how much they paid are displayed in the table. How much does each topping cost?

Name	Meat	Cheese	Veggie	Price
Ryan	1	2	5	\$5.70
Tyee	3	2	2	\$7.85
Jaleel	2	1	4	\$6.15

Example 5 Solve the system of equations.

$$\begin{aligned} x + 3y + 2z &= 1 \\ 2x + y - z &= 2 \\ x + y + z &= 2 \end{aligned}$$

Use elimination to make a system of two equations in two variables.

$$\begin{array}{rcl} 2x + 6y + 4z & = & 2 \quad \text{First equation} \times 2 \\ (-) 2x + y - z & = & 2 \quad \text{Second equation} \\ \hline 5y + 5z & = & 0 \quad \text{Subtract.} \end{array}$$

Do the same with the first and third equations to get $2y + z = -1$.

Solve the system of two equations.

$$\begin{array}{rcl} 5y + 5z & = & 0 \\ (-) 10y + 5z & = & -5 \\ \hline -5y & = & 5 \quad \text{Subtract to eliminate } z. \\ y & = & -1 \quad \text{Divide each side by } -5. \end{array}$$

Substitute -1 for y in one of the equations with two variables and solve for z .

Then, substitute -1 for y and the value you received for z into an equation from the original system to solve for x .

The solution is $(2, -1, 1)$.

Solve each system of equations.

1. $-4x + y = -5$
 $2x + y = 7$
2. $x + y = -8$
 $-3x + 2y = 9$
3. $3x + 2y = 18$
 $y = 6x - 6$
4. $-6x + 3y = 33$
 $-4x + y = 16$
5. $-7x + 6y = 42$
 $3x + 4y = 28$
6. $2y = 5x - 1$
 $x + y = -1$

Solve each system of inequalities by graphing.

7. $y \geq x - 3$
 $y \geq -x + 1$
8. $x + 2y \geq 7$
 $3x - 4y < 12$
9. $3x + y < -5$
 $2x - 4y \geq 6$
10. $2x + y \geq 7$
 $3y \leq 4x + 1$

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and the minimum values of the given function.

11. $5 \geq y \geq -3$
 $4x + y \leq 5$
 $-2x + y \leq 5$
 $f(x, y) = 4x - 3y$
12. $x \geq -10$
 $1 \geq y \geq -6$
 $3x + 4y \leq -8$
 $2y \geq x - 10$
 $f(x, y) = 2x + y$

13. **MULTIPLE CHOICE** Which statement best describes the graphs of the two equations?

$$16x - 2y = 24$$

$$12x = 3y - 36$$

- A The lines are parallel.
- B The lines are the same.
- C The lines intersect in only one point.
- D The lines intersect in more than one point, but are not the same.

Solve each system of equations.

14. $x + y + z = -1$
 $2x + 4y + z = 1$
 $x + 2y - 3z = -3$
15. $x + z = 7$
 $2y - z = -3$
 $-x - 3y + 2z = 11$

16. **MULTIPLE CHOICE** Carla, Meiko, and Kayla went shopping to get ready for college. Their purchases and total amounts spent are shown in the table below.

Person	Shirts	Pants	Shoes	Total Spent
Carla	3	4	2	\$149.79
Meiko	5	3	3	\$183.19
Kayla	6	5	1	\$181.14

Assume that all of the shirts were the same price, all of the pants were the same price, and all of the shoes were the same price. What was the price of each item?

- F shirt, \$12.95; pants, \$15.99; shoes, \$23.49
- G shirt, \$15.99; pants, \$12.95; shoes, \$23.49
- H shirt, \$15.99; pants, \$23.49; shoes, \$12.95
- J shirt, \$23.49; pants, \$15.99; shoes, \$12.95

MANUFACTURING For Exercises 17–19, use the following information.

A sporting goods manufacturer makes a \$5 profit on soccer balls and a \$4 profit on volleyballs. Cutting requires 2 hours to make 75 soccer balls and 3 hours to make 60 volleyballs. Sewing needs 3 hours to make 75 soccer balls and 2 hours to make 60 volleyballs. The cutting department has 500 hours available, and the sewing department has 450 hours available.

17. How many soccer balls and volleyballs should be made to maximize the company's profit?
18. What is the maximum profit the company can make from these two products?
19. What would the maximum profit be if Cutting and Sewing got new equipment that allowed them to produce soccer balls at the same rate, but allowed Cutting to produce 75 volleyballs in 3 hours and Sewing to make 75 volleyballs in 2 hours?

Standardized Test Practice

Cumulative, Chapters 1–3

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. At the Gallatin Valley Cinema, the cost of 2 boxes of popcorn and 1 soda is \$11.50. The cost of 3 boxes of popcorn and 4 sodas is \$27.25. Which pair of equations can be used to determine p , the cost of a box of popcorn, and s , the cost of a soda?

A $2p + s = 27.25$
 $3p + 4s = 11.50$

B $2p - s = 11.50$
 $3p - 4s = 27.25$

C $2p + s = 11.50$
 $3p + 4s = 27.25$

D $p + s = 11.50$
 $p + 4 = 27.25$

2. What are the x -intercepts of the graph of the equation $y = x^2 - 2x - 15$?

F $x = -3, x = 5$

G $x = -1, x = 15$

H $x = -5, x = 3$

J $x = -5, x = -3$

3. **GRIDDABLE** What is the y -coordinate of the solution to the system of equations below?

$$y = 4x - 7$$

$$y = -\frac{1}{2}x + 2$$

4. As a fund-raiser, the student council sold T-shirts and sweatshirts. They sold a total of 105 T-shirts and sweatshirts and raised \$1170. If the cost of a T-shirt t was \$10 and the cost of a sweatshirt s was \$15, what was the number of sweatshirts sold?

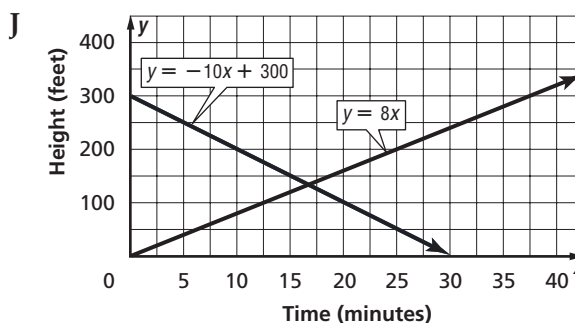
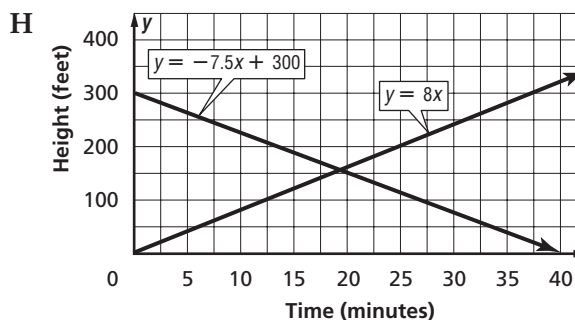
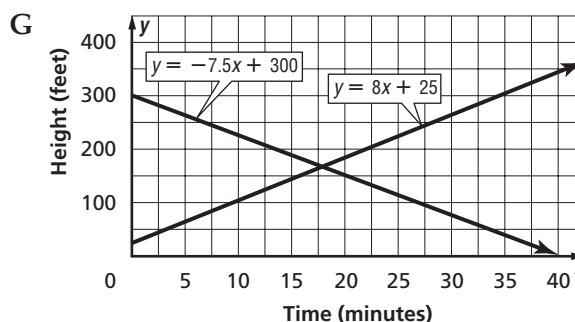
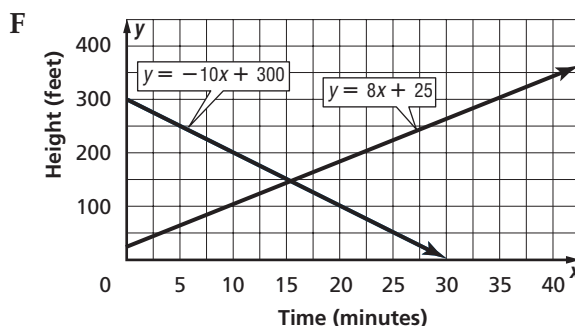
A 24

B 52

C 81

D 105

5. At the Carter County Fair, one hot air balloon is descending at a rate of 10 feet per minute from a height of 300 feet. At the same time, another hot air balloon is climbing from ground level at a rate of 8 feet per minute. Which graph shows when the two hot air balloons will be at the same altitude?



6. Which of the following best describes the graph of the equations below?

$$3y = 4x - 3$$

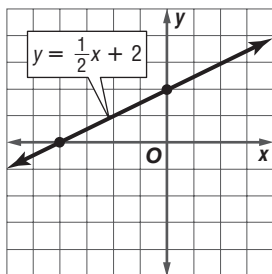
$$8y = -6x - 5$$

- A The lines have the same y -intercept.
B The lines have the same x -intercept.
C The lines are perpendicular.
D The lines are parallel.

TEST-TAKING TIP

QUESTION 6 This problem does not include a drawing. Make one. It can help you quickly see how to solve the problem.

7. The graph of the equation $y = \frac{1}{2}x + 2$ is given below. Suppose you graph $y = x - 1$ on the grid.



What is the solution to the system of equations?

- F $(0, -1)$ H $(6, 5)$
G $(7, 6)$ J no solution
8. The equations of two lines are $2x - y = 6$ and $4x - y = -2$. Which of the following describes their point of intersection?
- A $(2, -2)$
B $(-8, -38)$
C $(-4, -14)$
D no intersection

9. Let p represent the price that Ella charges for a necklace. Let $f(x)$ represent the total amount of money that Ella makes for selling x necklaces. The function $f(x)$ is best represented by

- F $x + p$
G xp^2
H px
J $x^2 + p$

10. **GRIDDABLE** Martha had some money saved for a week long vacation. The first day of the vacation she spent \$125 on food and a hotel. On the second day, she was given \$80 from her sister for expenses. Martha then had \$635 left for the rest of the vacation. How much money, in dollars, did she begin the vacation with?

Pre-AP

**Record your answers on a sheet of paper.
Show your work.**

11. Christine had one dress and three sweaters cleaned at the dry cleaner and the charge was \$19.50. The next week, she had two dresses and two sweaters cleaned for a total charge of \$23.00.
- a. Let d represent the price of cleaning a dress and s represent the price of cleaning a sweater. Write a system of linear equations to represent the prices of cleaning each item.
- b. Solve the system of equations using substitution or elimination. Explain your choice of method.
- c. What will the charge be if Christine takes two dresses and four sweaters to be cleaned?

NEED EXTRA HELP?											
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11
Go to Lesson...	3-1	2-4	3-1	3-1	3-2	3-1	3-1	3-1	2-4	3-2	3-2