UNIT 2

Quadratic, Polynomial, and Radical Equations and Inequalities

Focus

Use functions and equations as means for analyzing and understanding a broad variety of relationships.

CHAPTER 5

Quadratic Functions and Inequalities

BIG Idea Formulate equations and inequalities based on quadratic functions, use a variety of methods to solve them, and analyze the solutions in terms of the situation.

BIG Idea Interpret and describe the effects of changes in the parameters of quadratic functions.

CHAPTER 6

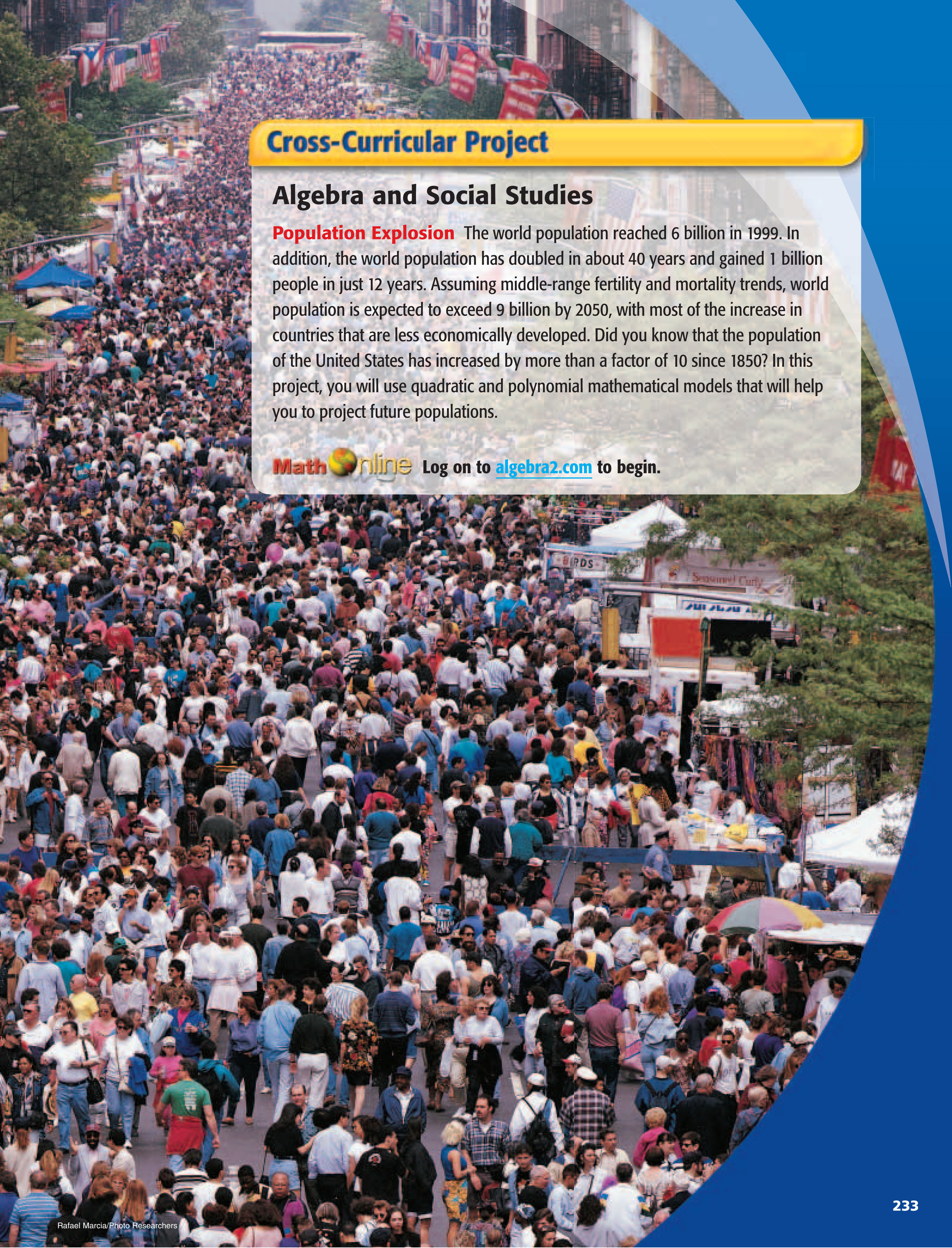
Polynomial Functions

BIG Idea Use properties and attributes of polynomial functions and apply functions to problem situations.

CHAPTER 7

Radical Equations and Inequalities

BIG Idea Formulate equations and inequalities based on square root functions, use a variety of methods to solve them, and analyze the solutions in terms of the situation.



Rafael Marcia/Photo Researchers

Cross-Curricular Project

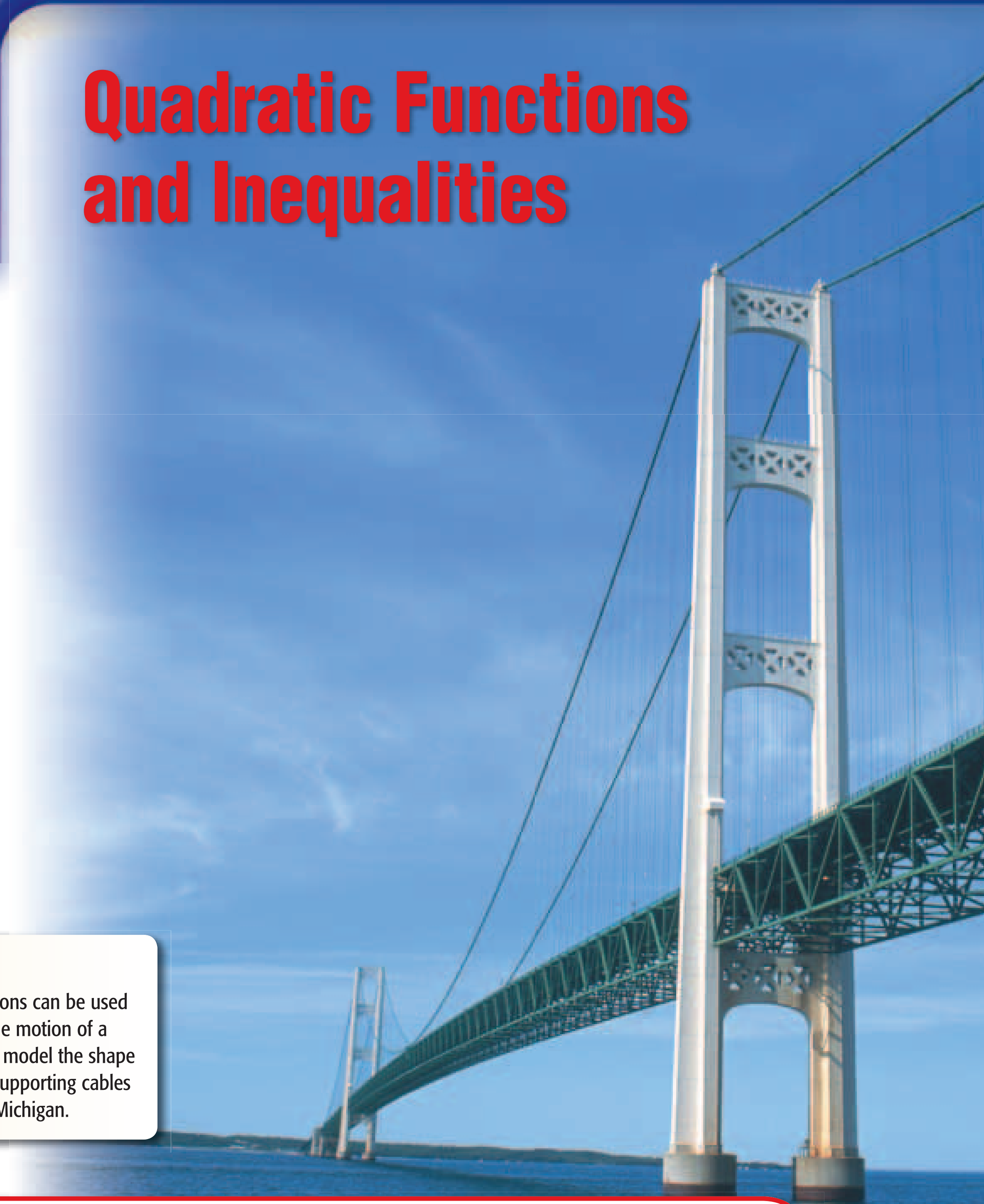
Algebra and Social Studies

Population Explosion The world population reached 6 billion in 1999. In addition, the world population has doubled in about 40 years and gained 1 billion people in just 12 years. Assuming middle-range fertility and mortality trends, world population is expected to exceed 9 billion by 2050, with most of the increase in countries that are less economically developed. Did you know that the population of the United States has increased by more than a factor of 10 since 1850? In this project, you will use quadratic and polynomial mathematical models that will help you to project future populations.

Math Online **Log on to algebra2.com to begin.**

CHAPTER 5

Quadratic Functions and Inequalities



BIG Ideas

- Graph quadratic functions.
- Solve quadratic equations.
- Perform operations with complex numbers.
- Graph and solve quadratic inequalities.

Key Vocabulary

discriminant (p. 279)

imaginary unit (p. 260)

root (p. 246)

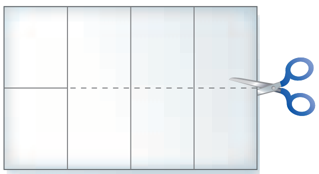
Real-World Link

Suspension Bridges Quadratic functions can be used to model real-world phenomena like the motion of a falling object. They can also be used to model the shape of architectural structures such as the supporting cables of the Mackinac Suspension Bridge in Michigan.

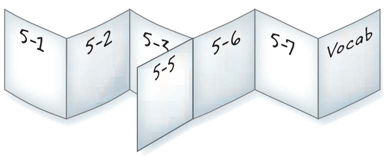
FOLDABLES Study Organizer

Quadratic Functions and Inequalities Make this Foldable to help you organize your notes. Begin with one sheet of 11″ by 17″ paper.

1. **Fold** in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.



2. **Refold** along the lengthwise fold and staple the uncut section at the top. Label each section with a lesson number and close to form a booklet.



Jon Arnold Images/Alamy Images

GET READY for Chapter 5

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at algebra2.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Given $f(x) = 2x^2 - 6$ and $g(x) = -x^2 + 4x - 4$, find each value. (Lesson 2-1)

1. $f(1)$ **2.** $f(4)$
3. $f(0)$ **4.** $f(-2)$
5. $g(0)$ **6.** $g(-1)$
7. $g(2)$ **8.** $g(0.5)$

FISH For Exercises 9 and 10, use the following information.

Tuna swim at a steady rate of 9 miles per hour until they die, and they never stop moving. (Lesson 2–1)

9. Write a function that is a model for the situation.

10. Evaluate the function to estimate how far a 2-year-old tuna has traveled.

Factor completely. If the polynomial is not factorable, write *prime*.
(Prerequisite Skills, p. 877)

11. $x^2 + 11x + 30$ **12.** $x^2 - 13x + 36$
13. $x^2 - x - 56$ **14.** $x^2 - 5x - 14$
15. $x^2 + x + 2$ **16.** $x^2 + 10x + 25$
17. $x^2 - 22x + 121$ **18.** $x^2 - 9$

19. **FLOOR PLAN** A living room has a floor space of $x^2 + 11x + 28$ square feet. If the width of the room is $(x + 4)$ feet, what is the length? (Prerequisite Skills, p. 877)

QUICKReview

EXAMPLE 1 **Given $f(x) = 3x^2 + 2$ and $g(x) = 0.5x^2 + 2x - 1$, find each value.**

a. $f(3)$

$f(x) = 3x^2 + 2$ Original function
$f(3) = 3(3)^2 + 2$ Substitute 3 for x.
$= 27 + 2$ or 29 Simplify.

b. $g(-4)$

$g(x) = 0.5x^2 + 2x - 1$ Original function
$g(-4) = 0.5(-4)^2 + 2(-4) - 1$ Substitute -4 for x.
$= 8 + (-8) - 1$ Multiply.
$= -1$ Simplify.

EXAMPLE 2 **Factor $x^2 - x - 2$ completely. If the polynomial is not factorable, write *prime*.**

To find the coefficients of the x-terms, you must find two numbers whose product is $(1)(-2)$ or -2, and whose sum is -1. The two coefficients must be 1 and -2 since $(1)(-2) = -2$ and $1 + (-2) = -1$. Rewrite the expression and factor by grouping.

$x^2 - x - 2$
$= x^2 + x - 2x - 2$ Substitute $x - 2x$ for $-x$.
$= (x^2 + x) + (-2x - 2)$ Associative Property
$= x(x + 1) - 2(x + 1)$ Factor out the GCF.
$= (x + 1)(x - 2)$ Distributive Property

5-1 Graphing Quadratic Functions

Main Ideas

- Graph quadratic functions.
- Find and interpret the maximum and minimum values of a quadratic function.

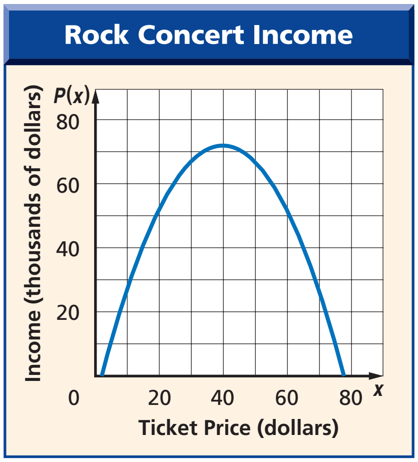
New Vocabulary

quadratic function
quadratic term
linear term
constant term
parabola
axis of symmetry
vertex
maximum value
minimum value

GET READY for the Lesson

Rock music managers handle publicity and other business issues for the artists they manage. One group's manager has found that based on past concerts, the predicted income for a performance is $P(x) = -50x^2 + 4000x - 7500$, where x is the price per ticket in dollars.

The graph of this quadratic function is shown at the right. At first the income increases as the price per ticket increases, but as the price continues to increase, the income declines.



Graph Quadratic Functions A **quadratic function** is described by an equation of the following form.

$$f(x) = \underbrace{ax^2}_{\text{quadratic term}} + \underbrace{bx}_{\text{linear term}} + \underbrace{c}_{\text{constant term}}, \text{ where } a \neq 0$$

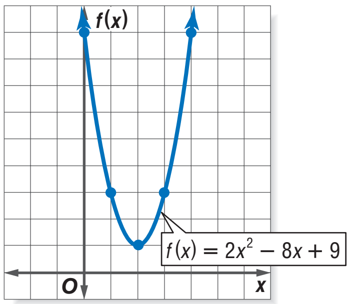
The graph of any quadratic function is called a **parabola**. To graph a quadratic function, graph ordered pairs that satisfy the function.

EXAMPLE 1 Graph a Quadratic Function

Graph $f(x) = 2x^2 - 8x + 9$ by making a table of values.

Choose integer values for x and evaluate the function for each value. Graph the resulting coordinate pairs and connect the points with a smooth curve.

x	$2x^2 - 8x + 9$	$f(x)$	$(x, f(x))$
0	$2(0)^2 - 8(0) + 9$	9	(0, 9)
1	$2(1)^2 - 8(1) + 9$	3	(1, 3)
2	$2(2)^2 - 8(2) + 9$	1	(2, 1)
3	$2(3)^2 - 8(3) + 9$	3	(3, 3)
4	$2(4)^2 - 8(4) + 9$	9	(4, 9)

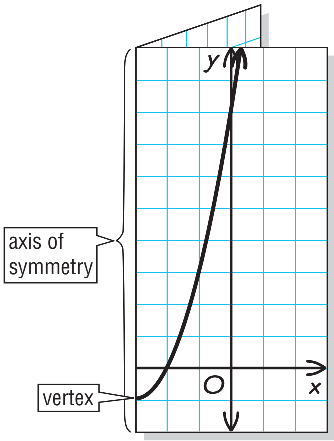


CHECK Your Progress

Graph each function by making a table of values.

1A. $g(x) = -x^2 + 2x - 6$ **1B.** $f(x) = x^2 - 8x + 15$

All parabolas have an **axis of symmetry**. If you were to fold a parabola along its axis of symmetry, the portions of the parabola on either side of this line would match.



The point at which the axis of symmetry intersects a parabola is called the **vertex**. The *y*-intercept of a quadratic function, the equation of the axis of symmetry, and the *x*-coordinate of the vertex are related to the equation of the function as shown below.

Study Tip

Graphing Quadratic Functions

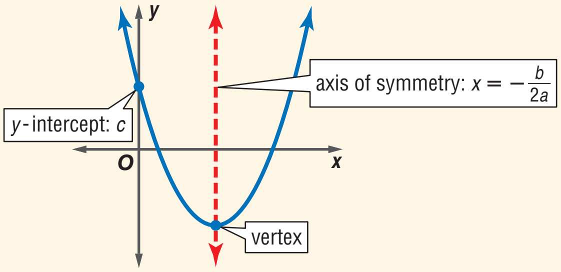
Knowing the location of the axis of symmetry, *y*-intercept, and vertex can help you graph a quadratic function.

KEY CONCEPT — *Graph of a Quadratic Equation*

Words Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The *y*-intercept is $a(0)^2 + b(0) + c$ or c.
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The *x*-coordinate of the vertex is $-\frac{b}{2a}$.

Model



EXAMPLE — Axis of Symmetry, *y*-Intercept, and Vertex

2 **Consider the quadratic function $f(x) = x^2 + 9 + 8x$.**

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

Begin by rearranging the terms of the function so that the quadratic term is first, the linear term is second, and the constant term is last. Then identify a, b, and c.

$$f(x) = ax^2 + bx + c$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$f(x) = x^2 + 9 + 8x \quad \rightarrow \quad f(x) = 1x^2 + 8x + 9 \quad \rightarrow \quad a = 1, b = 8, \text{ and } c = 9$$

The *y*-intercept is 9. Use a and b to find the equation of the axis of symmetry.

$$x = -\frac{b}{2a} \qquad \text{Equation of the axis of symmetry}$$

$$= -\frac{8}{2(1)} \qquad a = 1, b = 8$$

$$= -4 \qquad \text{Simplify.}$$

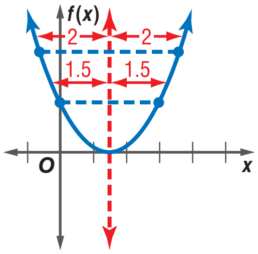
The equation of the axis of symmetry is $x = -4$. Therefore, the *x*-coordinate of the vertex is -4.

(continued on the next page)

Study Tip

Symmetry

Sometimes it is convenient to use symmetry to help find other points on the graph of a parabola. Each point on a parabola has a mirror image located the same distance from the axis of symmetry on the other side of the parabola.



b. Make a table of values that includes the vertex.

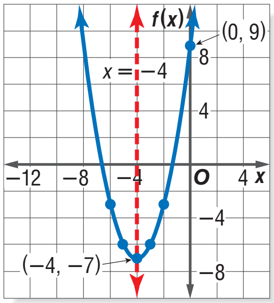
Choose some values for x that are less than -4 and some that are greater than -4. This ensures that points on each side of the axis of symmetry are graphed.

x	$x^2 + 8x + 9$	$f(x)$	$(x, f(x))$
-6	$(-6)^2 + 8(-6) + 9$	-3	$(-6, -3)$
-5	$(-5)^2 + 8(-5) + 9$	-6	$(-5, -6)$
-4	$(-4)^2 + 8(-4) + 9$	-7	$(-4, -7)$ ← Vertex
-3	$(-3)^2 + 8(-3) + 9$	-6	$(-3, -6)$
-2	$(-2)^2 + 8(-2) + 9$	-3	$(-2, -3)$

c. Use this information to graph the function.

Graph the vertex and y-intercept. Then graph the points from your table, connecting them and the y-intercept with a smooth curve.

As a check, draw the axis of symmetry, $x = -4$, as a dashed line. The graph of the function should be symmetrical about this line.



CHECK Your Progress

Consider the quadratic function $g(x) = 3 - 6x + x^2$.

2A. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

2B. Make a table of values that includes the vertex.

2C. Use this information to graph the function.

Maximum and Minimum Values The y-coordinate of the vertex of a quadratic function is the **maximum value** or **minimum value** attained by the function.

Study Tip

Domain

The domain of a quadratic function is all real numbers.

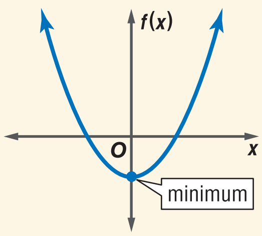
KEY CONCEPT *Maximum and Minimum Value*

Words The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

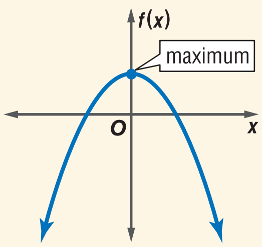
- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.
- The range of a quadratic function is all real numbers greater than or equal to the minimum, or all real numbers less than or equal to the maximum.

Models

***a* is positive.**



***a* is negative.**



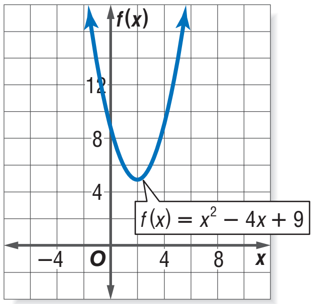
EXAMPLE 3 Maximum or Minimum Value

3 **Consider the function $f(x) = x^2 - 4x + 9$.**

a. Determine whether the function has a maximum or a minimum value.

For this function, $a = 1$, $b = -4$, and $c = 9$. Since $a > 0$, the graph opens up and the function has a minimum value.

b. State the maximum or minimum value of the function.



The minimum value of the function is the y-coordinate of the vertex.

The x-coordinate of the vertex is $-\frac{-4}{2(1)}$ or 2.

Find the y-coordinate of the vertex by evaluating the function for $x = 2$.

$f(x) = x^2 - 4x + 9$ — Original function

$f(2) = (2)^2 - 4(2) + 9$ or 5 — $x = 2$

Therefore, the minimum value of the function is 5.

c. State the domain and range of the function.

The domain is all real numbers. The range is all reals greater than or equal to the minimum value. That is, $\{f(x) \mid f(x) \geq 5\}$.

Study Tip

Common Misconception

The terms *minimum point* and *minimum value* are not interchangeable. The minimum point is the set of coordinates that describe the location of the vertex. The minimum value of a function is the y-coordinate of the minimum point. It is the least value obtained when $f(x)$ is evaluated for all values of x.

CHECK Your Progress

Consider $g(x) = 2x^2 - 4x - 3$.

3A. Determine whether the function has a maximum or minimum value.

3B. State the maximum or minimum value of the function.

3C. What are the domain and range of the function?

When quadratic functions are used to model real-world situations, their maximum or minimum values can have real-world meaning.

Personal Tutor

Real-World EXAMPLE

4 **TOURISM** **A tour bus in Boston serves 400 customers a day. The charge is \$5 per person. The owner of the bus service estimates that the company would lose 10 passengers a day for each \$0.50 fare increase.**

a. How much should the fare be in order to maximize the income for the company?

Words The income is the number of passengers multiplied by the price per ticket.

Variables Let x = the number of \$0.50 fare increases.
Then $5 + 0.50x$ = the price per passenger and
$400 - 10x$ = the number of passengers.

Let $I(x)$ = income as a function of x.

(continued on the next page)

	The income	is	the number of passengers	multiplied by	the price per passenger.
Equation	$I(x)$	$=$	$(400 - 10x)$	$\cdot$	$(5 + 0.50x)$

$$= 400(5) + 400(0.50x) - 10x(5) - 10x(0.50x)$$

$$= 2000 + 200x - 50x - 5x^2 \quad \text{Multiply.}$$

$$= 2000 + 150x - 5x^2 \quad \text{Simplify.}$$

$$= -5x^2 + 150x + 2000 \quad \text{Rewrite in } ax^2 + bx + c \text{ form.}$$

$I(x)$ is a quadratic function with $a = -5$, $b = 150$, and $c = 2000$. Since $a < 0$, the function has a maximum value at the vertex of the graph. Use the formula to find the x-coordinate of the vertex.

$$x\text{-coordinate of the vertex} = -\frac{b}{2a} \quad \text{Formula for the } x\text{-coordinate of the vertex}$$

$$= -\frac{150}{2(-5)} \quad a = -5, b = 150$$

$$= 15 \quad \text{Simplify.}$$

This means the company should make 15 fare increases of \$0.50 to maximize its income. Thus, the ticket price should be 5 + 0.50(15) or \$12.50.

The domain of the function is all real numbers, but negative values of x would correspond to a decreased fare. Therefore, a value of 15 fare increases is reasonable.

Real-World Link

Known as "Beantown," Boston is the largest city and unofficial capital of New England.

Source: boston-online.com

b. What is the maximum income the company can expect to make?

To determine maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 15$.

$$I(x) = -5x^2 + 150x + 2000 \quad \text{Income function}$$

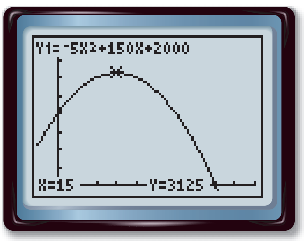
$$I(15) = -5(15)^2 + 150(15) + 2000 \quad x = 15$$

$$= 3125 \quad \text{Use a calculator.}$$

Thus, the maximum income the company can expect is \$3125. The increased fare would produce greater income. The income from the lower fare was \$5(400), or \$2000. So an answer of \$3125 is reasonable.

CHECK Graph this function on a graphing calculator and use the **CALC** menu to confirm this solution.

KEYSTROKES: 2nd [CALC] 4 0 ENTER 25 ENTER ENTER



[−5, 50] scl: 5 by [−100, 4000] scl: 500

At the bottom of the display are the coordinates of the maximum point on the graph. The y-value is the maximum value of the function, or 3125. The graph shows the range of the function as all reals less than or equal to 3125. ✓

CHECK Your Progress

4. Suppose that for each \$0.50 increase in the fare, the company will lose 8 passengers. Determine how much the fare should be in order to maximize the income, and then determine the maximum income.

Online **Personal Tutor at** algebra2.com

CHECK Your Understanding

Examples 1, 2
(pp. 236–238)

Complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**

b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.**

1. $f(x) = -4x^2$
2. $f(x) = x^2 + 2x$
3. $f(x) = -x^2 + 4x - 1$
4. $f(x) = x^2 + 8x + 3$
5. $f(x) = 2x^2 - 4x + 1$
6. $f(x) = 3x^2 + 10x$

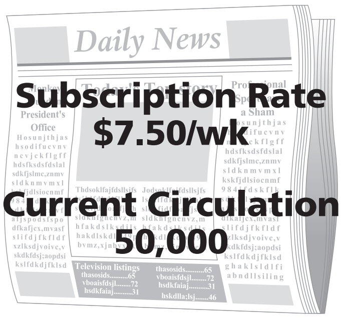
Example 3
(p. 239)

Determine whether each function has a maximum or a minimum value and find the maximum or minimum value. Then state the domain and range of the function.

7. $f(x) = -x^2 + 7$
8. $f(x) = x^2 - x - 6$
9. $f(x) = 4x^2 + 12x + 9$
10. $f(x) = -x^2 - 4x + 1$

Example 4
(pp. 239–240)

11. **NEWSPAPERS** Due to increased production costs, the *Daily News* must increase its subscription rate. According to a recent survey, the number of subscriptions will decrease by about 1250 for each 25¢ increase in the subscription rate. What weekly subscription rate will maximize the newspaper's income from subscriptions?



Exercises

HOMEWORK HELP	
For Exercises	See Examples
12–21	1, 2
22–31	3
32–36	4

Complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**

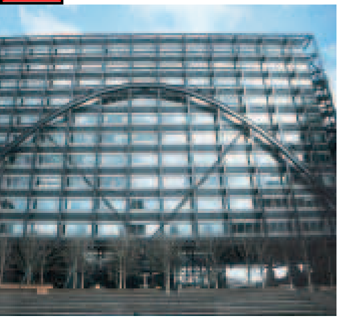
b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.**

12. $f(x) = 2x^2$
13. $f(x) = -5x^2$
14. $f(x) = x^2 + 4$
15. $f(x) = x^2 - 9$
16. $f(x) = 2x^2 - 4$
17. $f(x) = 3x^2 + 1$
18. $f(x) = x^2 - 4x + 4$
19. $f(x) = x^2 - 9x + 9$
20. $f(x) = x^2 - 4x - 5$
21. $f(x) = x^2 + 12x + 36$

Determine whether each function has a maximum or a minimum value and find the maximum or minimum value. Then state the domain and range of the function.

22. $f(x) = 3x^2$
23. $f(x) = -x^2 - 9$
24. $f(x) = x^2 - 8x + 2$
25. $f(x) = x^2 + 6x - 2$
26. $f(x) = 4x - x^2 + 1$
27. $f(x) = 3 - x^2 - 6x$
28. $f(x) = x^2 - 10x - 1$
29. $f(x) = x^2 + 8x + 15$
30. $f(x) = -x^2 + 12x - 28$
31. $f(x) = -14x - x^2 - 109$



Real-World Link

The Exchange House in London, England, is supported by two interior and two exterior steel arches. V-shaped braces add stability to the structure.

Source: Council on Tall Buildings and Urban Habitat

ARCHITECTURE **For Exercises 32 and 33, use the following information.**
The shape of each arch supporting the Exchange House can be modeled by $h(x) = -0.025x^2 + 2x$, where $h(x)$ represents the height of the arch and x represents the horizontal distance from one end of the base in meters.

32. Write the equation of the axis of symmetry and find the coordinates of the vertex of the graph of $h(x)$.

33. According to this model, what is the maximum height of the arch?

PHYSICS **For Exercises 34–36, use the following information.**
An object is fired straight up from the top of a 200-foot tower at a velocity of 80 feet per second. The height $h(t)$ of the object t seconds after firing is given by $h(t) = -16t^2 + 80t + 200$.

34. What are the domain and range of the function? What domain and range values are reasonable in the given situation?

35. Find the maximum height reached by the object and the time that the height is reached.

36. Interpret the meaning of the y-intercept in the context of this problem.

Complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**

b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.**

37. $f(x) = 3x^2 + 6x - 1$

38. $f(x) = -2x^2 + 8x - 3$

39. $f(x) = -3x^2 - 4x$

40. $f(x) = 2x^2 + 5x$

41. $f(x) = 0.5x^2 - 1$

42. $f(x) = -0.25x^2 - 3x$

43. $f(x) = \frac{1}{2}x^2 + 3x + \frac{9}{2}$

44. $f(x) = x^2 - \frac{2}{3}x - \frac{8}{9}$

Determine whether each function has a maximum or a minimum value and find the maximum or minimum value. Then state the domain and range of the function.

45. $f(x) = 2x + 2x^2 + 5$

46. $f(x) = x - 2x^2 - 1$

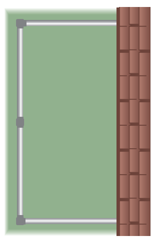
47. $f(x) = -7 - 3x^2 + 12x$

48. $f(x) = -20x + 5x^2 + 9$

49. $f(x) = -\frac{1}{2}x^2 - 2x + 3$

50. $f(x) = \frac{3}{4}x^2 - 5x - 2$

CONSTRUCTION **For Exercises 51–54, use the following information.**
Jaime has 120 feet of fence to make a rectangular kennel for his dogs. He will use his house as one side.



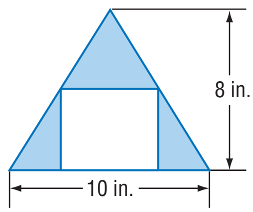
51. Write an algebraic expression for the kennel's length.

52. What are reasonable values for the domain of the area function?

53. What dimensions produce a kennel with the greatest area?

54. Find the maximum area of the kennel.

55. **GEOMETRY** A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (*Hint:* Use similar triangles.)



Aidan O'Rourke

FUND-RAISING **For Exercises 56 and 57, use the following information.**
Last year, 300 people attended the Sunnybrook High School Drama Club's winter play. The ticket price was \$8. The advisor estimates that 20 fewer people would attend for each \$1 increase in ticket price.

56. What ticket price would give the most income for the Drama Club?

57. If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

Graphing Calculator

MAXIMA AND MINIMA You can use the MINIMUM or MAXIMUM feature on a graphing calculator to find the minimum or maximum of a quadratic function. This involves defining an interval that includes the vertex of the parabola. A lower bound is an x-value left of the vertex, and an upper bound is an x-value right of the vertex.

Step 1 Graph the function so that the vertex of the parabola is visible.

Step 2 Select 3:minimum or 4:maximum from the CALC menu.

Step 3 Using the arrow keys, locate a left bound and press ENTER.

Step 4 Locate a right bound and press ENTER twice. The cursor appears on the maximum or minimum of the function. The maximum or minimum value is the y-coordinate of that point.

EXTRA PRACTICE
See page 899, 930.
Math Online
Self-Check Quiz at algebra2.com

Find the value of the maximum or minimum of each quadratic function to the nearest hundredth.

58. $f(x) = 3x^2 - 7x + 2$

59. $f(x) = -5x^2 + 8x$

60. $f(x) = 2x^2 - 3x + 2$

61. $f(x) = -6x^2 + 9x$

62. $f(x) = 7x^2 + 4x + 1$

63. $f(x) = -4x^2 + 5x$

H.O.T. Problems

64. **OPEN ENDED** Give an example of a quadratic function that has a domain of all real numbers and a range of all real numbers less than a maximum value. State the maximum value and sketch the graph of the function.

65. **CHALLENGE** Write an expression for the minimum value of a function of the form $y = ax^2 + c$, where $a > 0$. Explain your reasoning. Then use this function to find the minimum value of $y = 8.6x^2 - 12.5$.

66. ***Writing in Math*** Use the information on page 236 to explain how income from a rock concert can be maximized. Include an explanation of how to algebraically and graphically determine what ticket price should be charged to achieve maximum income.

STANDARDIZED TEST PRACTICE

67. **ACT/SAT** The graph of which of the following equations is symmetrical about the y-axis?

A $y = x^2 + 3x - 1$

B $y = -x^2 + x$

C $y = 6x^2 + 9$

D $y = 3x^2 - 3x + 1$

68. **REVIEW** In which equation does every real number x correspond to a nonnegative real number y?

F $y = -x^2$

G $y = -x$

H $y = x$

J $y = x^2$

Spiral Review

Solve each system of equations by using inverse matrices. (Lesson 4-8)

69. $2x + 3y = 8$
$x - 2y = -3$

70. $x + 4y = 9$
$3x + 2y = -3$

Find the inverse of each matrix, if it exists. (Lesson 4-7)

71. $\begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$

72. $\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$

Perform the indicated operation, if possible. (Lesson 4-5)

73. $\begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix}$

74. $\begin{bmatrix} 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 1 \\ -3 & 2 & 0 \end{bmatrix}$

Perform the indicated operations. (Lesson 4-2)

75. $\begin{bmatrix} 4 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 6 & -5 & 8 \end{bmatrix}$

76. $\begin{bmatrix} 2 & -5 & 7 \end{bmatrix} - \begin{bmatrix} -3 & 8 & -1 \end{bmatrix}$

77. $4\begin{bmatrix} -7 & 5 & -11 \\ 2 & -4 & 9 \end{bmatrix}$

78. $-2\begin{bmatrix} -3 & 0 & 12 \\ -7 & \frac{1}{3} & 4 \end{bmatrix}$

79. CONCERTS The price of two lawn seats and a pavilion seat at an outdoor amphitheater is \$75. The price of three lawn seats and two pavilion seats is \$130. How much do lawn and pavilion seats cost? (Lesson 3-2)

Solve each system of equations. (Lesson 3-2)

80. $4a - 3b = -4$
$3a - 2b = -4$

81. $2r + s = 1$
$r - s = 8$

82. $3x - 2y = -3$
$3x + y = 3$

83. Graph the system of equations $y = -3x$ and $y - x = 4$. State the solution. Is the system of equations *consistent and independent*, *consistent and dependent*, or *inconsistent*? (Lesson 3-1)

Find the slope of the line that passes through each pair of points. (Lesson 2-3)

84. $(6, 7), (0, -5)$

85. $(-3, -2), (-1, -4)$

86. $(-3, 2), (5, 6)$

87. $(-2, 8), (1, -7)$

88. $(3, 8), (7, 22)$

89. $(4, 21), (9, 12)$

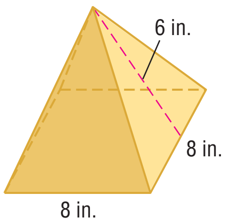
Solve each equation. Check your solutions. (Lesson 1-4)

90. $|x - 3| = 7$

91. $-4|d + 2| = -12$

92. $5|k - 4| = k + 8$

93. GEOMETRY The formula for the surface area of a regular pyramid is $S = \frac{1}{2}P\ell + B$ where P is the perimeter of the base, ℓ is the slant height of the pyramid, and B is the area of the base. Find the surface area of the pyramid shown. (Lesson 1-1)



GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each function for the given value. (Lesson 2-1)

94. $f(x) = x^2 + 2x - 3, x = 2$

95. $f(x) = -x^2 - 4x + 5, x = -3$

96. $f(x) = 3x^2 + 7x, x = -2$

97. $f(x) = \frac{2}{3}x^2 + 2x - 1, x = -3$

READING MATH

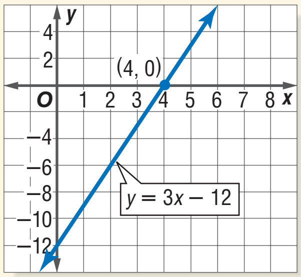
Roots of Equations and Zeros of Functions

The *solution* of an equation is called the *root* of the equation.

Example **Find the root of $0 = 3x - 12$.**

$0 = 3x - 12$	Original equation
$12 = 3x$	Add 12 to each side.
$4 = x$	Divide each side by 4.

The root of the equation is 4.



You can also find the root of an equation by finding the *zero* of its related function. Values of x for which $f(x) = 0$ are called *zeros* of the function f.

Linear Equation	Related Linear Function
$0 = 3x - 12$	$f(x) = 3x - 12$ or $y = 3x - 12$

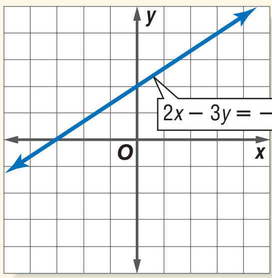
The zero of a function is the *x-intercept* of its graph. Since the graph of $y = 3x - 12$ intercepts the x-axis at 4, the zero of the function is 4.

You will learn about roots of quadratic equations and zeros of quadratic functions in Lesson 5-2.

Reading to Learn

1. Use $0 = 2x - 9$ and $f(x) = 2x - 9$ to distinguish among roots, solutions, and zeros.
2. Relate x-intercepts of graphs and solutions of equations.

Determine whether each statement is *true* or *false*. Explain your reasoning.



3. The function graphed at the right has two zeros, −3 and 2.
4. The root of $4x + 7 = 0$ is -1.75.
5. $f(0)$ is a zero of the function $f(x) = -\frac{1}{2}x + 5$.
6. **PONDS** The function $y = 24 - 2x$ represents the inches of water in a pond y after it is drained for x minutes. Find the zero and describe what it means in the context of this situation. Make a connection between the zero of the function and the root of $0 = 24 - 2x$.

5-2 Solving Quadratic Equations by Graphing

Main Ideas

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

New Vocabulary

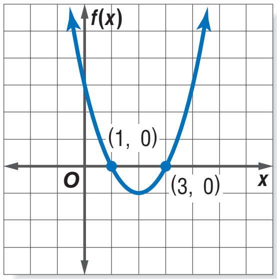
quadratic equation
standard form
root
zero

GET READY for the Lesson

As you speed to the top of a free-fall ride, you are pressed against your seat so that you feel like you're being pushed downward. Then as you free-fall, you fall at the same rate as your seat. Without the force of your seat pressing on you, you *feel* weightless. The height above the ground (in feet) of an object in free-fall can be determined by the quadratic function $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height is h_0 feet.

Solve Quadratic Equations When a quadratic function is set equal to a value, the result is a quadratic equation. A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. When a quadratic equation is written in this way, and a, b, and c are all integers, it is in **standard form**.

The solutions of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic equation is to find the **zeros** of the related quadratic function. The zeros of the function are the x-intercepts of its graph. These are the solutions of the related equation because $f(x) = 0$ at those points. The zeros of the function graphed at the right are 1 and 3.



Reading Math

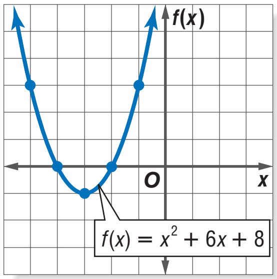
Roots, Zeros, Intercepts In general, equations have roots, functions have zeros, and graphs of functions have x-intercepts.

EXAMPLE 1 Two Real Solutions

Solve $x^2 + 6x + 8 = 0$ by graphing.

Graph the related quadratic function $f(x) = x^2 + 6x + 8$. The equation of the axis of symmetry is $x = -\frac{6}{2(1)}$ or -3. Make a table using x values around -3. Then, graph each point.

x	-5	-4	-3	-2	-1
$f(x)$	3	0	-1	0	3



We can see that the zeros of the function are -4 and -2. Therefore, the solutions of the equation are -4 and -2.

CHECK Your Progress **Solve each equation by graphing.**

1A. $x^2 - x - 6 = 0$ **1B.** $x^2 + x = 2$

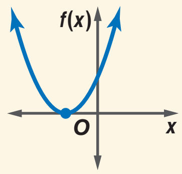
There are three possible outcomes when solving a quadratic equation.

KEY CONCEPT — Solutions of a Quadratic Equation

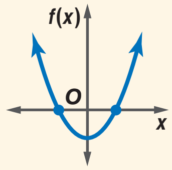
Words A quadratic equation can have one real solution, two real solutions, or no real solution.

Models

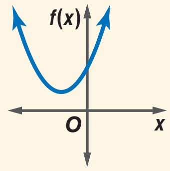
One Real Solution



Two Real Solutions



No Real Solution



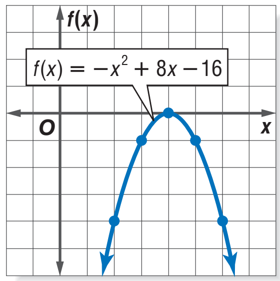
EXAMPLE 2 One Real Solution

Solve $8x - x^2 = 16$ by graphing.

$8x - x^2 = 16 \rightarrow -x^2 + 8x - 16 = 0$ Subtract 16 from each side.

Graph the related quadratic function $f(x) = -x^2 + 8x - 16$.

x	2	3	4	5	6
$f(x)$	−4	−1	0	−1	−4



Notice that the graph has only one x-intercept, 4. Thus, the equation's only solution is 4.

CHECK Your Progress Solve each equation by graphing.

2A. $10x = -25 - x^2$

2B. $-x^2 - 2x = 1$

Study Tip

One Real Solution

When a quadratic equation has one real solution, it really has two solutions that are the same number.

Personal Tutor

EXAMPLE 3 No Real Solution

NUMBER THEORY Find two real numbers with a sum of 6 and a product of 10 or show that no such numbers exist.

Explore Let x = one of the numbers. Then $6 - x$ = the other number.

Plan

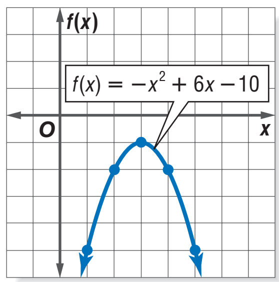
$x(6 - x) = 10$ The product is 10.

$6x - x^2 = 10$ Distributive Property

$-x^2 + 6x - 10 = 0$ Subtract 10 from each side.

Solve Graph the related function.

The graph has no x-intercepts. This means the original equation has no real solution. Thus, it is *not* possible for two numbers to have a sum of 6 and a product of 10.



Check Try finding the product of several pairs of numbers with sums of 6. Is each product less than 10 as the graph suggests?

CHECK Your Progress

3. Find two real numbers with a sum of 8 and a product of 12 or show that no such numbers exist.

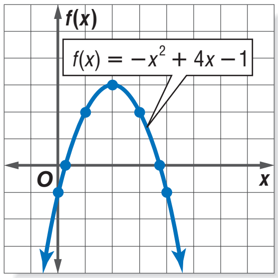
Online Personal Tutor at algebra2.com

Estimate Solutions Often exact roots cannot be found by graphing. You can estimate solutions by stating the integers between which the roots are located.

EXAMPLE 4 Estimate Roots

Solve $-x^2 + 4x - 1 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

x	0	1	2	3	4
$f(x)$	-1	2	3	2	-1



The x-intercepts of the graph indicate that one solution is between 0 and 1, and the other is between 3 and 4.

Study Tip

Location of Roots

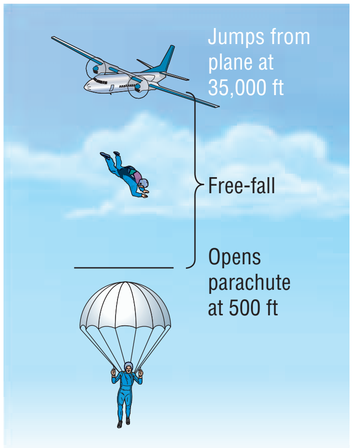
Notice in the table of values that the value of the function changes from negative to positive between the x-values of 0 and 1, and 3 and 4.

CHECK Your Progress

4. Solve $x^2 + 5x - 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

Real-World EXAMPLE 5

EXTREME SPORTS In 1999, Adrian Nicholas broke the world record for the longest human flight. He flew 10 miles from a drop point in 4 minutes 55 seconds using an aerodynamic suit. Using the information at the right and ignoring air resistance, how long would he have been in free-fall had he not used this suit? Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet.



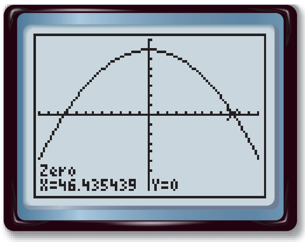
We need to find t when $h_0 = 35{,}000$ and $h(t) = 500$. Solve $500 = -16t^2 + 35{,}000$.

$500 = -16t^2 + 35{,}000$ Original equation

$0 = -16t^2 + 34{,}500$ Subtract 500 from each side.

Graph the related function $y = -16t^2 + 34{,}500$ on a graphing calculator.

Use the **Zero** feature, 2nd [CALC], to find the positive zero of the function, since time cannot be negative. Use the arrow keys to locate a left bound and press ENTER. Then, locate a right bound and press ENTER twice. The positive zero of the function is approximately 46.4. Mr. Nicholas would have been in free-fall for about 46 seconds.



[−60, 60] scl: 5 by [−40000, 40000] scl: 5000

CHECK Your Progress

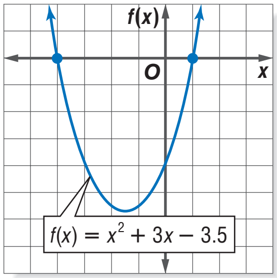
5. If Mr. Nicholas had jumped from the plane at 40,000 feet, how long would he have been in free-fall had he not used his special suit?

CHECK Your Understanding

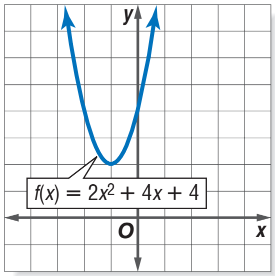
Examples 1–3
(pp. 246–247)

Use the related graph of each equation to determine its solutions.

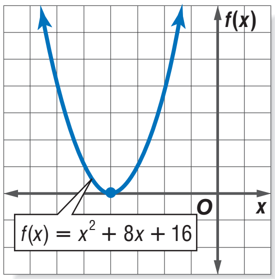
1. $x^2 + 3x - 3.5 = 0$



2. $2x^2 + 4x + 4 = 0$



3. $x^2 + 8x + 16 = 0$



Examples 1–4
(pp. 246–248)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $-x^2 - 7x = 0$

5. $x^2 - 2x - 24 = 0$

6. $25 + x^2 + 10x = 0$

7. $-14x + x^2 + 49 = 0$

8. $x^2 + 16x + 64 = -6$

9. $x^2 - 12x = -37$

10. $4x^2 - 7x - 15 = 0$

11. $2x^2 - 2x - 3 = 0$

Examples 1, 3
(pp. 246, 247)

12. NUMBER THEORY Use a quadratic equation to find two real numbers with a sum of 5 and a product of −14, or show that no such numbers exist.

Example 5
(p. 248)

13. ARCHERY An arrow is shot upward with a velocity of 64 feet per second. Ignoring the height of the archer, how long after the arrow is released does it hit the ground? Use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

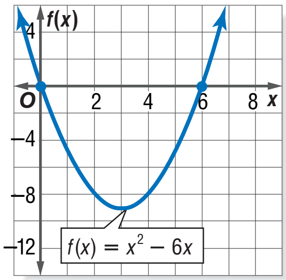
Exercises

HOMEWORK HELP

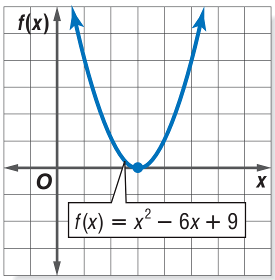
For Exercises	See Examples
14–19	1–3
20–29	1–4
30, 31	5

Use the related graph of each equation to determine its solutions.

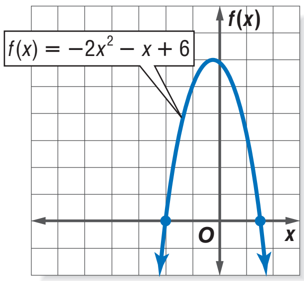
14. $x^2 - 6x = 0$



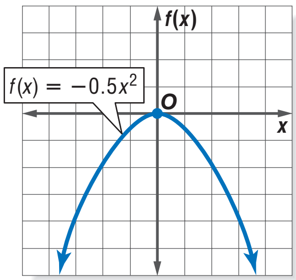
15. $x^2 - 6x + 9 = 0$



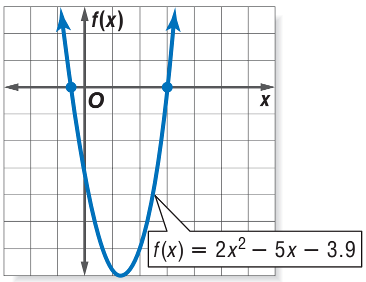
16. $-2x^2 - x + 6 = 0$



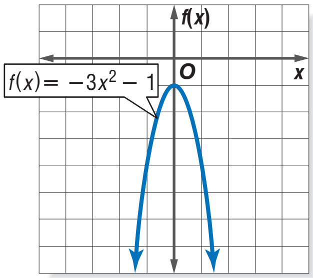
17. $-0.5x^2 = 0$



18. $2x^2 - 5x - 3.9 = 0$



19. $-3x^2 - 1 = 0$



Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

20. $x^2 - 3x = 0$

21. $-x^2 + 4x = 0$

22. $-x^2 + x = -20$

23. $x^2 - 9x = -18$

24. $14x + x^2 + 49 = 0$

25. $-12x + x^2 = -36$

26. $x^2 + 2x + 5 = 0$

27. $-x^2 + 4x - 6 = 0$

28. $x^2 + 4x - 4 = 0$

29. $x^2 - 2x - 1 = 0$

For Exercises 30 and 31, use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

30. **TENNIS** A tennis ball is hit upward with a velocity of 48 feet per second. Ignoring the height of the tennis player, how long does it take for the ball to fall to the ground?

31. **BOATING** A boat in distress launches a flare straight up with a velocity of 190 feet per second. Ignoring the height of the boat, how many seconds will it take for the flare to hit the water?

Real-World Link

Located on the 86th floor, 1050 feet (320 meters) above the streets of New York City, the Observatory offers panoramic views from within a glass-enclosed pavilion and from the surrounding open-air promenade.

Source: www.esbnyc.com

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

32. $2x^2 - 3x = 9$

33. $4x^2 - 8x = 5$

34. $2x^2 = -5x + 12$

35. $2x^2 = x + 15$

36. $x^2 + 3x - 2 = 0$

37. $x^2 - 4x + 2 = 0$

38. $-2x^2 + 3x + 3 = 0$

39. $0.5x^2 - 3 = 0$

NUMBER THEORY **Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.**

40. Their sum is -17 and their product is 72.

41. Their sum is 7 and their product is 14.

42. Their sum is -9 and their product is 24.

43. Their sum is 12 and their product is -28.

44. **LAW ENFORCEMENT** Police officers can use the length of skid marks to help determine the speed of a vehicle before the brakes were applied. If the skid marks are on dry concrete, the formula $\frac{s^2}{24} = d$ can be used. In the formula, s represents the speed in miles per hour and d represents the length of the skid marks in feet. If the length of the skid marks on dry concrete are 50 feet, how fast was the car traveling?

45. **PHYSICS** Suppose you could drop a small object from the Observatory of the Empire State Building. How long would it take for the object to reach the ground, assuming there is no air resistance? Use the information at the left and the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet.

EXTRA PRACTICE

See pages 900, 930.

Math Online

Self-Check Quiz at algebra2.com

H.O.T. Problems

46. **OPEN ENDED** Give an example of a quadratic equation with a double root, and state the relationship between the double root and the graph of the related function.

47. **REASONING** Explain how you can estimate the solutions of a quadratic equation by examining the graph of its related function.

48. CHALLENGE A quadratic function has values $f(-4) = -11$, $f(-2) = 9$, and $f(0) = 5$. Between which two x-values must $f(x)$ have a zero? Explain your reasoning.

49. ***Writing in Math*** Use the information on page 246 to explain how a quadratic function models a free-fall ride. Include a graph showing the height at any given time of a free-fall ride that lifts riders to a height of 185 feet and an explanation of how to use this graph to estimate how long the riders would be in free-fall if the ride were allowed to hit the ground before stopping.

STANDARDIZED TEST PRACTICE

50. ACT/SAT If one of the roots of the equation $x^2 + kx - 12 = 0$ is 4, what is the value of k?

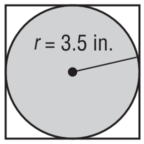
A -1

B 0

C 1

D 3

51. REVIEW What is the area of the square in square inches?



F 49

G 51

H 53

J 55

Spiral Review

Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for each quadratic function. Then graph the function by making a table of values. (Lesson 5-1)

52. $f(x) = x^2 - 6x + 4$

53. $f(x) = -4x^2 + 8x - 1$

54. $f(x) = \frac{1}{4}x^2 + 3x + 4$

55. Solve the system $4x - y = 0$, $2x + 3y = 14$ by using inverse matrices. (Lesson 4-8)

Evaluate the determinant of each matrix. (Lesson 4-3)

56. $\begin{bmatrix} 6 & 4 \\ -3 & 2 \end{bmatrix}$

57. $\begin{bmatrix} 2 & -1 & -6 \\ 5 & 0 & 3 \\ -3 & 2 & 11 \end{bmatrix}$

58. $\begin{bmatrix} 6 & 5 & 2 \\ -3 & 0 & -6 \\ 1 & 4 & 2 \end{bmatrix}$

59. COMMUNITY SERVICE A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised? (Lesson 3-4)

GET READY for the Next Lesson

PREREQUISITE SKILL Factor completely. (p. 753)

60. $x^2 + 5x$

61. $x^2 - 100$

62. $x^2 - 11x + 28$

63. $x^2 - 18x + 81$

64. $3x^2 + 8x + 4$

65. $6x^2 - 14x - 12$

EXTEND 5-2

Graphing Calculator Lab

Modeling Using Quadratic Functions

ACTIVITY

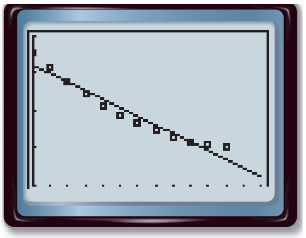
FALLING WATER **Water drains from a hole made in a 2-liter bottle. The table shows the level of the water y measured in centimeters from the bottom of the bottle after x seconds. Find and graph a linear regression equation and a quadratic regression equation. Determine which equation is a better fit for the data.**

Time (s)	0	20	40	60	80	100	120	140	160	180	200	220
Water level (cm)	42.6	40.7	38.9	37.2	35.8	34.3	33.3	32.3	31.5	30.8	30.4	30.1

Step 1 Find a linear regression equation.

- Enter the times in L1 and the water levels in L2. Then find a linear regression equation. Graph a scatter plot and the equation.

KEYSTROKES: *Review lists and finding and graphing a linear regression equation on page 92.*



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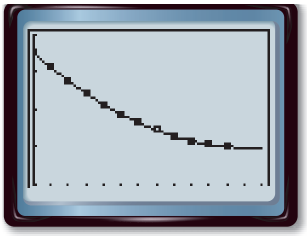
Step 2 Find a quadratic regression equation.

- Find the quadratic regression equation. Then copy the equation to the Y= list and graph.

KEYSTROKES: [STAT] [▶] 5 [ENTER] [Y=] [VARS] 5 [▶] [▶] [ENTER] [GRAPH]



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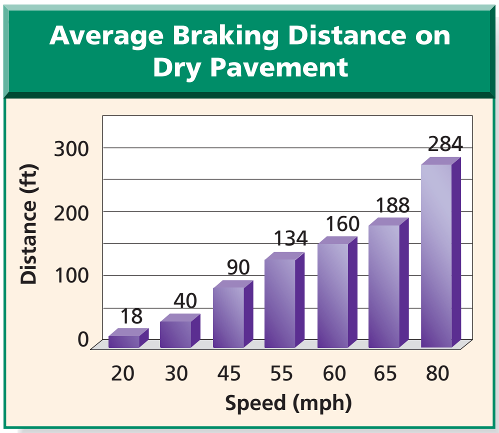
The graph of the linear regression equation appears to pass through just two data points. However, the graph of the quadratic regression equation fits the data very well.

Exercises

For Exercises 1–4, use the graph of the braking distances for dry pavement.

1. Find and graph a linear regression equation and a quadratic regression equation for the data. Determine which equation is a better fit for the data.
2. Use the CALC menu with each regression equation to estimate the braking distance at speeds of 100 and 150 miles per hour.
3. How do the estimates found in Exercise 2 compare?
4. How might choosing a regression equation that does not fit the data well affect predictions made by using the equation?



Source: Missouri Department of Revenue

Other Calculator Keystrokes at algebra2.com

5-3 Solving Quadratic Equations by Factoring

Main Ideas

- Write quadratic equations in intercept form.
- Solve quadratic equations by factoring.

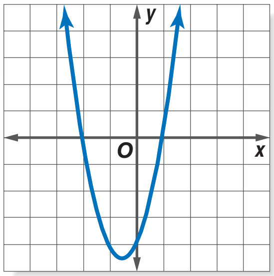
New Vocabulary

intercept form

FOIL method

GET READY for the Lesson

The **intercept form** of a quadratic equation is $y = a(x - p)(x - q)$. In the equation, p and q represent the x-intercepts of the graph corresponding to the equation. The intercept form of the equation shown in the graph is $y = 2(x - 1)(x + 2)$. The x-intercepts of the graph are 1 and -2. The standard form of the equation is $y = 2x^2 + 2x - 4$.



Intercept Form Changing a quadratic equation in intercept form to standard form requires the use of the FOIL method. The **FOIL method** uses the Distributive Property to multiply binomials.

KEY CONCEPT — FOIL Method for Multiplying Binomials

The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

To change $y = 2(x - 1)(x + 2)$ to standard form, use the FOIL method to find the product of $(x - 1)$ and $(x + 2)$, $x^2 + x - 2$, and then multiply by 2. The standard form of the equation is $y = 2x^2 + 2x - 4$.

You have seen that a quadratic equation of the form $(x - p)(x - q) = 0$ has roots p and q. You can use this pattern to find a quadratic equation for a given pair of roots.

EXAMPLE Write an Equation Given Roots

1 **Write a quadratic equation with $\frac{1}{2}$ and -5 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.**

$(x - p)(x - q) = 0$ Write the pattern.

$\left(x - \frac{1}{2}\right)[x - (-5)] = 0$ Replace p with $\frac{1}{2}$ and q with -5.

$\left(x - \frac{1}{2}\right)(x + 5) = 0$ Simplify.

$x^2 + \frac{9}{2}x - \frac{5}{2} = 0$ Use FOIL.

$2x^2 + 9x - 5 = 0$ Multiply each side by 2 so that b and c are integers.

Study Tip

Writing an Equation

The pattern $(x - p)(x - q) = 0$ produces one equation with roots p and q.

In fact, there are an infinite number of equations that have these same roots.

CHECK Your Progress

1. Write a quadratic equation with $-\frac{1}{3}$ and 4 as its roots. Write the equation in standard form.

Solve Equations by Factoring

In the last lesson, you learned to solve a quadratic equation by graphing. Another way to solve a quadratic equation is by factoring an equation in standard form. When an equation in standard form is factored and written in intercept form $y = a(x - p)(x - q)$, the solutions of the equation are p and q.

The following factoring techniques, or patterns, will help you factor polynomials. Then you can use the Zero Product Property to solve equations.

CONCEPT SUMMARY *Factoring Techniques*

Factoring Technique	General Case
Greatest Common Factor (GCF)	$a^3b^2 - 3ab^2 = ab^2(a^2 - 3)$
Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
General Trinomials	$acx^2 + (ad + bc)x + bd =$ $(ax + b)(cx + d)$

The FOIL method can help you factor a polynomial into the product of two binomials. Study the following example.

$$(ax + b)(cx + d) = \overbrace{ax \cdot cx}^{F} + \overbrace{ax \cdot d}^{O} + \overbrace{b \cdot cx}^{I} + \overbrace{b \cdot d}^{L}$$
$$= acx^2 + (ad + bc)x + bd$$

Notice that the product of the coefficient of x^2 and the constant term is $abcd$. The product of the two terms in the coefficient of x is also $abcd$.

EXAMPLE 2 Two or Three Terms

Factor each polynomial.

a. $5x^2 - 13x + 6$

To find the coefficients of the x-terms, you must find two numbers with a product of $5 \cdot 6$ or 30, and a sum of -13. The two coefficients must be -10 and -3 since $(-10)(-3) = 30$ and $-10 + (-3) = -13$.

Rewrite the expression using $-10x$ and $-3x$ in place of $-13x$ and factor by grouping.

$5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$ Substitute $-10x - 3x$ for $-13x$.
$= (5x^2 - 10x) + (-3x + 6)$ Associative Property
$= 5x(x - 2) - 3(x - 2)$ Factor out the GCF of each group.
$= (5x - 3)(x - 2)$ Distributive Property

b. $m^6 - n^6$

$m^6 - n^6 = (m^3 + n^3)(m^3 - n^3)$ Difference of two squares
$= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$ Sum and difference of two cubes

Study Tip

The difference of two squares should always be done before the difference of two cubes. This will make the next step of the factorization easier.

CHECK Your Progress

2A. $3xy^2 - 48x$ **2B.** $c^3d^3 + 27$

Solving quadratic equations by factoring is an application of the **Zero Product Property**.

KEY CONCEPT *Zero Product Property*

Words	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.
Example	If $(x + 5)(x - 7) = 0$, then $x + 5 = 0$ or $x - 7 = 0$.

Personal Tutor

EXAMPLE Two Roots

3 **Solve $x^2 = 6x$ by factoring. Then graph.**

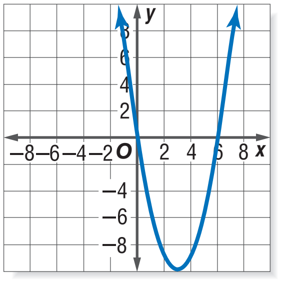
$x^2 = 6x$	Original equation
$x^2 - 6x = 0$	Subtract $6x$ from each side.
$x(x - 6) = 0$	Factor the binomial.
$x = 0$ or $x - 6 = 0$	Zero Product Property
$x = 6$	Solve the second equation.

The solution set is $\{0, 6\}$.

To complete the graph, find the vertex. Use the equation for the axis of symmetry.

$x = -\frac{b}{2a}$	Equation of the axis of symmmetry
$= -\frac{-6}{2}(1)$	$a = 1, b = -6$
$= 3$	Simplify.

Therefore, the x-coordinate of the vertex is 3. Substitute 3 into the equation to find the y-value.



$y = x^2 - 6x$	Original equation
$= 3^2 - 6(3)$	$x = 3$
$= 9 - 18$	Simplify.
$= -9$	Subtract.

The vertex is at $(3, -9)$. Graph the x-intercepts $(0, 0)$ and $(6, 0)$ and the vertex $(3, -9)$, connecting them with a smooth curve.

CHECK Your Progress **Solve each equation by factoring. Then graph.**

3A. $3x^2 = 9x$ **3B.** $6x^2 = 1 - x$

Online Personal Tutor at algebra2.com

Study Tip

Double Roots

The application of the Zero Product Property produced two identical equations, $x - 8 = 0$, both of which have a root of 8. For this reason, 8 is called the *double root* of the equation.

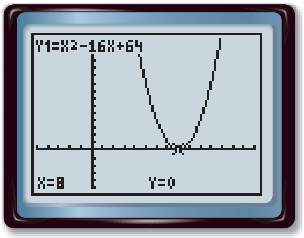
EXAMPLE Double Root

4 **Solve $x^2 - 16x + 64 = 0$ by factoring.**

$x^2 - 16x + 64 = 0$	Original equation
$(x - 8)(x - 8) = 0$	Factor.
$x - 8 = 0$ or $x - 8 = 0$	Zero Product Property
$x = 8$ $x = 8$	Solve each equation.

The solution set is $\{8\}$. *(continued on the next page)*

CHECK The graph of the related function, $f(x) = x^2 - 16x + 64$, intersects the x-axis only once. Since the zero of the function is 8, the solution of the related equation is 8.



CHECK Your Progress

Solve each equation by factoring.

4A. $x^2 + 12x + 36 = 0$ **4B.** $x^2 - 25 = 0$

CHECK Your Understanding

Example 1 (p. 253)

Write a quadratic equation with the given root(s). Write the equation in standard form.

1. $-4, 7$ **2.** $\frac{1}{2}, \frac{4}{3}$ **3.** $-\frac{3}{5}, -\frac{1}{3}$

Example 2 (p. 254)

Factor each polynomial.

4. $x^3 - 27$ **5.** $4xy^2 - 16x$ **6.** $3x^2 + 8x + 5$

Examples 3, 4 (pp. 255–256)

Solve each equation by factoring. Then graph.

7. $x^2 - 11x = 0$ **8.** $x^2 + 6x - 16 = 0$ **9.** $4x^2 - 13x = 12$

10. $x^2 - 14x = -49$ **11.** $x^2 + 9 = 6x$ **12.** $x^2 - 3x = -\frac{9}{4}$

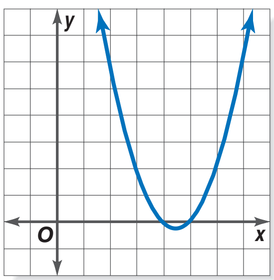
Exercises

HOMEWORK HELP

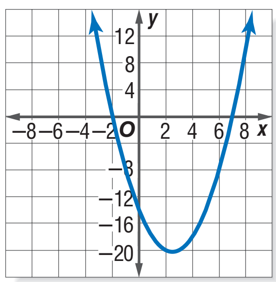
For Exercises	See Examples
13–16	1
17–20	2
21–32	3, 4

Write a quadratic equation in standard form for each graph.

13.



14.



Write a quadratic equation in standard form with the given roots.

15. $4, -5$ **16.** $-6, -8$

Factor each polynomial.

17. $x^2 - 7x + 6$ **18.** $x^2 + 8x - 9$

19. $3x^2 + 12x - 63$ **20.** $5x^2 - 80$

Solve each equation by factoring. Then graph.

21. $x^2 + 5x - 24 = 0$ **22.** $x^2 - 3x - 28 = 0$

23. $x^2 = 25$ **24.** $x^2 = 81$

25. $x^2 + 3x = 18$ **26.** $x^2 - 4x = 21$

27. $-2x^2 + 12x - 16 = 0$ **28.** $-3x^2 - 6x + 9 = 0$

29. $x^2 + 36 = 12x$ **30.** $x^2 + 64 = 16x$

31. NUMBER THEORY Find two consecutive even integers with a product of 224.

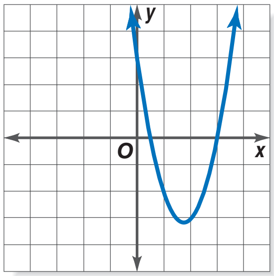
32. **PHOTOGRAPHY** A rectangular photograph is 8 centimeters wide and 12 centimeters long. The photograph is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new photograph?

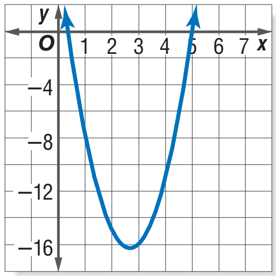
Solve each equation by factoring.

33. $3x^2 = 5x$
34. $4x^2 = -3x$
35. $4x^2 + 7x = 2$
36. $4x^2 - 17x = -4$
37. $4x^2 + 8x = -3$
38. $6x^2 + 6 = -13x$
39. $9x^2 + 30x = -16$
40. $16x^2 - 48x = -27$

41. Find the roots of $x(x + 6)(x - 5) = 0$.
42. Solve $x^3 = 9x$ by factoring.

Write a quadratic equation with the given graph or roots.

43.


44.


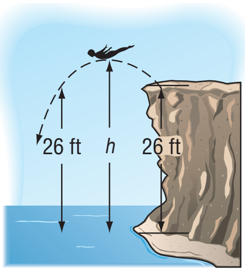
45. $-\frac{2}{3}, \frac{3}{4}$
46. $-\frac{3}{2}, -\frac{4}{5}$

Real-World Link

A board foot is a measure of lumber volume. One piece of lumber 1 foot long by 1 foot wide by 1 inch thick measures one board foot.

Source: www.wood-worker.com

47. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height h in feet t seconds after jumping. Find the time at which she returns to a height of 26 feet.



FORESTRY For Exercises 48 and 49, use the following information.

Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is the *Doyle Log Rule*, $B = \frac{L}{16}(D^2 - 8D + 16)$ where B is the number of board feet, D is the diameter in inches, and L is the length of the log in feet.

48. Rewrite Doyle's formula for logs that are 16 feet long.
49. Find the root(s) of the quadratic equation you wrote in Exercise 48. What do the root(s) tell you about the kinds of logs for which Doyle's rule makes sense?

EXTRA PRACTICE
See pages 900, 930.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

50. **FIND THE ERROR** Lina and Kristin are solving $x^2 + 2x = 8$. Who is correct? Explain your reasoning.

Lina

$x^2 + 2x = 8$
$x(x + 2) = 8$
$x = 8$ or $x + 2 = 8$
$x = 6$

Kristin

$x^2 + 2x = 8$
$x^2 + 2x - 8 = 0$
$(x + 4)(x - 2) = 0$
$x + 4 = 0$ or $x - 2 = 0$
$x = -4$ $\quad$ $x = 2$

51. **OPEN ENDED** Choose two integers. Then write an equation with those roots in standard form. How would the equation change if the signs of the two roots were switched?

52. **CHALLENGE** For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the x-intercepts p and q.

53. ***Writing in Math*** Use the information on page 253 to explain how to solve a quadratic equation using the Zero Product Property. Explain why you cannot solve $x(x + 5) = 24$ by solving $x = 24$ and $x + 5 = 24$.

STANDARDIZED TEST PRACTICE

54. **ACT/SAT** Which quadratic equation has roots $\frac{1}{2}$ and $\frac{1}{3}$?

A $5x^2 - 5x - 2 = 0$

B $5x^2 - 5x + 1 = 0$

C $6x^2 + 5x - 1 = 0$

D $6x^2 - 5x + 1 = 0$

55. **REVIEW** What is the solution set for the equation $3(4x + 1)^2 = 48$?

F $\left\{\frac{5}{4}, -\frac{3}{4}\right\}$

G $\left\{-\frac{5}{4}, \frac{3}{4}\right\}$

H $\left\{\frac{15}{4}, -\frac{17}{4}\right\}$

J $\left\{\frac{1}{3}, -\frac{4}{3}\right\}$

Spiral Review

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

56. $0 = -x^2 - 4x + 5$

57. $0 = 4x^2 + 4x + 1$

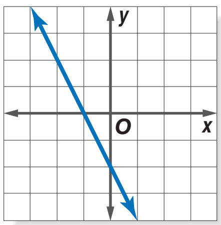
58. $0 = 3x^2 - 10x - 4$

59. Determine whether $f(x) = 3x^2 - 12x - 7$ has a maximum or a minimum value. Then find the maximum or minimum value. (Lesson 5-1)

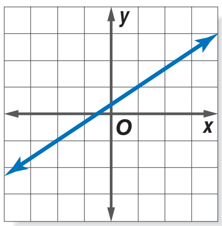
60. **CAR MAINTENANCE** Vince needs 12 quarts of a 60% anti-freeze solution. He will combine an amount of 100% anti-freeze with an amount of a 50% anti-freeze solution. How many quarts of each solution should be mixed to make the required amount of the 60% anti-freeze solution? (Lesson 4-8)

Write an equation in slope-intercept form for each graph. (Lesson 2-4)

61.



62.



GET READY for the Next Lesson

PREREQUISITE SKILL **Name the property illustrated by each equation.** (Lesson 1-2)

63. $2x + 4y + 3z = 2x + 3z + 4y$

64. $3(6x - 7y) = 3(6x) + 3(-7y)$

65. $(3 + 4) + x = 3 + (4 + x)$

66. $(5x)(-3y)(6) = (-3y)(6)(5x)$

5-4 Complex Numbers

Concepts in Motion

Interactive Lab algebra2.com

Main Ideas

- Find square roots and perform operations with pure imaginary numbers.
- Perform operations with complex numbers.

New Vocabulary

square root
imaginary unit
pure imaginary number
Square Root Property
complex number
complex conjugates

GET READY for the Lesson

Consider $2x^2 + 2 = 0$. One step in the solution of this equation is $x^2 = -1$. Since there is no real number that has a square of -1, there are no real solutions. French mathematician René Descartes (1596–1650) proposed that a number i be defined such that $i^2 = -1$.

Square Roots and Pure Imaginary Numbers A **square root** of a number n is a number with a square of n. For example, 7 is a square root of 49 because $7^2 = 49$. Since $(-7)^2 = 49$, -7 is also a square root of 49. Two properties will help you simplify expressions that contain square roots.

KEY CONCEPT — Product and Quotient Properties of Square Roots

Words For nonnegative real numbers a and b,

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, and

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $b \neq 0$.

Examples $\sqrt{3 \cdot 2} = \sqrt{3} \cdot \sqrt{2}$

$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}}$

Simplified square root expressions do not have radicals in the denominator, and any number remaining under the square root has no perfect square factor other than 1.

EXAMPLE 1 Properties of Square Roots

Simplify.

a. $\sqrt{50}$

$$\sqrt{50} = \sqrt{25 \cdot 2}$$
$$= \sqrt{25} \cdot \sqrt{2}$$
$$= 5\sqrt{2}$$

b. $\sqrt{\frac{11}{49}}$

$$\sqrt{\frac{11}{49}} = \frac{\sqrt{11}}{\sqrt{49}}$$
$$= \frac{\sqrt{11}}{7}$$

CHECK Your Progress

1A. $\sqrt{45}$

1B. $\sqrt{\frac{32}{81}}$

Since i is defined to have the property that $i^2 = -1$, the number i is the principal square root of -1; that is, $i = \sqrt{(-1)}$. i is called the **imaginary unit**. Numbers of the form $3i$, $-5i$, and $i\sqrt{2}$ are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

Reading Math

Imaginary Unit i is usually written before radical symbols to make it clear that it is not under the radical.

EXAMPLE 2 Square Roots of Negative Numbers

Simplify.

a. $\sqrt{-18}$

$$\sqrt{-18} = \sqrt{-1 \cdot 3^2 \cdot 2}$$
$$= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{2}$$
$$= i \cdot 3 \cdot \sqrt{2} \text{ or } 3i\sqrt{2}$$

b. $\sqrt{-125x^5}$

$$\sqrt{-125x^5} = \sqrt{-1 \cdot 5^2 \cdot x^4 \cdot 5x}$$
$$= \sqrt{-1} \cdot \sqrt{5^2} \cdot \sqrt{x^4} \cdot \sqrt{5x}$$
$$= i \cdot 5 \cdot x^2 \cdot \sqrt{5x} \text{ or } 5ix^2\sqrt{5x}$$

CHECK Your Progress

2A. $\sqrt{-27}$ **2B.** $\sqrt{-216y^4}$

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers.

EXAMPLE 3 Products of Pure Imaginary Numbers

Simplify.

a. $-2i \cdot 7i$

$$-2i \cdot 7i = -14i^2$$
$$= -14(-1) \quad i^2 = -1$$
$$= 14$$

b. $\sqrt{-10} \cdot \sqrt{-15}$

$$\sqrt{-10} \cdot \sqrt{-15} = i\sqrt{10} \cdot i\sqrt{15}$$
$$= i^2\sqrt{150}$$
$$= -1 \cdot \sqrt{25} \cdot \sqrt{6}$$
$$= -5\sqrt{6}$$

c. i^{45}

$$i^{45} = i \cdot i^{44} \quad \text{Multiplying powers}$$
$$= i \cdot (i^2)^{22} \quad \text{Power of a Power}$$
$$= i \cdot (-1)^{22} \quad i^2 = -1$$
$$= i \cdot 1 \text{ or } i \quad (-1)^{22} = 1$$

CHECK Your Progress

3A. $3i \cdot 4i$ **3B.** $\sqrt{-20} \cdot \sqrt{-12}$ **3C.** i^{31}

You can solve some quadratic equations by using the **Square Root Property**.

Reading Math

Plus or Minus $\pm\sqrt{n}$ is read *plus or minus the square root of n.*

KEY CONCEPT *Square Root Property*

For any real number n, if $x^2 = n$, then $x = \pm\sqrt{n}$.

EXAMPLE 4 Equation with Pure Imaginary Solutions

Solve $3x^2 + 48 = 0$.

$$3x^2 + 48 = 0 \quad \text{Original equation}$$
$$3x^2 = -48 \quad \text{Subtract 48 from each side.}$$
$$x^2 = -16 \quad \text{Divide each side by 3.}$$
$$x = \pm\sqrt{-16} \quad \text{Square Root Property}$$
$$x = \pm 4i \quad \sqrt{-16} = \sqrt{16} \cdot \sqrt{-1}$$

CHECK Your Progress **Solve each equation.**

4A. $4x^2 + 100 = 0$ **4B.** $x^2 + 4 = 0$

Operations with Complex Numbers Consider $5 + 2i$. Since 5 is a real number and $2i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

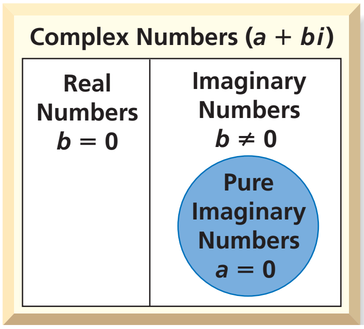
KEY CONCEPT — Complex Numbers

Words A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.

Examples $7 + 4i$ and $2 - 6i = 2 + (-6)i$

The Venn diagram shows the complex numbers.

- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

Reading Math

Complex Numbers The form $a + bi$ is sometimes called the *standard form* of a complex number.

EXAMPLE 5 Equate Complex Numbers

Find the values of x and y that make the equation $2x - 3 + (y - 4)i = 3 + 2i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x - 3 = 3$ Real parts
$2x = 6$ Add 3 to each side.
$x = 3$ Divide each side by 2.

$y - 4 = 2$ Imaginary parts
$y = 6$ Add 4 to each side.

CHECK Your Progress

5. Find the values of x and y that make the equation $5x + 1 + (3 + 2y)i = 2x - 2 + (y - 6)i$ true.

To add or subtract complex numbers, combine like terms. That is, combine the real parts and combine the imaginary parts.

EXAMPLE 6 Add and Subtract Complex Numbers

Simplify.

a. $(6 - 4i) + (1 + 3i)$

$(6 - 4i) + (1 + 3i) = (6 + 1) + (-4 + 3)i$ Commutative and Associative Properties

$= 7 - i$ Simplify.

b. $(3 - 2i) - (5 - 4i)$

$(3 - 2i) - (5 - 4i) = (3 - 5) + [-2 - (-4)]i$ Commutative and Associative Properties

$= -2 + 2i$ Simplify.

CHECK Your Progress

6A. $(-2 + 5i) + (1 - 7i)$ **6B.** $(4 + 6i) - (-1 + 2i)$

Study Tip

Complex Numbers

While all real numbers are also complex, the term *Complex Numbers* usually refers to a number that is not real.

One difference between real and complex numbers is that complex numbers cannot be represented by lines on a coordinate plane. However, complex numbers can be graphed on a *complex plane.* A complex plane is similar to a coordinate plane, except that the horizontal axis represents the real part a of the complex number, and the vertical axis represents the imaginary part b of the complex number.

You can also use a complex plane to model the addition of complex numbers.

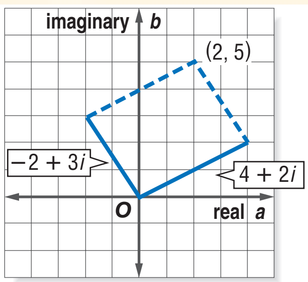
ALGEBRA LAB

Adding Complex Numbers Graphically

Use a complex plane to find $(4 + 2i) + (-2 + 3i)$.

- Graph $4 + 2i$ by drawing a segment from the origin to (4, 2) on the complex plane.
- Graph $-2 + 3i$ by drawing a segment from the origin to (−2 , 3) on the complex plane.
- Given three vertices of a parallelogram, complete the parallelogram.
- The fourth vertex at (2, 5) represents the complex number $2 + 5i$.

So, $(4 + 2i) + (-2 + 3i) = 2 + 5i$.



MODEL AND ANALYZE

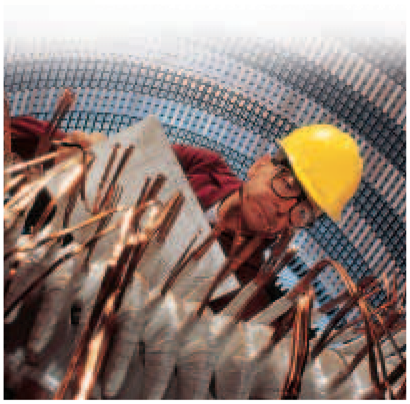
1. Model $(-3 + 2i) + (4 - i)$ on a complex plane.

2. Describe how you could model the difference $(-3 + 2i) - (4 - i)$ on a complex plane.

Complex numbers are used with electricity. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers. To multiply these numbers, use the FOIL method.

Study Tip

Electrical engineers use j as the imaginary unit to avoid confusion with the I for current.



Real-World Career

Electrical Engineer

The chips and circuits in computers are designed by electrical engineers.

Math Online

For more information, go to algebra2.com.

Real-World EXAMPLE

Personal Tutor

7 ELECTRICITY **In an AC circuit, the voltage E, current I, and impedance Z are related by the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 3j$ amps and impedance $7 - 5j$ ohms.**

$E = I \cdot Z$	Electricity formula
$= (1 + 3j) \cdot (7 - 5j)$	$I = 1 + 3j$, $Z = 7 - 5j$
$= 1(7) + 1(-5j) + (3j)7 + 3j(-5j)$	FOIL
$= 7 - 5j + 21j - 15j^2$	Multiply.
$= 7 + 16j - 15(-1)$	$j^2 = -1$
$= 22 + 16j$	Add.

The voltage is $22 + 16j$ volts.

CHECK Your Progress

7. Find the voltage in a circuit with current $2 - 4j$ amps and impedance $3 - 2j$ ohms.

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Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number. You can use this fact to simplify the quotient of two complex numbers.

EXAMPLE Divide Complex Numbers

8 Simplify.

a. $\dfrac{3i}{2 + 4i}$

$\dfrac{3i}{2 + 4i} = \dfrac{3i}{2 + 4i} \cdot \dfrac{2 - 4i}{2 - 4i}$	$2 + 4i$ and $2 + 4i$ are conjugates.
$= \dfrac{6i - 12i^2}{4 - 16i^2}$	Multiply.
$= \dfrac{6i + 12}{20}$	$i^2 = -1$
$= \dfrac{3}{5} + \dfrac{3}{10}i$	Standard form

b. $\dfrac{5 + i}{2i}$

$\dfrac{5 + i}{2i} = \dfrac{5 + i}{2i} \cdot \dfrac{i}{i}$	Why multiply by $\frac{i}{i}$ instead of $\frac{-2i}{-2i}$?
$= \dfrac{5i + i^2}{2i^2}$	Multiply.
$= \dfrac{5i - 1}{-2}$	$i^2 = -1$
$= \dfrac{1}{2} - \dfrac{5}{2}i$	Standard form

CHECK Your Progress

8A. $\dfrac{-2i}{3 + 5i}$ **8B.** $\dfrac{2 + i}{1 - i}$

CHECK Your Understanding

Examples 1–3
(pp. 259–260)

Simplify.

1. $\sqrt{56}$
2. $\sqrt{80}$
3. $\sqrt{\frac{48}{49}}$
4. $\sqrt{\frac{120}{9}}$
5. $\sqrt{-36}$
6. $\sqrt{-50x^2y^2}$
7. $(6i)(-2i)$
8. $5\sqrt{-24} \cdot 3\sqrt{-18}$
9. i^{29}
10. i^{80}

Example 4
(p. 260)

Solve each equation.

11. $2x^2 + 18 = 0$
12. $-5x^2 - 25 = 0$

Example 5
(p. 261)

Find the values of m and n that make each equation true.

13. $2m + (3n + 1)i = 6 - 8i$
14. $(2n - 5) + (-m - 2)i = 3 - 7i$

Example 6
(p. 262)

15. **ELECTRICITY** The current in one part of a series circuit is $4 - j$ amps. The current in another part of the circuit is $6 + 4j$ amps. Add these complex numbers to find the total current in the circuit.

Examples 7, 8
(p. 263)

Simplify.

16. $(-2 + 7i) + (-4 - 5i)$
17. $(8 + 6i) - (2 + 3i)$
18. $(3 - 5i)(4 + 6i)$
19. $(1 + 2i)(-1 + 4i)$
20. $\frac{2 - i}{5 + 2i}$
21. $\frac{3 + i}{1 + 4i}$

Exercises

HOMEWORK HELP

For Exercises	See Examples
22–25	1
26–29	2
30–33	3
34–37	6
38, 39, 50	7
40, 41, 51	8
42–45	4
46–49	5

Simplify.

22. $\sqrt{125}$
23. $\sqrt{147}$
24. $\sqrt{\frac{192}{121}}$
25. $\sqrt{\frac{350}{81}}$
26. $\sqrt{-144}$
27. $\sqrt{-81}$
28. $\sqrt{-64x^4}$
29. $\sqrt{-100a^4b^2}$
30. $(-2i)(-6i)(4i)$
31. $3i(-5i)^2$
32. i^{13}
33. i^{24}
34. $(5 - 2i) + (4 + 4i)$
35. $(-2 + i) + (-1 - i)$
36. $(15 + 3i) - (9 - 3i)$
37. $(3 - 4i) - (1 - 4i)$
38. $(3 + 4i)(3 - 4i)$
39. $(1 - 4i)(2 + i)$
40. $\frac{4i}{3 + i}$
41. $\frac{4}{5 + 3i}$

Solve each equation.

42. $5x^2 + 5 = 0$
43. $4x^2 + 64 = 0$
44. $2x^2 + 12 = 0$
45. $6x^2 + 72 = 0$

Find the values of m and n that make each equation true.

46. $8 + 15i = 2m + 3ni$
47. $(m + 1) + 3ni = 5 - 9i$
48. $(2m + 5) + (1 - n)i = -2 + 4i$
49. $(4 + n) + (3m - 7)i = 8 - 2i$

ELECTRICITY **For Exercises 50 and 51, use the formula $E = I \cdot Z$.**

50. The current in a circuit is $2 + 5j$ amps, and the impedance is $4 - j$ ohms. What is the voltage?

51. The voltage in a circuit is $14 - 8j$ volts, and the impedance is $2 - 3j$ ohms. What is the current?

52. Find the sum of $ix^2 - (2 + 3i)x + 2$ and $4x^2 + (5 + 2i)x - 4i$.

53. Simplify $[(3 + i)x^2 - ix + 4 + i] - [(-2 + 3i)x^2 + (1 - 2i)x - 3]$.

Simplify.

54. $\sqrt{-13} \cdot \sqrt{-26}$

55. $(4i)\left(\frac{1}{2}i\right)^2(-2i)^2$

56. i^{38}

57. $(3 - 5i) + (3 + 5i)$

58. $(7 - 4i) - (3 + i)$

59. $(-3 - i)(2 - 2i)$

60. $\dfrac{(10 + i)^2}{4 - i}$

61. $\dfrac{2 - i}{3 - 4i}$

62. $(-5 + 2i)(6 - i)(4 + 3i)$

63. $(2 + i)(1 + 2i)(3 - 4i)$

64. $\dfrac{5 - i\sqrt{3}}{5 + i\sqrt{3}}$

65. $\dfrac{1 - i\sqrt{2}}{1 + i\sqrt{2}}$

Solve each equation, and locate the complex solutions in the complex plane.

66. $-3x^2 - 9 = 0$

67. $-2x^2 - 80 = 0$

68. $\frac{2}{3}x^2 + 30 = 0$

69. $\frac{4}{5}x^2 + 1 = 0$

EXTRA PRACTICE
See pages 900, 930.
Math Online
Self-Check Quiz at algebra2.com

Find the values of m and n that make each equation true.

70. $(m + 2n) + (2m - n)i = 5 + 5i$

71. $(2m - 3n)i + (m + 4n) = 13 + 7i$

72. ELECTRICITY The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$. Add these complex numbers to find the total impedance in the circuit.

H.O.T. Problems

73. OPEN ENDED Write two complex numbers with a product of 10.

74. CHALLENGE Copy and complete the table. Explain how to use the exponent to determine the simplified form of any power of i.

Power of i	Simplified Expression
i^6	?
i^7	?
i^8	?
i^9	?
i^{10}	?
i^{11}	?
i^{12}	?
i^{13}	?

75. Which One Doesn't Belong? Identify the expression that does not belong with the other three. Explain your reasoning.

$(3i)^2$ | $(2i)(3i)(4i)$ | $(6 + 2i) - (4 + 2i)$ | $(2i)^4$

76. REASONING Determine if each statement is *true* or *false*. If false, find a counterexample.

a. Every real number is a complex number.

b. Every imaginary number is a complex number.

77. ***Writing in Math*** Use the information on page 261 to explain how complex numbers are related to quadratic equations. Explain how the a and c must be related if the equation $ax^2 + c = 0$ has complex solutions and give the solutions of the equation $2x^2 + 2 = 0$.

STANDARDIZED TEST PRACTICE

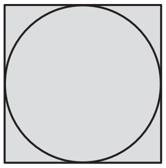
78. ACT/SAT The area of the square is 16 square units. What is the area of the circle?

A 2π units^2

B 12 units^2

C 4π units^2

D 16π units^2



79. If $i^2 = -1$, then what is the value of i^{71}?

F -1

G 0

H $-i$

J i

Spiral Review

Write a quadratic equation with the given root(s). Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers. (Lesson 5-3)

80. $-3, 9$

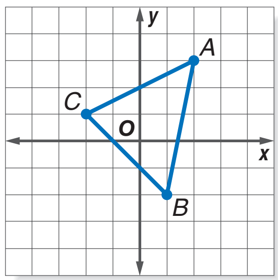
81. $-\frac{1}{3}, -\frac{3}{4}$

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

82. $3x^2 = 4 - 8x$

83. $2x^2 + 11x = -12$

Triangle ABC is reflected over the x-axis. (Lesson 4-4)



84. Write a vertex matrix for the triangle.

85. Write the reflection matrix.

86. Write the vertex matrix for $\triangle A'B'C'$.

87. Graph $\triangle A'B'C'$.

88. FURNITURE A new sofa, love seat, and coffee table cost $2050. The sofa costs twice as much as the love seat. The sofa and the coffee table together cost $1450. How much does each piece of furniture cost? (Lesson 3-5)

89. DECORATION Samantha is going to use more than 75 but less than 100 bricks to make a patio off her back porch. If each brick costs $2.75, write and solve a compound inequality to determine the amount she will spend on bricks. (Lesson 1-6)

GET READY for the Next Lesson

Determine whether each polynomial is a perfect square trinomial. (Lesson 5-3)

90. $x^2 - 10x + 16$

91. $x^2 + 18x + 81$

92. $x^2 - 9$

93. $x^2 - 12x - 36$

94. $x^2 - x + \frac{1}{4}$

95. $2x^2 - 15x + 25$

CHAPTER 5 Mid-Chapter Quiz

Lessons 5-1 through 5-4

1. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for $f(x) = 3x^2 - 12x + 4$. Then graph the function by making a table of values. (Lesson 5-1)

2. MULTIPLE CHOICE For which function is the x-coordinate of the vertex at 4? (Lesson 5-1)

A $f(x) = x^2 - 8x + 15$

B $f(x) = -x^2 - 4x + 12$

C $f(x) = x^2 + 6x + 8$

D $f(x) = -x^2 - 2x + 2$

3. Determine whether $f(x) = 3 - x^2 + 5x$ has a maximum or minimum value. Then find this maximum or minimum value and state the domain and range of the function. (Lesson 5-1)

4. BASEBALL From 2 feet above home plate, Grady hits a baseball upward with a velocity of 36 feet per second. The height $h(t)$ of the baseball t seconds after Grady hits it is given by $h(t) = -16t^2 + 36t + 2$. Find the maximum height reached by the baseball and the time that this height is reached. (Lesson 5-1)

5. Solve $2x^2 - 11x + 12 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. (Lesson 5-2)

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist. (Lesson 5-2)

6. Their sum is 12, and their product is 20.

7. Their sum is 5 and their product is 9.

8. MULTIPLE CHOICE For what value of x does $f(x) = x^2 + 5x + 6$ reach its minimum value? (Lesson 5-2)

F -5

G -3

H $-\frac{5}{2}$

J -2

9. FOOTBALL A place kicker kicks a ball upward with a velocity of 32 feet per second. Ignoring the height of the kicking tee, how long after the football is kicked does it hit the ground? Use the formula $h(t) = v_0t - 16t^2$ where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds. (Lesson 5-2)

Solve each equation by factoring. (Lesson 5-3)

10. $2x^2 - 5x - 3 = 0$

11. $6x^2 + 4x - 2 = 0$

12. $3x^2 - 6x - 24 = 0$

13. $x^2 + 12x + 20 = 0$

REMODELING For Exercises 14 and 15, use the following information. (Lesson 5-3)

Sandy'closet was supposed to be 10 feet by 12 feet. The architect decided that this would not work and reduced the dimensions by the same amount x on each side. The area of the new closet is 63 square feet.

14. Write a quadratic equation that represents the area of Sandy's closet now.

15. Find the new dimensions of her closet.

16. Write a quadratic equation in standard form with roots -4 and $\frac{1}{3}$. (Lesson 5-3)

Simplify. (Lesson 5-4)

17. $\sqrt{-49}$

18. $\sqrt{-36a^3b^4}$

19. $(28 - 4i) - (10 - 30i)$

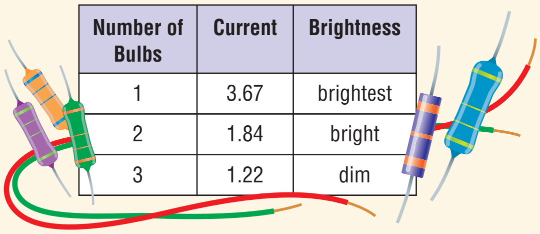
20. i^{89}

21. $(6 - 4i)(6 + 4i)$

22. $\frac{2 - 4i}{1 + 3i}$

23. ELECTRICITY The impedance in one part of a series circuit is $2 + 5j$ ohms and the impedance in another part of the circuit is $7 - 3j$ ohms. Add these complex numbers to find the total impedance in the circuit. (Lesson 5-4)

Series Circuit



Number of Bulbs	Current	Brightness
1	3.67	brightest
2	1.84	bright
3	1.22	dim

5-5 Completing the Square

Main Ideas

- Solve quadratic equations by using the Square Root Property.
- Solve quadratic equations by completing the square.

New Vocabulary

completing the square

GET READY for the Lesson

Under a yellow caution flag, race car drivers slow to a speed of 60 miles per hour. When the green flag is waved, the drivers can increase their speed.

Suppose the driver of one car is 500 feet from the finish line. If the driver accelerates at a constant rate of 8 feet per second squared, the equation $t^2 + 22t + 121 = 246$ represents the time t it takes the driver to reach this line. To solve this equation, you can use the Square Root Property.

Square Root Property You have solved equations like $x^2 - 25 = 0$ by factoring. You can also use the Square Root Property to solve such an equation. This method is useful with equations like the one above that describes the race car's speed. In this case, the quadratic equation contains a perfect square trinomial set equal to a constant.

EXAMPLE 1 Equation with Rational Roots

Solve $x^2 + 10x + 25 = 49$ by using the Square Root Property.

$x^2 + 10x + 25 = 49$	Original equation
$(x + 5)^2 = 49$	Factor the perfect square trinomial.
$x + 5 = \pm\sqrt{49}$	Square Root Property
$x + 5 = \pm 7$	$\sqrt{49} = 7$
$x = -5 \pm 7$	Add -5 to each side.
$x = -5 + 7$ or $x = -5 - 7$	Write as two equations.
$x = 2$ $\quad$ $x = -12$	Solve each equation.

The solution set is $\{2, -12\}$. You can check this result by using factoring to solve the original equation.

CHECK Your Progress

Solve each equation by using the Square Root Property.

1A. $x^2 - 12x + 36 = 25$ $\qquad$ **1B.** $x^2 - 16x + 64 = 49$

Roots that are irrational numbers may be written as exact answers in radical form or as *approximate* answers in decimal form when a calculator is used.

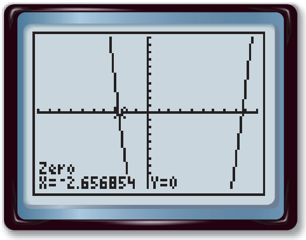
EXAMPLE Equation with Irrational Roots

2 **Solve $x^2 - 6x + 9 = 32$ by using the Square Root Property.**

$x^2 - 6x + 9 = 32$	Original equation
$(x - 3)^2 = 32$	Factor the perfect square trinomial.
$x - 3 = \pm\sqrt{32}$	Square Root Property
$x = 3 \pm 4\sqrt{2}$	Add 3 to each side; $-\sqrt{32} = 4\sqrt{2}$
$x = 3 + 4\sqrt{2}$ or $x = 3 - 4\sqrt{2}$	Write as two equations.
$x \approx 8.7$ $\quad$ $x \approx -2.7$	Use a calculator.

The exact solutions of this equation are $3 - 4\sqrt{2}$ and $3 + 4\sqrt{2}$. The approximate solutions are -2.7 and 8.7. Check these results by finding and graphing the related quadratic function.

$x^2 - 6x + 9 = 32$	Original equation
$x^2 - 6x - 23 = 0$	Subtract 32 from each side.
$y = x^2 - 6x - 23$	Related quadratic function



CHECK Use the ZERO function of a graphing calculator. The approximate zeros of the related function are -2.7 and 8.7.

Study Tip

Plus or Minus

When using the Square Root Property, remember to put a $\pm$ sign before the radical.

CHECK Your Progress

Solve each equation by using the Square Root Property.

2A. $x^2 + 8x + 16 = 20$ **2B.** $x^2 - 6x + 9 = 32$

Complete the Square

The Square Root Property can only be used to solve quadratic equations when the quadratic expression is a perfect square. However, few quadratic expressions are perfect squares. To make a quadratic expression a perfect square, a method called **completing the square** may be used.

In a perfect square trinomial, there is a relationship between the coefficient of the linear term and the constant term. Consider the following pattern.

$$(x + 7)^2 = x^2 + 2(7)x + 7^2 \quad \text{Square of a sum pattern}$$

$$= x^2 + 14x + 49 \quad \text{Simplify.}$$

$$\left(\frac{14}{2}\right)^2 \rightarrow 7^2 \quad \text{Notice that 49 is } 7^2 \text{ and 7 is one half of 14.}$$

Use this pattern of coefficients to complete the square of a quadratic expression.

KEY CONCEPT — *Completing the Square*

Words To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

Symbols $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

EXAMPLE Complete the Square

3 **Find the value of c that makes $x^2 + 12x + c$ a perfect square. Then write the trinomial as a perfect square.**

Step 1 Find one half of 12. $\frac{12}{2} = 6$

Step 2 Square the result of Step 1. $6^2 = 36$

Step 3 Add the result of Step 2 to $x^2 + 12x$. $x^2 + 12x + 36$

The trinomial $x^2 + 12x + 36$ can be written as $(x + 6)^2$.

CHECK Your Progress

3. Find the value of c that makes $x^2 - 14x + c$ a perfect square. Then write the trinomial as a perfect square.

Concepts in Motion
Animation
algebra2.com

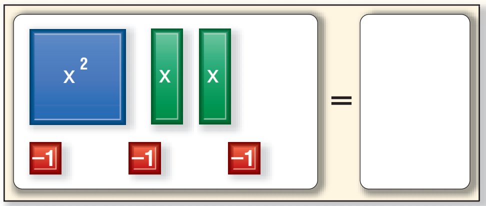
You can solve any quadratic equation by completing the square. Because you are solving an equation, add the value you use to complete the square to each side.

ALGEBRA LAB

Completing the Square

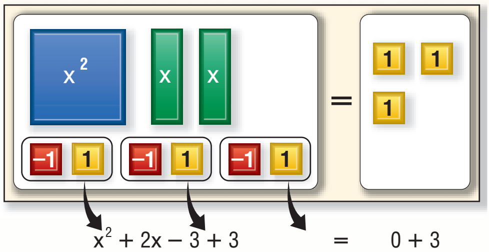
Use algebra tiles to complete the square for the equation $x^2 + 2x - 3 = 0$.

Step 1 Represent $x^2 + 2x - 3 = 0$ on an equation mat.



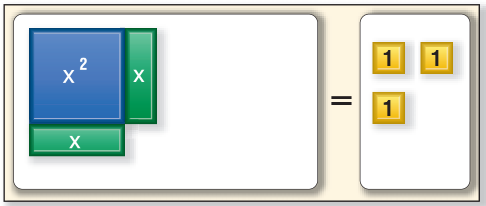
$x^2 + 2x - 3 = 0$

Step 2 Add 3 to each side of the mat. Remove the zero pairs.



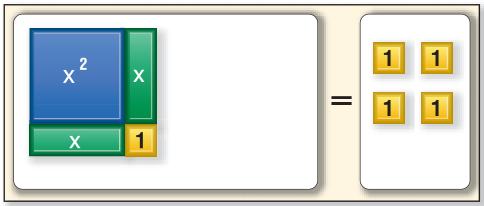
$x^2 + 2x - 3 + 3 = 0 + 3$

Step 3 Begin to arrange the x^2- and x-tiles into a square.



$x^2 + 2x = 3$

Step 4 To complete the square, add 1 yellow 1-tile to each side. The completed equation is $x^2 + 2x + 1 = 4$ or $(x + 1)^2 = 4$.



$x^2 + 2x + 1 = 3 + 1$

MODEL

Use algebra tiles to complete the square for each equation.

1. $x^2 + 2x - 4 = 0$

2. $x^2 + 4x + 1 = 0$

3. $x^2 - 6x = -5$

4. $x^2 - 2x = -1$

EXAMPLE Solve an Equation by Completing the Square

4 **Solve $x^2 + 8x - 20 = 0$ by completing the square.**

$x^2 + 8x - 20 = 0$	Notice that $x^2 + 8x - 20$ is not a perfect square.
$x^2 + 8x = 20$	Rewrite so the left side is of the form $x^2 + bx$.
$x^2 + 8x + 16 = 20 + 16$	Since $\left(\frac{8}{2}\right)^2 = 16$, add 16 to each side.
$(x + 4)^2 = 36$	Write the left side as a perfect square by factoring.
$x + 4 = \pm 6$	Square Root Property
$x = -4 \pm 6$	Add -4 to each side.
$x = -4 + 6$ or $x = -4 - 6$	Write as two equations.
$x = 2$ $\quad$ $x = -10$	The solution set is $\{-10, 2\}$.

You can check this result by using factoring to solve the original equation.

Study Tip

Common Misconception

When solving equations by completing the square, don't forget to add $\left(\frac{b}{2}\right)^2$ to *each* side of the equation.

CHECK Your Progress

Solve each equation by completing the square.

4A. $x^2 - 10x + 24 = 0$ **4B.** $x^2 + 10x + 9 = 0$

When the coefficient of the quadratic term is not 1, you must divide the equation by that coefficient before completing the square.

EXAMPLE Equation with $a \neq 1$

5 **Solve $2x^2 - 5x + 3 = 0$ by completing the square.**

$2x^2 - 5x + 3 = 0$	Notice that $2x^2 - 5x + 3$ is not a perfect square.
$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$	Divide by the coefficient of the quadratic term, 2.
$x^2 - \frac{5}{2}x = -\frac{3}{2}$	Subtract $\frac{3}{2}$ from each side.
$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{3}{2} + \frac{25}{16}$	Since $\left(-\frac{5}{2} \div 2\right)^2 = \frac{25}{16}$, add $\frac{25}{16}$ to each side.
$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$	Write the left side as a perfect square by factoring. Simplify the right side.
$x - \frac{5}{4} = \pm \frac{1}{4}$	Square Root Property
$x = \frac{5}{4} \pm \frac{1}{4}$	Add $\frac{5}{4}$ to each side.
$x = \frac{5}{4} + \frac{1}{4}$ or $x = \frac{5}{4} - \frac{1}{4}$	Write as two equations.
$x = \frac{3}{2}$ $\quad$ $x = 1$	The solution set is $\left\{1, \frac{3}{2}\right\}$.

Study Tip

Mental Math

Use mental math to find a number to add to each side to complete the square.

$\left(-\frac{5}{2} \div 2\right)^2 = \frac{25}{16}$

CHECK Your Progress

Solve each equation by completing the square.

5A. $3x^2 + 10x - 8 = 0$ **5B.** $3x^2 - 14x + 16 = 0$

Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex numbers of the form $a + bi$, where $b \neq 0$.

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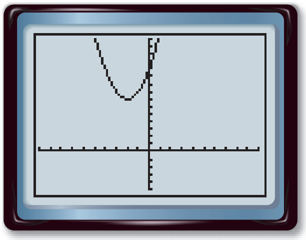
EXAMPLE Equation with Complex Solutions

6 **Solve $x^2 + 4x + 11 = 0$ by completing the square.**

$x^2 + 4x + 11 = 0$	Notice that $x^2 + 4x + 11$ is not a perfect square.
$x^2 + 4x = -11$	Rewrite so the left side is of the form $x^2 + bx$.
$x^2 + 4x + 4 = -11 + 4$	Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side.
$(x + 2)^2 = -7$	Write the left side as a perfect square by factoring.
$x + 2 = \pm\sqrt{-7}$	Square Root Property
$x + 2 = \pm\, i\sqrt{7}$	$\sqrt{-1} = i$
$x = -2 \pm i\sqrt{7}$	Subtract 2 from each side.

The solution set is $\{-2 + i\sqrt{7}, -2 - i\sqrt{7}\}$. Notice that these are imaginary solutions.

CHECK A graph of the related function shows that the equation has no real solutions since the graph has no x-intercepts. Imaginary solutions must be checked algebraically by substituting them in the original equation.



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CHECK Your Progress

Solve each equation by completing the square.

6A. $x^2 + 2x + 2 = 0$ **6B.** $x^2 - 6x + 25 = 0$

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CHECK Your Understanding

Examples 1 and 2 (pp. 268–269)

Solve each equation by using the Square Root Property.

1. $x^2 + 14x + 49 = 9$ **2.** $x^2 - 12x + 36 = 25$

3. $x^2 + 16x + 64 = 7$ **4.** $9x^2 - 24x + 16 = 2$

Example 2 (p. 269)

ASTRONOMY For Exercises 5–7, use the following information.
The height h of an object t seconds after it is dropped is given by $h = -\frac{1}{2}gt^2 + h_0$, where h_0 is the initial height and g is the acceleration due to gravity. The acceleration due to gravity near Earth's surface is 9.8 m/s^2, while on Jupiter it is 23.1 m/s^2. Suppose an object is dropped from an initial height of 100 meters from the surface of each planet.

5. On which planet should the object reach the ground first?

6. Find the time it takes for the object to reach the ground on each planet to the nearest tenth of a second.

7. Do the times to reach the ground seem reasonable? Explain.

Example 3 (p. 270)

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

8. $x^2 - 12x + c$

9. $x^2 - 3x + c$

Examples 4–6 (pp. 271–272)

Solve each equation by completing the square.

10. $x^2 + 3x - 18 = 0$

11. $x^2 - 8x + 11 = 0$

12. $2x^2 - 3x - 3 = 0$

13. $3x^2 + 12x - 18 = 0$

14. $x^2 + 2x + 6 = 0$

15. $x^2 - 6x + 12 = 0$

Exercises

HOMEWORK HELP

For Exercises	See Examples
16–19, 40, 41	1
20–23	2
24–27	3
28–31	4
32–35	5
36–39	6

Solve each equation by using the Square Root Property.

16. $x^2 + 4x + 4 = 25$

17. $x^2 - 10x + 25 = 49$

18. $x^2 - 9x + \frac{81}{4} = \frac{1}{4}$

19. $x^2 + 7x + \frac{49}{4} = 4$

20. $x^2 + 8x + 16 = 7$

21. $x^2 - 6x + 9 = 8$

22. $x^2 + 12x + 36 = 5$

23. $x^2 - 3x + \frac{9}{4} = 6$

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

24. $x^2 + 16x + c$

25. $x^2 - 18x + c$

26. $x^2 - 15x + c$

27. $x^2 + 7x + c$

Solve each equation by completing the square.

28. $x^2 - 8x + 15 = 0$

29. $x^2 + 2x - 120 = 0$

30. $x^2 + 2x - 6 = 0$

31. $x^2 - 4x + 1 = 0$

32. $2x^2 + 3x - 5 = 0$

33. $2x^2 - 3x + 1 = 0$

34. $2x^2 + 7x + 6 = 0$

35. $9x^2 - 6x - 4 = 0$

36. $x^2 - 4x + 5 = 0$

37. $x^2 + 6x + 13 = 0$

38. $x^2 - 10x + 28 = 0$

39. $x^2 + 8x + 9 = -9$

40. MOVIE SCREENS The area A in square feet of a projected picture on a movie screen is given by $A = 0.16d^2$, where d is the distance from the projector to the screen in feet. At what distance will the projected picture have an area of 100 square feet?

41. FRAMING A picture has a square frame that is 2 inches wide. The area of the picture is one third of the total area of the picture and frame. What are the dimensions of the picture to the nearest quarter of an inch?

Solve each equation by using the Square Root Property.

42. $x^2 + x + \frac{1}{4} = \frac{9}{16}$

43. $x^2 + 1.4x + 0.49 = 0.81$

44. $4x^2 - 28x + 49 = 5$

45. $9x^2 + 30x + 25 = 11$

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

46. $x^2 + 0.6x + c$

47. $x^2 - 2.4x + c$

48. $x^2 - \frac{8}{3}x + c$

49. $x^2 + \frac{5}{2}x + c$

Solve each equation by completing the square.

50. $x^2 + 1.4x = 1.2$

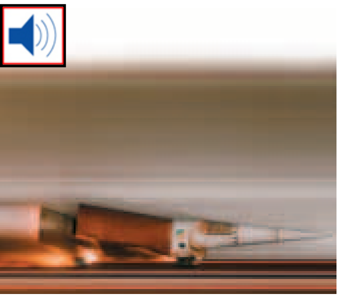
51. $x^2 - 4.7x = -2.8$

52. $x^2 - \frac{2}{3}x - \frac{26}{9} = 0$

53. $x^2 - \frac{3}{2}x - \frac{23}{16} = 0$

54. $3x^2 - 4x = 2$

55. $2x^2 - 7x = -12$



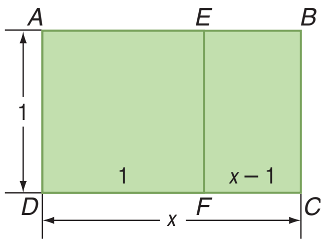
Real-World Link

Reverse ballistic testing—accelerating a target on a sled to impact a stationary test item at the end of the track—was pioneered at the Sandia National Laboratories' Rocket Sled Track Facility in Albuquerque, New Mexico. This facility provides a 10,000-foot track for testing items at very high speeds.

Source: sandia.gov

56. ENGINEERING In an engineering test, a rocket sled is propelled into a target. The sled's distance d in meters from the target is given by the formula $d = -1.5t^2 + 120$, where t is the number of seconds after rocket ignition. How many seconds have passed since rocket ignition when the sled is 10 meters from the target?

GOLDEN RECTANGLE For Exercises 57–59, use the following information.
A *golden rectangle* is one that can be divided into a square and a second rectangle that is geometrically similar to the original rectangle. The ratio of the length of the longer side to the shorter side of a golden rectangle is called the *golden ratio*.

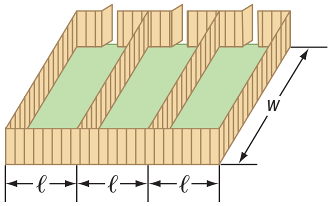


57. Find the ratio of the length of the longer side to the length of the shorter side for rectangle *ABCD* and for rectangle *EBCF*.

58. Find the exact value of the golden ratio by setting the two ratios in Exercise 57 equal and solving for x. (*Hint:* The golden ratio is a positive value.)

59. RESEARCH Use the Internet or other reference to find examples of the golden rectangle in architecture. What applications does the golden ratio have in music?

60. KENNEL A kennel owner has 164 feet of fencing with which to enclose a rectangular region. He wants to subdivide this region into three smaller rectangles of equal length, as shown. If the total area to be enclosed is 576 square feet, find the dimensions of the enclosed region. (*Hint:* Write an expression for ℓ in terms of w.)



EXTRA PRACTICE
See pages 901, 930.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

61. OPEN ENDED Write a perfect square trinomial equation in which the linear coefficient is negative and the constant term is a fraction. Then solve the equation.

62. FIND THE ERROR Rashid and Tia are solving $2x^2 - 8x + 10 = 0$ by completing the square. Who is correct? Explain your reasoning.

Rashid

$2x^2 - 8x + 10 = 0$
$2x^2 - 8x = -10$
$2x^2 - 8x + 16 = -10 + 16$
$(x - 4)^2 = 6$
$x - 4 = \pm\sqrt{6}$
$x = 4 \pm \sqrt{6}$

Tia

$2x^2 - 8x + 10 = 0$
$x^2 - 4x = 0 - 5$
$x^2 - 4x + 4 = -5 + 4$
$(x - 2)^2 = -1$
$x - 2 = \pm i$
$x = 2 \pm i$

63. REASONING Determine whether the value of c that makes $ax^2 + bx + c$ a perfect square trinomial is *sometimes*, *always*, or *never* negative. Explain your reasoning.

64. CHALLENGE Find all values of n such that $x^2 + bx + \left(\frac{b}{2}\right)^2 = n$ has

a. one real root. **b.** two real roots. **c.** two imaginary roots.

65. ***Writing in Math*** Use the information on page 268 to explain how you can find the time it takes an accelerating car to reach the finish line. Include an explanation of why $t^2 + 22t + 121 = 246$ cannot be solved by factoring and a description of the steps you would take to solve the equation.

STANDARDIZED TEST PRACTICE

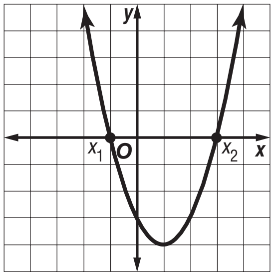
66. ACT/SAT The two zeros of a quadratic function are labeled x_1 and x_2 on the graph. Which expression has the greatest value?

A $2x_1$

B x_2

C $x_2 - x_1$

D $x_2 + x_1$



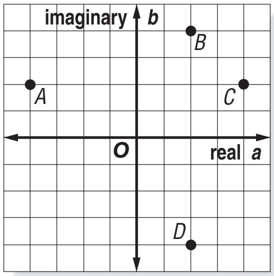
67. REVIEW If $i = \sqrt{-1}$ which point shows the location of $2 - 4i$ on the plane?

F point A

G point B

H point C

J point D



Spiral Review

Simplify. (Lesson 5-4)

68. i^{14} **69.** $(4 - 3i) - (5 - 6i)$ **70.** $(7 + 2i)(1 - i)$

Solve each equation by factoring. (Lesson 5-3)

71. $4x^2 + 8x = 0$ **72.** $x^2 - 5x = 14$ **73.** $3x^2 + 10 = 17x$

Solve each system of equations by using inverse matrices. (Lesson 4-8)

74. $5x + 3y = -5$
$7x + 5y = -11$

75. $6x + 5y = 8$
$3x - y = 7$

CHEMISTRY For Exercises 76 and 77, use the following information.
For hydrogen to be a liquid, its temperature must be within 2°C of −257°C. (Lesson 1-4)

76. Write an equation to determine the least and greatest temperatures for this substance.

77. Solve the equation.

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate $b^2 - 4ac$ for the given values of a, b, and c. (Lesson 1-1)

78. $a = 1, b = 7, c = 3$ **79.** $a = 1, b = 2, c = 5$

80. $a = 2, b = -9, c = -5$ **81.** $a = 4, b = -12, c = 9$

5-6 The Quadratic Formula and the Discriminant

Main Ideas

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number and type of roots of a quadratic equation.

New Vocabulary

Quadratic Formula
discriminant

GET READY for the Lesson

Competitors in the 10-meter platform diving competition jump upward and outward before diving into the pool below. The height h of a diver in meters above the pool after t seconds can be approximated by the equation $h = -4.9t^2 + 3t + 10$.

Quadratic Formula You have seen that exact solutions to some quadratic equations can be found by graphing, by factoring, or by using the Square Root Property. While completing the square can be used to solve any quadratic equation, the process can be tedious if the equation contains fractions or decimals. Fortunately, a formula exists that can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$. This formula can be derived by solving the general form of a quadratic equation.

$$ax^2 + bx + c = 0 \qquad \text{General quadratic equation}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{Divide each side by } a.$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Subtract } \frac{c}{a} \text{ from each side.}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \text{Complete the square.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{Factor the left side. Simplify the right side.}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Square Root Property}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Subtract } \frac{b}{2a} \text{ from each side.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Simplify.}$$

This equation is known as the **Quadratic Formula**.

Reading Math

Quadratic Formula The Quadratic Formula is read *x equals the opposite of b, plus or minus the square root of b squared minus 4ac, all divided by 2a.*

KEY CONCEPT — Quadratic Formula

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1 Two Rational Roots

Solve $x^2 - 12x = 28$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$\begin{array}{ccccccc} & & ax^2 & + & bx & + & c = 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ x^2 - 12x = 28 & \rightarrow & 1x^2 & - & 12x & - & 28 = 0 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)}$ Replace a with 1, b with -12, and c with -28.

$= \dfrac{12 \pm \sqrt{144 + 112}}{2}$ Simplify.

$= \dfrac{12 \pm \sqrt{256}}{2}$ Simplify.

$= \dfrac{12 \pm 16}{2}$ $\sqrt{256} = 16$

$x = \dfrac{12 + 16}{2}$ or $x = \dfrac{12 - 16}{2}$ Write as two equations.

$= 14$ $= -2$ Simplify.

The solutions are -2 and 14. Check by substituting each of these values into the original equation.

> **Study Tip**
>
> **Quadratic Formula**
>
> Although factoring may be an easier method to solve the equations in Examples 1 and 2, the Quadratic Formula can be used to solve any quadratic equation.

CHECK Your Progress

Solve each equation by using the Quadratic Formula.

1A. $x^2 + 6x = 16$ **1B.** $2x^2 + 25x + 33 = 0$

When the value of the radicand in the Quadratic Formula is 0, the quadratic equation has exactly one rational root.

EXAMPLE 2 One Rational Root

Solve $x^2 + 22x + 121 = 0$ by using the Quadratic Formula.

Identify a, b, and c. Then, substitute these values into the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

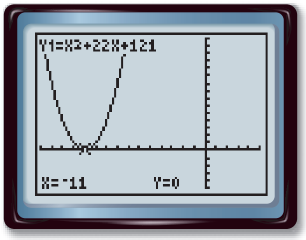
$= \dfrac{-22 \pm \sqrt{(22)^2 - 4(1)(121)}}{2(1)}$ Replace a with 1, b with 22, and c with 121.

$= \dfrac{-22 \pm \sqrt{0}}{2}$ Simplify.

$= \dfrac{-22}{2}$ or -11 $\sqrt{0} = 0$

The solution is -11.

CHECK A graph of the related function shows that there is one solution at $x = -11$.



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> **Study Tip**
>
> **Constants**
>
> The constants a, b, and c are not limited to being integers. They can be irrational or complex.

CHECK Your Progress

Solve each equation by using the Quadratic Formula.

2A. $x^2 - 16x + 64 = 0$ **2B.** $x^2 + 34x + 289 = 0$

You can express irrational roots exactly by writing them in radical form.

EXAMPLE 3 Irrational Roots

Solve $2x^2 + 4x - 5 = 0$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

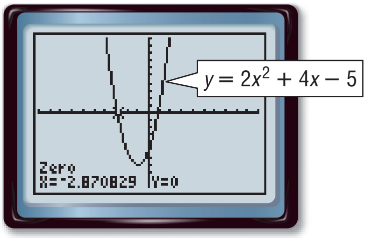
$$= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-5)}}{2(2)} \qquad \text{Replace } a \text{ with 2, } b \text{ with 4, and } c \text{ with } -5.$$

$$= \frac{-4 \pm \sqrt{56}}{4} \qquad \text{Simplify.}$$

$$= \frac{-4 \pm 2\sqrt{14}}{4} \text{ or } \frac{-2 \pm \sqrt{14}}{2} \qquad \sqrt{56} = \sqrt{4 \cdot 14} \text{ or } 2\sqrt{14}$$

The approximate solutions are -2.9 and 0.9.

CHECK Check these results by graphing the related quadratic function, $y = 2x^2 + 4x - 5$. Using the ZERO function of a graphing calculator, the approximate zeros of the related function are -2.9 and 0.9.



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CHECK Your Progress

Solve each equation by using the Quadratic Formula.

3A. $3x^2 + 5x + 1 = 0$ **3B.** $x^2 - 8x + 9 = 0$

When using the Quadratic Formula, if the radical contains a negative value, the solutions will be complex. Complex solutions of quadratic equations with real coefficients always appear in conjugate pairs.

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Study Tip

Using the Quadratic Formula

Remember that to correctly identify *a, b,* and *c* for use in the Quadratic Formula, the equation must be written in the form $ax^2 + bx + c = 0$.

EXAMPLE 4 Complex Roots

Solve $x^2 - 4x = -13$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic Formula}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} \qquad \text{Replace } a \text{ with 1, } b \text{ with } -4\text{, and } c \text{ with 13.}$$

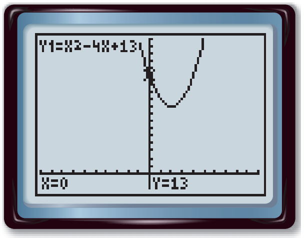
$$= \frac{4 \pm \sqrt{-36}}{2} \qquad \text{Simplify.}$$

$$= \frac{4 \pm 6i}{2} \qquad \sqrt{-36} = \sqrt{36(-1)} \text{ or } 6i$$

$$= 2 \pm 3i \qquad \text{Simplify.}$$

The solutions are the complex numbers $2 + 3i$ and $2 - 3i$.

A graph of the related function shows that the solutions are complex, but it cannot help you find them.



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CHECK The check for $2 + 3i$ is shown below.

$x^2 - 4x = -13$	Original equation
$(2 + 3i)^2 - 4(2 + 3i) \stackrel{?}{=} -13$	$x = 2 + 3i$
$4 + 12i + 9i^2 - 8 - 12i \stackrel{?}{=} -13$	Square of a sum; Distributive Property
$-4 + 9i^2 \stackrel{?}{=} -13$	Simplify.
$-4 - 9 = -13$ ✓	$i^2 = -1$

CHECK Your Progress

Solve each equation by using the Quadratic Formula.

4A. $3x^2 + 5x + 4 = 0$ **4B.** $x^2 - 6x + 10 = 0$

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Reading Math

Roots Remember that the solutions of an equation are called *roots*.

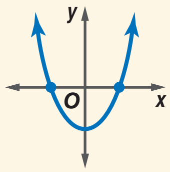
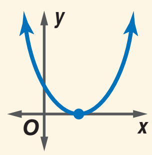
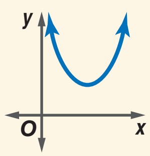
Roots and the Discriminant In Examples 1, 2, 3, and 4, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longleftarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation. The following table summarizes the possible types of roots.

KEY CONCEPT *Discriminant*

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real, rational root	
$b^2 - 4ac < 0$	2 complex roots	

The discriminant can help you check the solutions of a quadratic equation. Your solutions must match in number and in type to those determined by the discriminant.

EXAMPLE 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $9x^2 - 12x + 4 = 0$

$a = 9, b = -12, c = 4$ — Substitution

$b^2 - 4ac = (-12)^2 - 4(9)(4)$ — Simplify.

$= 144 - 144$ — Subtract.

$= 0$

The discriminant is 0, so there is one rational root.

b. $2x^2 - 16x + 33 = 0$

$a = 2, b = 16, c = 33$ — Substitution

$b^2 - 4ac = (16)^2 - 4(2)(33)$ — Simplify.

$= 256 - 264$ — Subtract.

$= -8$

The discriminant is negative, so there are two complex roots.

CHECK Your Progress

5A. $-5x^2 + 8x - 1 = 0$

5B. $-7x + 15x^2 - 4 = 0$

You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

CONCEPT SUMMARY — *Solving Quadratic Equations*

Method	Can be Used	When to Use
Graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
Factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 3x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x + 13)^2 = 9$
Completing the Square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 14x - 9 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $3.4x^2 - 2.5x + 7.9 = 0$

Study Tip

Study Notebook

You may wish to copy this list of methods to your math notebook or Foldable to keep as a reference as you study.

CHECK Your Understanding

Examples 1–4 (pp. 277–279)

Find the exact solutions by using the Quadratic Formula.

1. $8x^2 + 18x - 5 = 0$

2. $x^2 + 8x = 0$

3. $4x^2 + 4x + 1 = 0$

4. $x^2 + 6x + 9 = 0$

5. $2x^2 - 4x + 1 = 0$

6. $x^2 - 2x - 2 = 0$

7. $x^2 + 3x + 8 = 5$

8. $4x^2 + 20x + 25 = -2$

Examples 3 and 4 (pp. 278–279)

PHYSICS For Exercises 9 and 10, use the following information.
The height $h(t)$ in feet of an object t seconds after it is propelled straight up from the ground with an initial velocity of 85 feet per second is modeled by the equation $h(t) = -16t^2 + 85t$.

9. When will the object be at a height of 50 feet?

10. Will the object ever reach a height of 120 feet? Explain your reasoning.

Example 5 (p. 280)

Complete parts a and b for each quadratic equation.

a. **Find the value of the discriminant.**

b. **Describe the number and type of roots. Do your answers for Exercises 1, 3, 5, and 7 fit these descriptions, respectively?**

11. $8x^2 + 18x - 5 = 0$

12. $4x^2 + 4x + 1 = 0$

13. $2x^2 - 4x + 1 = 0$

14. $x^2 + 3x + 8 = 5$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
15, 16	1, 5
17, 18	2, 5
19–22	3, 5
23, 24	4, 5
25–33	1–4

Complete parts a–c for each quadratic equation.

a. **Find the value of the discriminant.**

b. **Describe the number and type of roots.**

c. **Find the exact solutions by using the Quadratic Formula.**

15. $-12x^2 + 5x + 2 = 0$

16. $-3x^2 - 5x + 2 = 0$

17. $9x^2 - 6x - 4 = -5$

18. $25 + 4x^2 = -20x$

19. $x^2 + 3x - 3 = 0$

20. $x^2 - 16x + 4 = 0$

21. $x^2 + 4x + 3 = 4$

22. $2x - 5 = -x^2$

23. $x^2 - 2x + 5 = 0$

24. $x^2 - x + 6 = 0$

Solve each equation by using the method of your choice. Find exact solutions.

25. $x^2 - 30x - 64 = 0$

26. $7x^2 + 3 = 0$

27. $x^2 - 4x + 7 = 0$

28. $2x^2 + 6x - 3 = 0$

29. $4x^2 - 8 = 0$

30. $4x^2 + 81 = 36x$

FOOTBALL For Exercises 31 and 32, use the following information.
The average NFL salary $A(t)$ (in thousands of dollars) can be estimated using $A(t) = 2.3t^2 - 12.4t + 73.7$, where t is the number of years since 1975.

31. Determine a domain and range for which this function makes sense.

32. According to this model, in what year did the average salary first exceed one million dollars?

33. **HIGHWAY SAFETY** Highway safety engineers can use the formula $d = 0.05s^2 + 1.1s$ to estimate the minimum stopping distance d in feet for a vehicle traveling s miles per hour. The speed limit on Texas highways is 70 mph. If a car is able to stop after 300 feet, was the car traveling faster than the Texas speed limit? Explain your reasoning.

Complete parts a–c for each quadratic equation.

a. **Find the value of the discriminant.**

b. **Describe the number and type of roots.**

c. **Find the exact solutions by using the Quadratic Formula.**

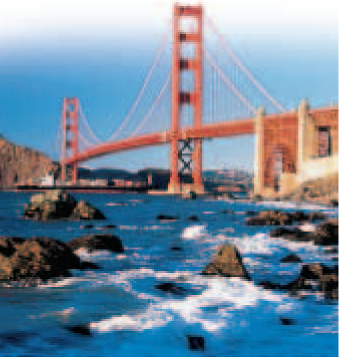
34. $x^2 + 6x = 0$ **35.** $4x^2 + 7 = 9x$ **36.** $3x + 6 = -6x^2$

37. $\frac{3}{4}x^2 - \frac{1}{3}x - 1 = 0$ **38.** $0.4x^2 + x - 0.3 = 0$ **39.** $0.2x^2 + 0.1x + 0.7 = 0$

Solve each equation by using the method of your choice. Find exact solutions.

40. $-4(x + 3)^2 = 28$ **41.** $3x^2 - 10x = 7$ **42.** $x^2 + 9 = 8x$

43. $10x^2 + 3x = 0$ **44.** $2x^2 - 12x + 7 = 5$ **45.** $21 = (x - 2)^2 + 5$



Real-World Link

The Golden Gate, located in San Francisco, California, is the tallest bridge in the world, with its towers extending 746 feet above the water and the floor of the bridge extending 220 feet above the water.

Source: www.goldengatebridge.org

BRIDGES **For Exercises 46 and 47, use the following information.**
The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by $y = 0.00012x^2 + 6$, where x represents the distance from the axis of symmetry and y represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$.

46. Calculate the value of the discriminant.

47. What does the discriminant tell you about the supporting cables of the Golden Gate Bridge?

48. **ENGINEERING** Civil engineers are designing a section of road that is going to dip below sea level. The road's curve can be modeled by the equation $y = 0.00005x^2 - 0.06x$, where x is the horizontal distance in feet between the points where the road is at sea level and y is the elevation (a positive value being above sea level and a negative being below). The engineers want to put stop signs at the locations where the elevation of the road is equal to sea level. At what horizontal distances will they place the stop signs?

H.O.T. Problems

EXTRA PRACTICE
See pages 901, 930.

Math Online
Self-Check Quiz at algebra2.com

49. **OPEN ENDED** Graph a quadratic equation that has a

a. positive discriminant. **b.** negative discriminant. **c.** zero discriminant.

50. **REASONING** Explain why the roots of a quadratic equation are complex if the value of the discriminant is less than 0.

51. **CHALLENGE** Find the exact solutions of $2ix^2 - 3ix - 5i = 0$ by using the Quadratic Formula.

52. **REASONING** Given the equation $x^2 + 3x - 4 = 0$,

a. Find the exact solutions by using the Quadratic Formula.

b. Graph $f(x) = x^2 + 3x - 4$.

c. Explain how solving with the Quadratic Formula can help graph a quadratic function.

53. ***Writing in Math*** Use the information on page 276 to explain how a diver's height above the pool is related to time. Explain how you could determine how long it will take the diver to hit the water after jumping from the platform.

STANDARDIZED TEST PRACTICE

54. ACT/SAT If $2x^2 - 5x - 9 = 0$, then x could be approximately equal to which of the following?

A -1.12

B 1.54

C 2.63

D 3.71

55. REVIEW What are the x-intercepts of the graph of $y = -2x^2 - 5x + 12$?

F $-\frac{3}{2}, 4$

G $-4, \frac{3}{2}$

H $-2, \frac{1}{2}$

J $-\frac{1}{2}, 2$

Spiral Review

Solve each equation by using the Square Root Property. (Lesson 5-5)

56. $x^2 + 18x + 81 = 25$

57. $x^2 - 8x + 16 = 7$

58. $4x^2 - 4x + 1 = 8$

Simplify. (Lesson 5-4)

59. $\frac{2i}{3 + i}$

60. $\frac{4}{5 - i}$

61. $\frac{1 + i}{3 - 2i}$

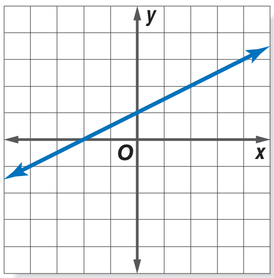
Solve each system of inequalities. (Lesson 3-3)

62. $x + y \leq 9$
$x - y \leq 3$
$y - x \geq 4$

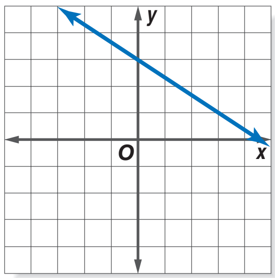
63. $x \geq 1$
$y \leq -1$
$y \leq x$

Write the slope-intercept form of the equation of the line with each graph shown. (Lesson 2-4)

64.



65.



66. PHOTOGRAPHY Desiree works in a photography studio and makes a commission of \$8 per photo package she sells. On Tuesday, she sold 3 more packages than she sold on Monday. For the two days, Victoria earned \$264. How many photo packages did she sell on these two days? (Lesson 1-3)

GET READY for the Next Lesson

PREREQUISITE SKILL **State whether each trinomial is a perfect square. If so, factor it.** (Lesson 5-3.)

67. $x^2 - 5x - 10$

68. $x^2 - 14x + 49$

69. $4x^2 + 12x + 9$

70. $25x^2 + 20x + 4$

71. $9x^2 - 12x + 16$

72. $36x^2 - 60x + 25$

EXPLORE 5-7

Graphing Calculator Lab

The Family of Parabolas

The general form of a quadratic function is $y = a(x - h)^2 + k$. Changing the values of a, h, and k results in a different parabola in the family of quadratic functions. The parent graph of the family of parabolas is the graph of $y = x^2$.

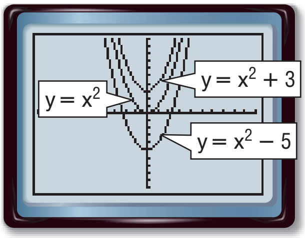
You can use a TI-83/84 Plus graphing calculator to analyze the effects that result from changing each of the parameters a, h, and k.

ACTIVITY 1

Graph the set of equations on the same screen in the standard viewing window.

$\boldsymbol{y = x^2, y = x^2 + 3, y = x^2 - 5}$

Describe any similarities and differences among the graphs.



[−10, 10] scl: 1 by [−10, 10] scl: 1

Activity 1 shows how changing the value of k in the equation $y = a(x - h)^2 + k$ *translates* the parabola along the y-axis. If $k > 0$, the parabola is translated k units up, and if $k < 0$, it is translated k units down.

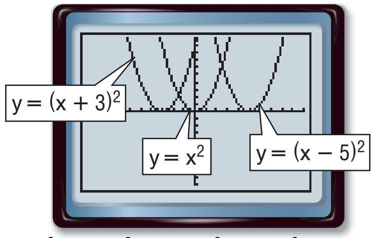
How do you think changing the value of h will affect the graph of $y = (x - h)^2$ as compared to the graph of $y = x^2$?

ACTIVITY 2

Graph the set of equations on the same screen in the standard viewing window.

$\boldsymbol{y = x^2, y = (x + 3)^2, \ y = (x - 5)^2}$

Describe any similarities and differences among the graphs.



[−10, 10] scl: 1 by [−10, 10] scl: 1

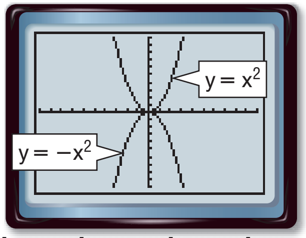
Activity 2 shows how changing the value of h in the equation $y = a(x - h)^2 + k$ *translates* the graph horizontally. If $h > 0$, the graph translates to the right h units. If $h < 0$, the graph translates to the left $|h|$ units.

ACTIVITY 3

Graph each set of equations on the same screen. Describe any similarities and differences among the graphs.

a. $y = x^2, y = -x^2$

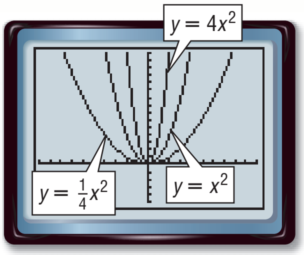
The graphs have the same vertex and the same shape. However, the graph of $y = x^2$ opens up and the graph of $y = -x^2$ opens down.



[−10, 10] scl: 1 by [−10, 10] scl: 1

b. $y = x^2, y = 4x^2, y = \frac{1}{4}x^2$

The graphs have the same vertex, (0, 0), but each has a different shape. The graph of $y = 4x^2$ is narrower than the graph of $y = x^2$. The graph of $y = \frac{1}{4}x^2$ is wider than the graph of $y = x^2$.



[−10, 10] scl: 1 by [−5, 15] scl: 1

Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph. If $a > 0$, the graph opens up, and if $a < 0$, the graph opens down or is *reflected* over the x-axis. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$. Thus, a change in the absolute value of a results in a *dilation* of the graph of $y = x^2$.

Analyze the Results

Consider $y = a(x - h)^2 + k$, where $a \neq 0$.

1. How does changing the value of h affect the graph? Give an example.
2. How does changing the value of k affect the graph? Give an example.
3. How does using $-a$ instead of a affect the graph? Give an example.

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs.

4. $y = x^2, y = x^2 + 2.5$
5. $y = -x^2, y = x^2 - 9$
6. $y = x^2, y = 3x^2$
7. $y = x^2, y = -6x^2$
8. $y = x^2, y = (x + 3)^2$
9. $y = -\frac{1}{3}x^2, y = -\frac{1}{3}x^2 + 2$
10. $y = x^2, y = (x - 7)^2$
11. $y = x^2, y = 3(x + 4)^2 - 7$
12. $y = x^2, y = -\frac{1}{4}x^2 + 1$
13. $y = (x + 3)^2 - 2, y = (x + 3)^2 + 5$
14. $y = 3(x + 2)^2 - 1,$ $y = 6(x + 2)^2 - 1$
15. $y = 4(x - 2)^2 - 3,$ $y = \frac{1}{4}(x - 2)^2 - 1$

5-7 Analyzing Graphs of Quadratic Functions

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Main Ideas

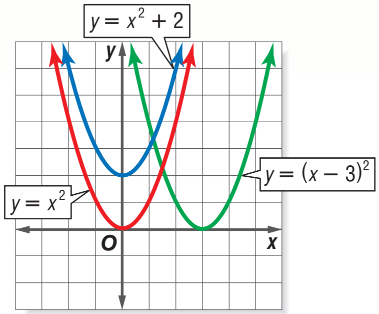
- Analyze quadratic functions of the form $y = a(x - h)^2 + k$.
- Write a quadratic function in the form $y = a(x - h)^2 + k$.

New Vocabulary

vertex form

GET READY for the Lesson

A *family of graphs* is a group of graphs that displays one or more similar characteristics. The graph of $y = x^2$ is called the *parent graph* of the family of quadratic functions.



The graphs of other quadratic functions such as $y = x^2 + 2$ and $y = (x - 3)^2$ can be found by transforming the graph of $y = x^2$.

Analyze Quadratic Functions Each function above can be written in the form $y = (x - h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is its axis of symmetry. This is often referred to as the **vertex form** of a quadratic function.

Equation		Axis of
$y = x^2$ or $y = (x - 0)^2 + 0$	(0, 0)	$x = 0$
$y = x^2 + 2$ or $y = (x - 0)^2 + 2$	(0, 2)	$x = 0$
$y = (x - 3)^2$ or $y = (x - 3)^2 + 0$	(3, 0)	$x = 3$

Recall that a *translation* slides a figure without changing its shape or size. As the values of h and k change, the graph of $y = a(x - h)^2 + k$ is the graph of $y = x^2$ translated:

- $|h|$ units *left* if h is negative or $|h|$ units *right* if h is positive, and
- $|k|$ units *up* if k is positive or $|k|$ units *down* if k is negative.

EXAMPLE Graph a Quadratic Equation in Vertex Form

1 **Analyze $y = (x + 2)^2 + 1$. Then draw its graph.**

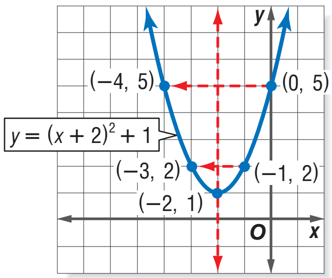
This function can be rewritten as $y = [x - (-2)]^2 + 1$. Then $h = -2$ and $k = 1$. The vertex is at (h, k) or $(-2, 1)$, and the axis of symmetry is $x = -2$. The graph is the graph of $y = x^2$ translated 2 units left and 1 unit up.

Now use this information to draw the graph.

Step 1 Plot the vertex, $(-2, 1)$.

Step 2 Draw the axis of symmetry, $x = -2$.

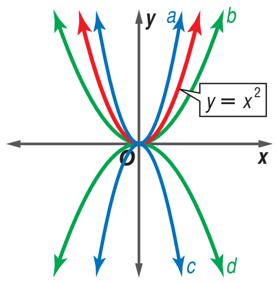
Step 3 Use symmetry to complete the graph.



CHECK Your Progress

1. Analyze $y = (x - 3)^2 - 2$. Then draw its graph.

How does the value of a in the general form $y = a(x - h)^2 + k$ affect a parabola? Compare the graphs of the following functions to the parent function, $y = x^2$.



a. $y = 2x^2$ **b.** $y = \frac{1}{2}x^2$

c. $y = -2x^2$ **d.** $y = -\frac{1}{2}x^2$

All of the graphs have the vertex (0, 0) and axis of symmetry $x = 0$.

Notice that the graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ are *dilations* of the graph of $y = x^2$. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$, while the graph of $y = \frac{1}{2}x^2$ is wider. The graphs of $y = -2x^2$ and $y = 2x^2$ are *reflections* of each other over the x-axis, as are the graphs of $y = -\frac{1}{2}x^2$ and $y = \frac{1}{2}x^2$.

Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph.

- If $a > 0$, the graph opens up.
- If $a < 0$, the graph opens down.
- If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
- If $0 < |a| < 1$, the graph is wider than the graph of $y = x^2$.

Study Tip

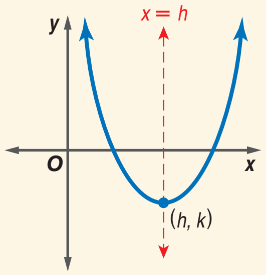
$0 < |a| < 1$ means that a is a real number between 0 and 1, such as $\frac{2}{5}$, or a real number between -1 and 0, such as $-\frac{\sqrt{2}}{2}$.

CONCEPT SUMMARY *Quadratic Functions in Vertex Form*

The vertex form of a quadratic function is $y = a(x - h)^2 + k$.

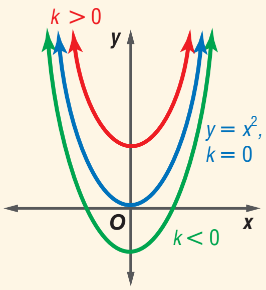
h* and *k

Vertex and Axis of Symmetry



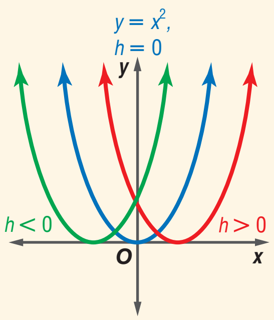
k

Vertical Translation



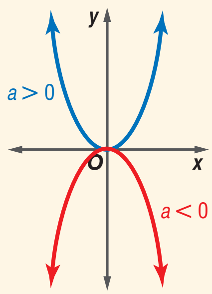
h

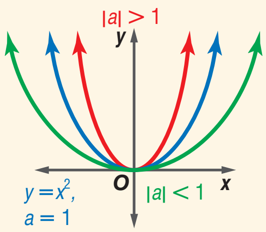
Horizontal Translation



a

Direction of Opening and Shape of Parabola





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STANDARDIZED TEST EXAMPLE — Vertex Form Parameters

2 Which function has the widest graph?

A $y = -2.5x^2$ **B** $y = -0.3x^2$ **C** $y = 2.5x^2$ **D** $y = 5x^2$

Read the Test Item

You are given four answer choices, each of which is in vertex form.

Solve the Test Item

The value of a determines the width of the graph. Since $|-2.5| = |2.5| > 1$ and $|5| > 1$, choices A, C, and D produce graphs that are narrower than $y = x^2$. Since $|-0.3| < 1$, choice B produces a graph that is wider than $y = x^2$. The answer is B.

Test-Taking Tip

The sign of a in the vertex form does not determine how wide the parabola will be. The sign determines whether the parabola opens up or down. The width is determined by the absolute value of a.

CHECK Your Progress

2. Which function has the narrowest graph?

F $y = -0.1x^2$ **G** $y = x^2$ **H** $y = 0.5x^2$ **J** $y = 2.3x^2$

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Write Quadratic Equations in Vertex Form Given a function of the form $y = ax^2 + bx + c$, you can complete the square to write the function in vertex form. If the coefficient of the quadratic term is not 1, the first step is to factor that coefficient from the quadratic and linear terms.

EXAMPLE — Write Equations in Vertex Form

3 **Write each equation in vertex form. Then analyze the function.**

a. $y = x^2 + 8x - 5$

$y = x^2 + 8x - 5$ — Notice that $x^2 + 8x - 5$ is not a perfect square.

$y = (x^2 + 8x + 16) - 5 - 16$ — Complete the square by adding $\left(\frac{8}{2}\right)^2$ or 16. Balance this addition by subtracting 16.

$y = (x + 4)^2 - 21$ — Write $x^2 + 8x + 16$ as a perfect square.

Since $h = -4$ and $k = -21$, the vertex is at $(-4, -21)$ and the axis of symmetry is $x = -4$. Since $a = 1$, the graph opens up and has the same shape as the graph of $y = x^2$, but it is translated 4 units left and 21 units down.

b. $y = -3x^2 + 6x - 1$

$y = -3x^2 + 6x - 1$ — Original equation

$y = -3(x^2 - 2x) - 1$ — Group $ax^2 - bx$ and factor, dividing by a.

$y = -3(x^2 - 2x + 1) - 1 - (-3)(1)$ — Complete the square by adding 1 inside the parentheses. Notice that this is an overall addition of $-3(1)$. Balance this addition by subtracting $-3(1)$.

$y = -3(x - 1)^2 + 2$ — Write $x^2 - 2x + 1$ as a perfect square.

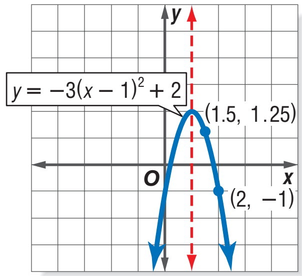
Study Tip

Check

As a check, graph the function in Example 3 to verify the location of its vertex and axis of symmetry.

The vertex is at (1, 2), and the axis of symmetry is $x = 1$. Since $a = -3$, the graph opens downward and is narrower than the graph of $y = x^2$. It is also translated 1 unit right and 2 units up.

Now graph the function. Two points on the graph to the right of $x = 1$ are (1.5, 1.25) and (2, −1). Use symmetry to complete the graph.



CHECK Your Progress

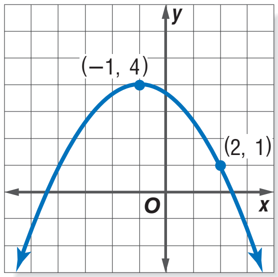
3A. $y = x^2 + 4x + 6$

3B. $y = 2x^2 + 12x + 17$

If the vertex and one other point on the graph of a parabola are known, you can write the equation of the parabola in vertex form.

EXAMPLE 4 Write an Equation Given a Graph

Write an equation for the parabola shown in the graph.



The vertex of the parabola is at (−1, 4), so $h = -1$ and $k = 4$. Since (2, 1) is a point on the graph of the parabola, let $x = 2$ and $y = 1$. Substitute these values into the vertex form of the equation and solve for a.

$y = a(x - h)^2 + k$	Vertex form
$1 = a[2 - (-1)]^2 + 4$	Substitute 1 for y, 2 for x, −1 for h, and 4 for k.
$1 = a(9) + 4$	Simplify.
$-3 = 9a$	Subtract 4 from each side.
$-\frac{1}{3} = a$	Divide each side by 9.

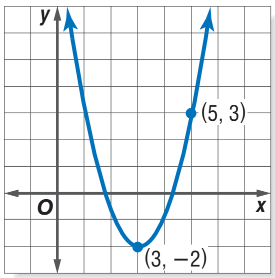
The equation of the parabola in vertex form is $y = -\frac{1}{3}(x + 1)^2 + 4$.

Cross-Curricular Project

Math Online You can use a quadratic function to model the world population. Visit algebra2.com to continue work on your project.

CHECK Your Progress

4. Write an equation for the parabola shown in the graph.



CHECK Your Understanding

Examples 1, 3 (pp. 286, 288)

Graph each function.

1. $y = 3(x + 3)^2$

2. $y = \frac{1}{3}(x - 1)^2 + 3$

3. $y = -2x^2 + 16x - 31$

Example 2 (p. 288)

4. STANDARDIZED TEST PRACTICE Which function has the widest graph?

A $y = -4x^2$ **B** $y = -1.2x^2$ **C** $y = 3.1x^2$ **D** $y = 11x^2$

Example 3 (pp. 288–289)

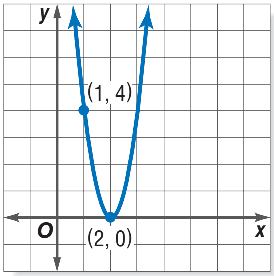
Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

5. $y = 5(x + 3)^2 - 1$ **6.** $y = x^2 + 8x - 3$ **7.** $y = -3x^2 - 18x + 11$

Example 4 (p. 289)

Write an equation in vertex form for the parabola shown in each graph.

8.



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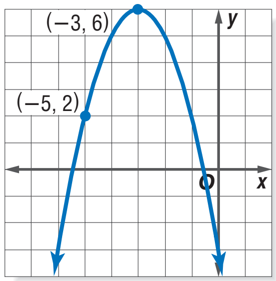

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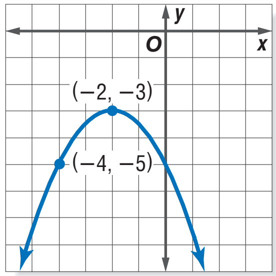
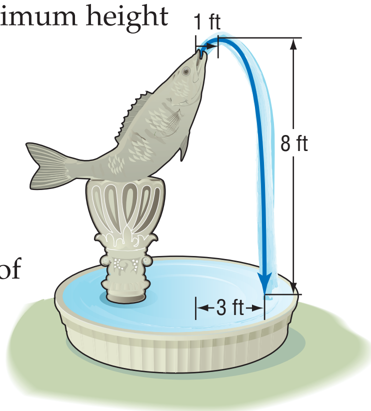


FOUNTAINS The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet.



11. If the water lands 3 feet away from the jet, find a quadratic function that models the height $H(d)$ of the water at any given distance d feet from the jet. Then compare the graph of the function to the parent function.

12. Suppose a worker increases the water pressure so that the stream reaches a maximum height of 12.5 feet at a distance of 15 inches from the jet. The water now lands 3.75 feet from the jet. Write a new quadratic function for $H(d)$. How do the changes in h and k affect the shape of the graph?

Exercises

HOMEWORK HELP

For Exercises	See Examples
13–16, 21, 22	1
17–18	1, 3
19, 20	2
23–26, 31, 32	3
27–30	4

Graph each function.

13. $y = 4(x + 3)^2 + 1$ **14.** $y = -(x - 5)^2 - 3$ **15.** $y = \frac{1}{4}(x - 2)^2 + 4$

16. $y = \frac{1}{2}(x - 3)^2 - 5$ **17.** $y = x^2 + 6x + 2$ **18.** $y = x^2 - 8x + 18$

19. What is the effect on the graph of the equation $y = x^2 + 2$ when the equation is changed to $y = x^2 - 5$?

20. What is the effect on the graph of the equation $y = x^2 + 2$ when the equation is changed to $y = 3x^2 - 5$?

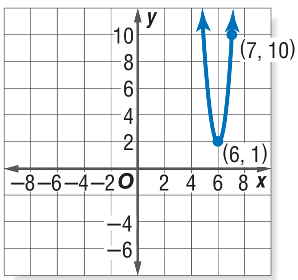
Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

21. $y = -2(x + 3)^2$ **22.** $y = \frac{1}{3}(x - 1)^2 + 2$ **23.** $y = -x^2 - 4x + 8$

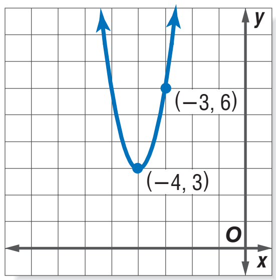
24. $y = x^2 - 6x + 1$ **25.** $y = 5x^2 - 6$ **26.** $y = -8x^2 + 3$

Write an equation in vertex form for the parabola shown in each graph.

27.



28.



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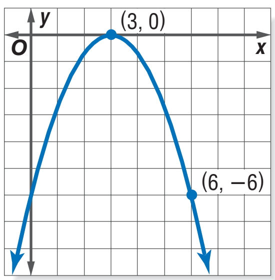
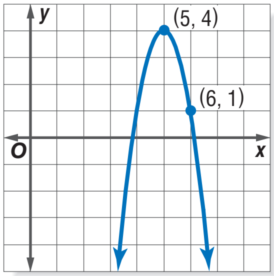
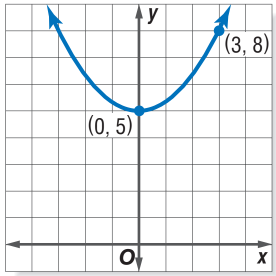


Write an equation in vertex form for the parabola shown in each graph.

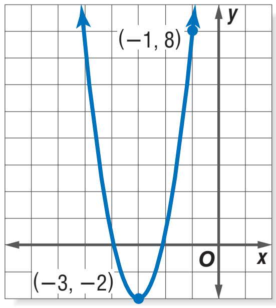
30.



31.



32.



LAWN CARE For Exercises 33 and 34, use the following information.
The path of water from a sprinkler can be modeled by a quadratic function. The three functions below model paths for three different angles of the water.

Angle A: $y = -0.28(x - 3.09)^2 + 3.27$
Angle B: $y = -0.14(x - 3.57)^2 + 2.39$
Angle C: $y = -0.09(x - 3.22)^2 + 1.53$

33. Which sprinkler angle will send water the highest? Explain your reasoning.
34. Which sprinkler angle will send water the farthest? Explain your reasoning.
35. Which sprinkler angle will produce the widest path? The narrowest path?

Graph each function.

36. $y = -4x^2 + 16x - 11$
37. $y = -5x^2 - 40x - 80$
38. $y = -\frac{1}{2}x^2 + 5x - \frac{27}{2}$
39. $y = \frac{1}{3}x^2 - 4x + 15$

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

40. $y = -3x^2 + 12x$
41. $y = 4x^2 + 24x$
42. $y = 4x^2 + 8x - 3$
43. $y = -2x^2 + 20x - 35$
44. $y = 3x^2 + 3x - 1$
45. $y = 4x^2 - 12x - 11$
46. Write an equation for a parabola with vertex at the origin and that passes through $(2, -8)$.
47. Write an equation for a parabola with vertex at $(-3, -4)$ and y-intercept 8.
48. Write one sentence that compares the graphs of $y = 0.2(x + 3)^2 + 1$ and $y = 0.4(x + 3)^2 + 1$.
49. Compare the graphs of $y = 2(x - 5)^2 + 4$ and $y = 2(x - 4)^2 - 1$.

50. AEROSPACE NASA's KC135A aircraft flies in parabolic arcs to simulate the weightlessness experienced by astronauts in space. The height h of the aircraft (in feet) t seconds after it begins its parabolic flight can be modeled by the equation $h(t) = -9.09(t - 32.5)^2 + 34{,}000$. What is the maximum height of the aircraft during this maneuver and when does it occur?

Real-World Link
The KC135A has the nickname "Vomit Comet." It starts its ascent at 24,000 feet. As it approaches maximum height, the engines are stopped and the aircraft is allowed to free-fall at a determined angle. Zero gravity is achieved for 25 seconds as the plane reaches the top of its flight and begins its descent.

DIVING For Exercises 49–51, use the following information.
The distance of a diver above the water $d(t)$ (in feet) t seconds after diving off a platform is modeled by the equation $d(t) = -16t^2 + 8t + 30$.

51. Find the time it will take for the diver to hit the water.
52. Write an equation that models the diver's distance above the water if the platform were 20 feet higher.
53. Find the time it would take for the diver to hit the water from this new height.

EXTRA PRACTICE
See pages 901, 930.
Math Online
Self-Check Quiz at algebra2.com

NASA

H.O.T. Problems

54. OPEN ENDED Write the equation of a parabola with a vertex of (2, −1) and which opens downward.

55. CHALLENGE Given $y = ax^2 + bx + c$ with $a \neq 0$, derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form $y = a(x - h)^2 + k$.

56. FIND THE ERROR Jenny and Ruben are writing $y = x^2 - 2x + 5$ in vertex form. Who is correct? Explain your reasoning.

Jenny	Ruben
$y = x^2 - 2x + 5$	$y = x^2 - 2x + 5$
$y = (x^2 - 2x + 1) + 5 - 1$	$y = (x^2 - 2x + 1) + 5 + 1$
$y = (x - 1)^2 + 4$	$y = (x - 1)^2 + 6$

57. CHALLENGE Explain how you can find an equation of a parabola using the coordinates of three points on its graph.

58. *Writing in Math* Use the information on page 286 to explain how the graph of $y = x^2$ can be used to graph any quadratic function. Include a description of the effects produced by changing a, h, and k in the equation $y = a(x - h)^2 + k$, and a comparison of the graph of $y = x^2$ and the graph of $y = a(x - h)^2 + k$ using values of your own choosing for a, h, and k.

STANDARDIZED TEST PRACTICE

59. ACT/SAT If $f(x) = x^2 - 5x$ and $f(n) = -4$, which of the following could be n?

A −5

B −4

C −1

D 1

60. REVIEW Which of the following most accurately describes the translation of the graph of $y = (x + 5)^2 - 1$ to the graph of $y = (x - 1)^2 + 3$?

F up 4 and 6 to the right

G up 4 and 1 to the left

H down 1 and 1 to the right

J down 1 and 5 to the left

Spiral Review

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation. (Lesson 5-6)

61. $3x^2 - 6x + 2 = 0$

62. $4x^2 + 7x = 11$

63. $2x^2 - 5x + 6 = 0$

Solve each equation by completing the square. (Lesson 5-5)

64. $x^2 + 10x + 17 = 0$

65. $x^2 - 6x + 18 = 0$

66. $4x^2 + 8x = 9$

GET READY for the Next Lesson

PREREQUISITE SKILL **Determine whether the given value satisfies the inequality.** (Lesson 1-6)

67. $-2x^2 + 3 < 0$; $x = 5$

68. $4x^2 + 2x - 3 \geq 0$; $x = -1$

69. $4x^2 - 4x + 1 \leq 10$; $x = 2$

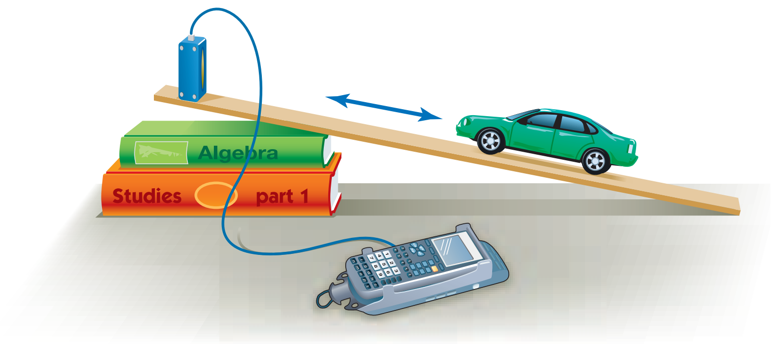
70. $6x^2 + 3x > 8$; $x = 0$

EXTEND 5-7

Graphing Calculator Lab
Modeling Motion

SET UP the Lab

- Place a board on a stack of books to create a ramp.
- Connect the data collection device to the graphing calculator. Place at the top of the ramp so that the data collection device can read the motion of the car on the ramp.



- Hold the car still about 6 inches up from the bottom of the ramp and zero the collection device.

ACTIVITY 1

Step 1 One group member should press the button to start collecting data.

Step 2 Another group member places the car at the bottom of the ramp. After data collection begins, gently but quickly push the car so it travels up the ramp toward the motion detector.

Step 3 Stop collecting data when the car returns to the bottom of the ramp. Save the data as Trial 1.

Step 4 Remove one book from the stack. Then repeat the experiment. Save the data as Trial 2. For Trial 3, create a steeper ramp and repeat the experiment.

ANALYZE THE RESULTS

1. What type of function could be used to represent the data? Justify your answer.
2. Use the CALC menu to find the vertex of the graph. Record the coordinates in a table like the one at the right.
3. Use the TRACE feature of the calculator to find the coordinates of another point on the graph. Then use the coordinates of the vertex and the point to find an equation of the graph.

Trial	Vertex (h, k)	Point (x, y)	Equation
1			
2			
3			

4. Find an equation for each of the graphs of Trials 2 and 3.
5. How do the equations for Trials 1, 2, and 3 compare? Which graph is widest and which is most narrow? Explain what this represents in the context of the situation. How is this represented in the equations?
6. What do the x-intercepts and vertex of each graph represent?
7. Why were the values of h and k different in each trial?

5-8 Graphing and Solving Quadratic Inequalities

Main Ideas

- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in one variable.

New Vocabulary

quadratic inequality

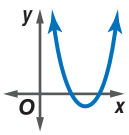
GET READY for the Lesson

Californian Jennifer Parilla is the only athlete from the United States to qualify for and compete in the Olympic trampoline event.

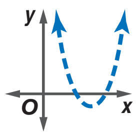
Suppose the height $h(t)$ in feet of a trampolinist above the ground during one bounce is modeled by the quadratic function $h(t) = -16t^2 + 42t + 3.75$. We can solve a quadratic inequality to determine how long this performer is more than a certain distance above the ground.

Graph Quadratic Inequalities You can graph **quadratic inequalities** in two variables using the same techniques you used to graph linear inequalities in two variables.

Step 1 Graph the related quadratic function, $y = ax^2 + bx + c$. Decide if the parabola should be solid or dashed.

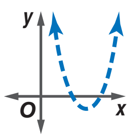


$\leq$ or $\geq$



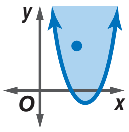
$<$ or $>$

Step 2 Test a point (x_1, y_1) inside the parabola. Check to see if this point is a solution of the inequality.

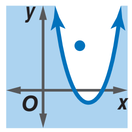


$y_1 \overset{?}{\geq} a(x_1)^2 + b(x_1) + c$

Step 3 If (x_1, y_1) is a solution, shade the region *inside* the parabola. If (x_1, y_1) is *not* a solution, shade the region *outside* the parabola.



(x_1, y_1) is a solution.



(x_1, y_1) is not a solution.

Study Tip

Look Back
For review of **graphing linear inequalities,** see Lesson 2-7.

EXAMPLE Graph a Quadratic Inequality

1 **Use a table to graph $y > -x^2 - 6x - 7$.**

Step 1 Graph the related quadratic function, $y = -x^2 - 6x - 7$.

Since the inequality symbol is $>$, the parabola should be dashed.

x	-5	-4	-3	-2	-1
y	-2	1	2	1	-2

Step 2 Test a point inside the parabola, such as $(-3, 0)$.

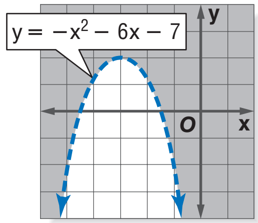
$y > -x^2 - 6x - 7$

$0 \overset{?}{>} -(-3)^2 - 6(-3) - 7$

$0 \overset{?}{>} -9 + 18 - 7$

$0 \overset{?}{>} 2$ ✗

So, $(-3, 0)$ is *not* a solution of the inequality.



Step 3 Shade the region outside the parabola.

CHECK Your Progress **Graph each inequality.**

1A. $y \le x^2 + 2x + 4$ **1B.** $y < -2x^2 + 3x + 5$

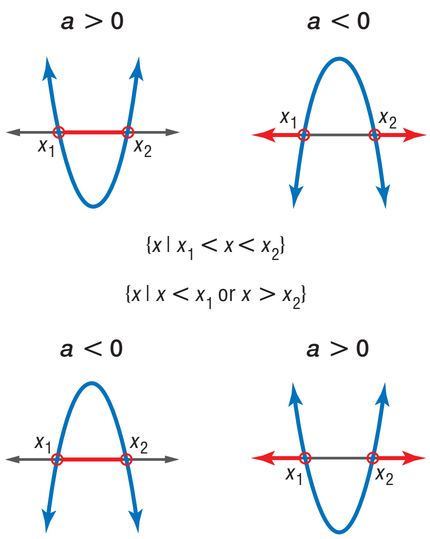
Solve Quadratic Inequalities To solve a quadratic inequality in one variable, you can use the graph of the related quadratic function.

To solve $ax^2 + bx + c < 0$, graph $y = ax^2 + bx + c$. Identify the x-values for which the graph lies *below* the x-axis.

For $\le$, include the x-intercepts in the solution.

To solve $ax^2 + bx + c > 0$, graph $y = ax^2 + bx + c$. Identify the x-values for which the graph lies *above* the x-axis.

For $\ge$, include the x-intercepts in the solution.



EXAMPLE Solve $ax^2 + bx + c > 0$

2 **Solve $x^2 + 2x - 3 > 0$ by graphing.**

The solution consists of the x-values for which the graph of the related quadratic function lies *above* the x-axis. Begin by finding the roots.

$x^2 + 2x - 3 = 0$	Related equation
$(x + 3)(x - 1) = 0$	Factor.
$x + 3 = 0$ or $x - 1 = 0$	Zero Product Property
$x = -3$ $\quad$ $x = 1$	Solve each equation.

(continued on the next page)

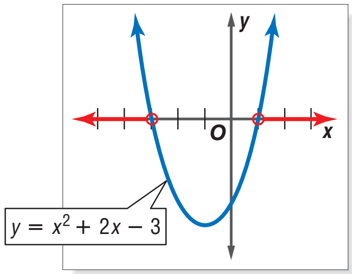
Study Tip

Solving Quadratic Inequalities by Graphing

A precise graph of the related quadratic function is not necessary since the zeros of the function were found algebraically.

Sketch the graph of a parabola that has x-intercepts at -3 and 1. The graph should open up since $a > 0$.

The graph lies above the x-axis to the left of $x = -3$ and to the right of $x = 1$. Therefore, the solution set is $\{x \mid x < -3 \text{ or } x > 1\}$.



CHECK Your Progress

Solve each inequality by graphing.

2A. $x^2 - 3x + 2 \geq 0$ **2B.** $0 \leq x^2 - 2x - 35$

EXAMPLE Solve $ax^2 + bx + c \leq 0$

3 **Solve $0 \geq 3x^2 - 7x - 1$ by graphing.**

This inequality can be rewritten as $3x^2 - 7x - 1 \leq 0$. The solution consists of the x-values for which the graph of the related quadratic function lies *on and below* the x-axis. Begin by finding the roots of the related equation.

$3x^2 - 7x - 1 = 0$ Related equation

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Use the Quadratic Formula.

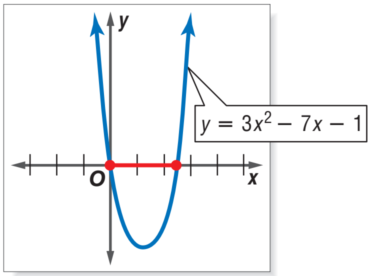
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)}$ Replace a with 3, b with -7, and c with -1.

$x = \dfrac{7 + \sqrt{61}}{6}$ or $x = \dfrac{7 - \sqrt{61}}{6}$ Simplify and write as two equations.

$x \approx 2.47$ $x \approx -0.14$ Simplify.

Sketch the graph of a parabola that has x-intercepts of 2.47 and -0.14. The graph should open up since $a > 0$.

The graph lies on and below the x-axis at $x = -0.14$ and $x = 2.47$ and between these two values. Therefore, the solution set of the inequality is approximately $\{x \mid -0.14 \leq x \leq 2.47\}$.



CHECK Test one value of x less than -0.14, one between -0.14 and 2.47, and one greater than 2.47 in the original inequality.

Test $x = -1$.	Test $x = 0$.	Test $x = 3$.
$0 \geq 3x^2 - 7x - 1$	$0 \geq 3x^2 - 7x - 1$	$0 \geq 3x^2 - 7x - 1$
$0 \overset{?}{\geq} 3(-1)^2 - 7(-1) - 1$	$0 \overset{?}{\geq} 3(0)^2 - 7(0) - 1$	$0 \overset{?}{\geq} 3(3)^2 - 7(3) - 1$
$0 \geq 9$ ✗	$0 \geq -1$ ✓	$0 \geq 5$ ✗

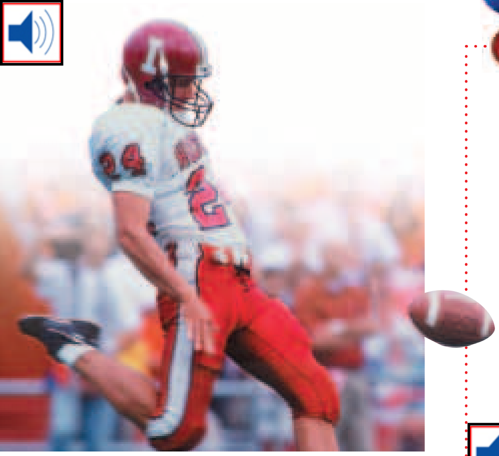
CHECK Your Progress

Solve each inequality by graphing.

3A. $0 > 2x^2 + 5x - 6$ **3B.** $5x^2 - 10x + 1 < 0$

Real-world problems that involve vertical motion can often be solved by using a quadratic inequality.

Real-World EXAMPLE



Real-World Link

A long hang time allows the kicking team time to provide good coverage on a punt return. The suggested hang time for high school and college punters is 4.5–4.6 seconds.

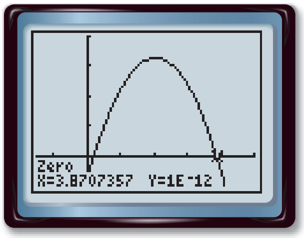
Source: www.takeaknee.com

4 **FOOTBALL** **The height of a punted football can be modeled by the function $H(x) = -4.9x^2 + 20x + 1$, where the height $H(x)$ is given in meters and the time x is in seconds. At what time in its flight is the ball within 5 meters of the ground?**

The function $H(x)$ describes the height of the football. Therefore, you want to find the values of x for which $H(x) \leq 5$.

$H(x) \leq 5$	Original inequality
$-4.9x^2 + 20x + 1 \leq 5$	$H(x) = -4.9x^2 + 20x + 1$
$-4.9x^2 + 20x - 4 \leq 0$	Subtract 5 from each side.

Graph the related function $y = -4.9x^2 + 20x - 4$ using a graphing calculator. The zeros of the function are about 0.21 and 3.87, and the graph lies below the x-axis when $x < 0.21$ or $x > 3.87$.



$[-1.5, 5]$ scl: 1 by $[-5, 20]$ scl: 5

Thus, the ball is within 5 meters of the ground for the first 0.21 second of its flight and again after 3.87 seconds until the ball hits the ground at 4.13 seconds.

CHECK The ball starts 1 meter above the ground, so $x < 0.21$ makes sense. Based on the given information, a punt stays in the air about 4.5 seconds. So, it is reasonable that the ball is back within 5 meters of the ground after 3.87 seconds.

CHECK Your Progress

4. Use the function $H(x)$ above to find at what time in its flight the ball is at least 7 meters above the ground.

Online Personal Tutor at algebra2.com

Study Tip

Solving Quadratic Inequalities Algebraically

As with linear inequalities, the solution set of a quadratic inequality is sometimes all real numbers or the empty set, $\varnothing$. The solution is all real numbers when all three test points satisfy the inequality. It is the empty set when none of the test points satisfy the inequality.

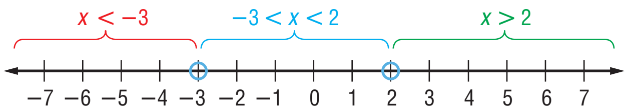
EXAMPLE Solve a Quadratic Inequality

5 **Solve $x^2 + x > 6$ algebraically.**

First solve the related quadratic equation $x^2 + x = 6$.

$x^2 + x = 6$	Related quadratic equation
$x^2 + x - 6 = 0$	Subtract 6 from each side.
$(x + 3)(x - 2) = 0$	Factor.
$x + 3 = 0$ or $x - 2 = 0$	Zero Product Property
$x = -3$ $\quad$ $x = 2$	Solve each equation.

Plot -3 and 2 on a number line. Use circles since these values are not solutions of the original inequality. Notice that the number line is now separated into three intervals.



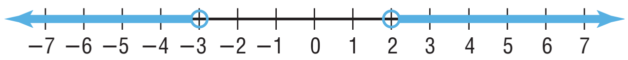
(continued on the next page)

Todd Rosenberg/Allsport/Getty Images

Test a value in each interval to see if it satisfies the original inequality.

$x < -3$	$-3 < x < 2$	$x > 2$
Test $x = -4$.	Test $x = 0$.	Test $x = 4$.
$x^2 + x > 6$	$x^2 + x > 6$	$x^2 + x > 6$
$(-4)^2 + (-4) \stackrel{?}{>} 6$	$0^2 + 0 \stackrel{?}{>} 6$	$4^2 + 4 \stackrel{?}{>} 6$
$12 > 6$ ✓	$0 > 6$ ✗	$20 > 6$ ✓

The solution set is $\{x \mid x < -3 \text{ or } x > 2\}$. This is shown on the number line below.



CHECK Your Progress

Solve each inequality algebraically.

5A. $x^2 + 5x < -6$

5B. $x^2 + 11x + 30 \leq 0$

CHECK Your Understanding

Example 1 (p. 295)

Graph each inequality.

1. $y \geq x^2 - 10x + 25$

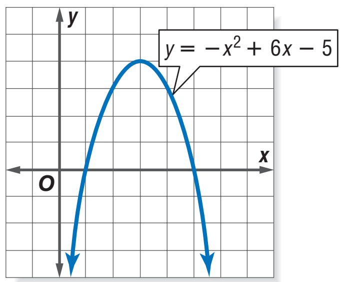
2. $y < x^2 - 16$

3. $y > -2x^2 - 4x + 3$

4. $y \leq -x^2 + 5x + 6$

Examples 2, 3 (pp. 295–296)

5. Use the graph of the related function of $-x^2 + 6x - 5 < 0$, which is shown at the right, to write the solutions of the inequality.



Examples 2, 3, 5 (pp. 295–298)

Solve each inequality using a graph, a table, or algebraically.

6. $x^2 - 6x - 7 < 0$

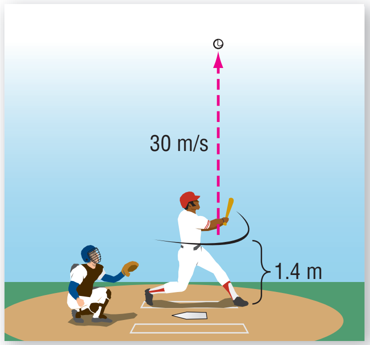
7. $x^2 - x - 12 > 0$

8. $x^2 < 10x - 25$

9. $x^2 \leq 3$

Example 4 (p. 297)

10. BASEBALL A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does a player on the opposing team have to get under the ball if he catches it 1.7 meters above the ground? Does your answer seem reasonable? Explain.



Exercises

Graph each inequality.

11. $y \geq x^2 + 3x - 18$

12. $y < -x^2 + 7x + 8$

13. $y \leq x^2 + 4x + 4$

14. $y \leq x^2 + 4x$

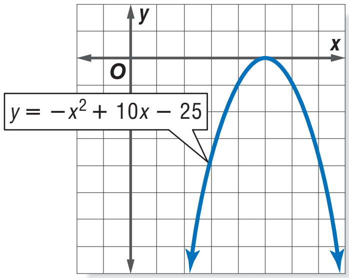
15. $y > x^2 - 36$

16. $y > x^2 + 6x + 5$

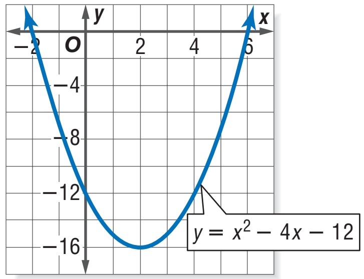
HOMEWORK HELP	
For Exercises	See Examples
11–16	1
17–20	2, 3
21–26	2, 3, 5
27, 28	4

Use the graph of the related function of each inequality to write its solutions.

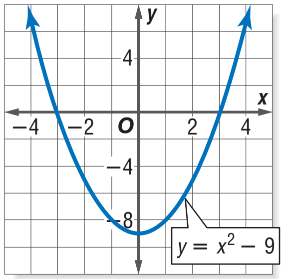
17. $-x^2 + 10x - 25 \geq 0$



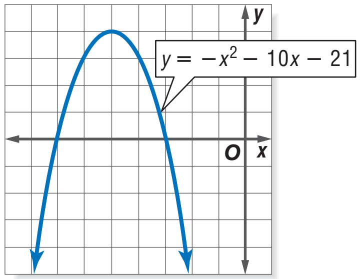
18. $x^2 - 4x - 12 \leq 0$



19. $x^2 - 9 > 0$



20. $-x^2 - 10x - 21 < 0$



Solve each inequality using a graph, a table, or algebraically.

21. $x^2 - 3x - 18 > 0$

22. $x^2 + 3x - 28 < 0$

23. $x^2 - 4x \leq 5$

24. $x^2 + 2x \geq 24$

25. $-x^2 - x + 12 \geq 0$

26. $-x^2 - 6x + 7 \leq 0$

27. LANDSCAPING Kinu wants to plant a garden and surround it with decorative stones. She has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but she wants the garden to cover no more than 240 square feet. What could the width of her garden be?

28. GEOMETRY A rectangle is 6 centimeters longer than it is wide. Find the possible dimensions if the area of the rectangle is more than 216 square centimeters.

Real-World Career

Landscape Architect

Landscape architects design outdoor spaces so that they are not only functional, but beautiful and compatible with the natural environment.

Math Online

For more information, go to algebra2.com.

Graph each inequality.

29. $y \leq -x^2 - 3x + 10$

30. $y \geq -x^2 - 7x + 10$

31. $y > -x^2 + 10x - 23$

32. $y < -x^2 + 13x - 36$

33. $y < 2x^2 + 3x - 5$

34. $y \geq 2x^2 + x - 3$

Solve each inequality using a graph, a table, or algebraically.

35. $9x^2 - 6x + 1 \leq 0$

36. $4x^2 + 20x + 25 \geq 0$

37. $x^2 + 12x < -36$

38. $-x^2 + 14x - 49 \geq 0$

39. $18x - x^2 \leq 81$

40. $16x^2 + 9 < 24x$

41. $(x - 1)(x + 4)(x - 3) > 0$

42. BUSINESS A mall owner has determined that the relationship between monthly rent charged for store space r (in dollars per square foot) and monthly profit $P(r)$ (in thousands of dollars) can be approximated by the function $P(r) = -8.1r^2 + 46.9r - 38.2$. Solve each quadratic equation or inequality. Explain what each answer tells about the relationship between monthly rent and profit for this mall.

a. $-8.1r^2 + 46.9r - 38.2 = 0$

b. $-8.1r^2 + 46.9r - 38.2 > 0$

c. $-8.1r^2 + 46.9r - 38.2 > 10$

d. $-8.1r^2 + 46.9r - 38.2 < 10$

EXTRA PRACTICE
See pages 902, 930.

Math Online
Self-Check Quiz at
algebra2.com

FUND-RAISING **For Exercises 43–45, use the following information.**
The girls' softball team is sponsoring a fund-raising trip to see a professional baseball game. They charter a 60-passenger bus for \$525. In order to make a profit, they will charge \$15 per person if all seats on the bus are sold, but for each empty seat, they will increase the price by \$1.50 per person.

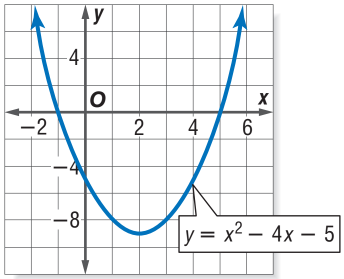
43. Write a quadratic function giving the softball team's profit $P(n)$ from this fund-raiser as a function of the number of passengers n.

44. What is the minimum number of passengers needed in order for the softball team not to lose money?

45. What is the maximum profit the team can make with this fund-raiser, and how many passengers will it take to achieve this maximum?

H.O.T. Problems

46. REASONING Examine the graph of $y = x^2 - 4x - 5$.



a. What are the solutions of $0 = x^2 - 4x - 5$?

b. What are the solutions of $x^2 - 4x - 5 \geq 0$?

c. What are the solutions of $x^2 - 4x - 5 \leq 0$?

47. OPEN ENDED List three points you might test to find the solution of $(x + 3)(x - 5) < 0$.

48. CHALLENGE Graph the intersection of the graphs of $y \leq -x^2 + 4$ and $y \geq x^2 - 4$.

49. *Writing in Math* Use the information on page 294 to explain how you can find the time a trampolinist spends above a certain height. Include a quadratic inequality that describes the time the performer spends more than 10 feet above the ground, and two approaches to solving this quadratic inequality.

STANDARDIZED TEST PRACTICE

50. ACT/SAT If $(x + 1)(x - 2)$ is positive, which statement must be true?

A $x < -1$ or $x > 2$

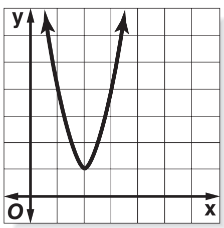
B $x > -1$ or $x < 2$

C $-1 < x < 2$

D $-2 < x < 1$

51. REVIEW Which is the graph of $y = -3(x - 2)^2 + 1$?

F



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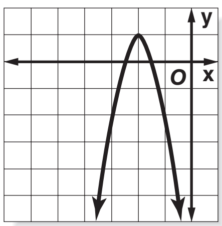

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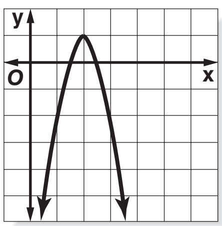

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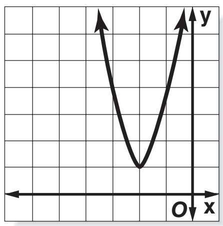


Spiral Review

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. (Lesson 5-7)

52. $y = x^2 - 2x + 9$

53. $y = -2x^2 + 16x - 32$

54. $y = \frac{1}{2}x^2 + 6x + 18$

Solve each equation by using the method of your choice. Find exact solutions. (Lesson 5-6)

55. $x^2 + 12x + 32 = 0$

56. $x^2 + 7 = -5x$

57. $3x^2 + 6x - 2 = 3$

Solve each matrix equation or system of equations by using inverse matrices. (Lesson 4-8)

58. $\begin{bmatrix} 3 & 6 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3 \\ 18 \end{bmatrix}$

59. $\begin{bmatrix} 5 & -7 \\ -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

60. $3j + 2k = 8$
$j - 7k = 18$

61. $5y + 2z = 11$
$10y - 4z = -2$

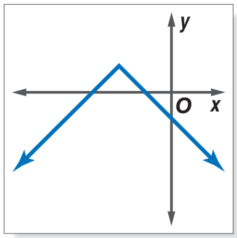
Find each product, if possible. (Lesson 4-3)

62. $\begin{bmatrix} -6 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}$

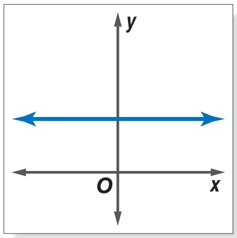
63. $\begin{bmatrix} 2 & -6 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 9 & 0 \\ -2 & 4 \end{bmatrix}$

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. (Lesson 2-6)

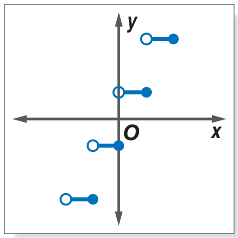
64.



65.



66.



67. EDUCATION The number of U.S. college students studying abroad in 2003 increased by about 8.57% over the previous year. The graph shows the number of U.S. students in study-abroad programs. (Lesson 2-5)

a. Write a prediction equation from the data given.

b. Use your equation to predict the number of students in these programs in 2010.

Source: Institute of International Education

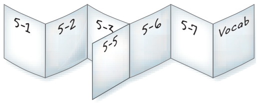
68. LAW ENFORCEMENT A certain laser device measures vehicle speed to within 3 miles per hour. If a vehicle's actual speed is 65 miles per hour, write and solve an absolute value equation to describe the range of speeds that might register on this device. (Lesson 1-6)

Chapter 5 Study Guide and Review

Study To Go: Download Vocabulary Review from algebra2.com

Foldables Study Organizer — Get Ready to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Graphing Quadratic Functions (Lesson 5-1)

- The graph of $y = ax^2 + bx + c$, $a \neq 0$, opens up, and the function has a minimum value when $a > 0$. The graph opens down, and the function has a maximum value when $a < 0$.

Solving Quadratic Equations (Lessons 5-2 and 5-3)

- The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the x-intercepts of its graph.

Complex Numbers (Lesson 5-4)

- i is the imaginary unit. $i^2 = -1$ and $i = \sqrt{-1}$.

Solving Quadratic Equations (Lessons 5-5 and 5-6)

- Completing the square: **Step 1** Find one half of b, the coefficient of x. **Step 2** Square the result in Step 1. **Step 3** Add the result of Step 2 to $x^2 + bx$.
- Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Analyzing Graphs (Lesson 5-7)

- As the values of h and k change, the graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated $|h|$ units left if h is negative or $|h|$ units right if h is positive and $|k|$ units up if k is positive or $|k|$ units down if k is negative.
- Consider the equation $y = a(x - h)^2 + k$, $a \neq 0$. If $a > 0$, the graph opens up; if $a < 0$ the graph opens down. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Key Vocabulary

axis of symmetry (p. 237)
completing the square (p. 269)
complex conjugates (p. 263)
complex number (p. 261)
constant term (p. 236)
discriminant (p. 279)
imaginary unit (p. 260)
linear term (p. 236)
maximum value (p. 238)
minimum value (p. 238)
parabola (p. 236)
pure imaginary number (p. 260)
quadratic equation (p. 246)
quadratic function (p. 236)
quadratic inequality (p. 294)
quadratic term (p. 236)
root (p. 246)
square root (p. 259)
vertex (p. 237)
vertex form (p. 286)
zero (p. 246)

Vocabulary Check

Choose the term from the list above that best matches each phrase.

1. the graph of any quadratic function
2. process used to create a perfect square trinomial
3. the line passing through the vertex of a parabola and dividing the parabola into two mirror images
4. a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
5. the solutions of an equation
6. $y = a(x - h)^2 + k$
7. in the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$
8. the square root of -1
9. a method used to solve a quadratic equation without using the quadratic formula
10. a number in the form $a + bi$

Online Vocabulary Review at algebra2.com

Lesson-by-Lesson Review

5–1 Graphing Quadratic Functions (pp. 236–244)

Complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**

b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.**

11. $f(x) = x^2 + 6x + 20$

12. $f(x) = x^2 - 8x + 7$

13. $f(x) = -2x^2 + 12x - 9$

14. **FRAMES** Josefina is making a rectangular picture frame. She has 72 inches of wood to make this frame. What dimensions will produce a picture frame that will frame the greatest area?

Example 1 **Find the maximum or minimum value of $f(x) = -x^2 + 4x - 12$.**

Since $a < 0$, the graph opens down and the function has a maximum value. The maximum value of the function is the y-coordinate of the vertex. The x-coordinate of the vertex is $x = \frac{-4}{2(-1)}$ or 2.

Find the y-coordinate by evaluating the function for $x = 2$.

$f(x) = -x^2 + 4x - 12$ Original function

$f(2) = -(2)^2 + 4(2) - 12$ Replace x with 2.

or -8

Therefore, the maximum value of the function is -8.

5–2 Solving Quadratic Equations by Graphing (pp. 246–251)

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

15. $x^2 - 36 = 0$

16. $-x^2 - 3x + 10 = 0$

17. $-3x^2 - 6x - 2 = 0$

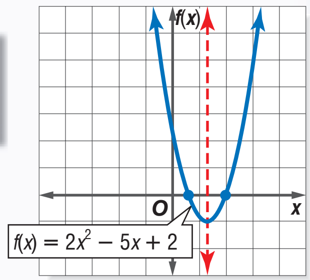
18. $\frac{1}{5}(x + 3)^2 - 5 = 0$

19. **BASEBALL** A baseball is hit upward at 100 feet per second. Use the formula $h(t) = v_o t - 16t^2$, where $h(t)$ is the height of an object in feet, v_o is the object's initial velocity in feet per second, and t is the time in seconds. Ignoring the height of the ball when it was hit, how long does it take for the ball to hit the ground?

Example 2 **Solve $2x^2 - 5x + 2 = 0$ by graphing.**

The equation of the axis of symmetry is $x = -\frac{-5}{2(2)}$ or $x = \frac{5}{4}$.

x	0	$\frac{1}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$
$f(x)$	2	0	$-\frac{9}{8}$	0	2



The zeros of the related function are $\frac{1}{2}$ and 2. Therefore, the solutions of the equation are $\frac{1}{2}$ and 2.

Study Guide and Review

5-3 Solving Quadratic Equations by Factoring (pp. 253–258)

Write a quadratic equation in standard form with the given root(s).

20. $-4, -25$ **21.** $10, -7$ **22.** $\frac{1}{3}, 2$

Solve each equation by factoring.

23. $x^2 - 4x - 32 = 0$ **24.** $3x^2 + 6x + 3 = 0$

25. $5y^2 = 80$ **26.** $25x^2 - 30x = -9$

27. $6x^2 + 7x = 3$ **28.** $2c^2 + 18c - 44 = 0$

29. TRIANGLES Find the dimensions of a triangle if the base is $\frac{2}{3}$ the length of the height and the area is 12 square centimeters.

Example 3 **Write a quadratic equation in standard form with the roots 3 and −5.**

$(x - p)(x - q) = 0$ Write the pattern.

$(x - 3)(x + 5) = 0$ $p = 3$ and $q = -5$

$x^2 + 2x - 15 = 0$ Use FOIL.

Example 4 **Solve $x^2 + 9x + 20 = 0$ by factoring.**

$x^2 + 9x + 20 = 0$ Original equation

$(x + 4)(x + 5) = 0$ Factor the trinomial.

$x + 4 = 0$ or $x + 5 = 0$ Zero Product Property

$x = -4$ $x = -5$

The solution set is $\{-5, -4\}$.

5-4 Complex Numbers (pp. 259–266)

Simplify.

30. $\sqrt{45}$ **31.** $\sqrt{64n^3}$

32. $\sqrt{-64m^{12}}$

33. $(7 - 4i) - (-3 + 6i)$

34. $(3 + 4i)(5 - 2i)$ **35.** $(\sqrt{6} + i)(\sqrt{6} - i)$

36. $\frac{1 + i}{1 - i}$ **37.** $\frac{4 - 3i}{1 + 2i}$

38. ELECTRICITY The impedance in one part of a series circuit is $2 + 3j$ ohms, and the impedance in the other part of the circuit is $4 - 2j$. Add these complex numbers to find the total impedance in the circuit.

Example 5 **Simplify $(15 - 2i) + (-11 + 5i)$.**

$(15 - 2i) + (-11 + 5i)$

$= [15 + (-11)] + (-2 + 5)i$ Group the real and imaginary parts.

$= 4 + 3i$ Add.

Example 6 **Simplify $\frac{7i}{2 + 3i}$.**

$\frac{7i}{2 + 3i} = \frac{7i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$ $2 + 3i$ and $2 - 3i$ are conjugates.

$= \frac{14i - 21i^2}{4 - 9i^2}$ Multiply.

$= \frac{21 + 14i}{13}$ or $\frac{21}{13} + \frac{14}{13}i$ $i^2 = 1$

Mixed Problem Solving
For mixed problem-solving practice, see page 930.

5–5 Completing the Square (pp. 268–275)

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

39. $x^2 + 34x + c$ **40.** $x^2 - 11x + c$

Solve each equation by completing the square.

41. $2x^2 - 7x - 15 = 0$

42. $2x^2 - 5x + 7 = 3$

43. GARDENING Antoinette has a rectangular rose garden with the length 8 feet longer than the width. If the area of her rose garden is 128 square feet, find the dimensions of the garden.

Example 7 **Solve $x^2 + 10x - 39 = 0$ by completing the square.**

$$x^2 + 10x - 39 = 0$$
$$x^2 + 10x = 39$$
$$x^2 + 10x + 25 = 39 + 25$$
$$(x + 5)^2 = 64$$
$$x + 5 = \pm 8$$
$$x + 5 = 8 \text{ or } x + 5 = -8$$
$$x = 3 \qquad x = -13$$

The solution set is $\{-13, 3\}$.

5–6 The Quadratic Formula and the Discriminant (pp. 276–283)

Complete parts a–c for each quadratic equation.

a. **Find the value of the discriminant.**
b. **Describe the number and type of roots.**
c. **Find the exact solutions by using the Quadratic Formula.**

44. $x^2 + 2x + 7 = 0$

45. $-2x^2 + 12x - 5 = 0$

46. $3x^2 + 7x - 2 = 0$

47. FOOTBALL The path of a football thrown across a field is given by the equation $y = -0.005x^2 + x + 5$, where x represents the distance, in feet, the ball has traveled horizontally and y represents the height, in feet, of the ball above ground level. About how far has the ball traveled horizontally when it returns to the ground?

Example 8 **Solve $x^2 - 5x - 66 = 0$ by using the Quadratic Formula.**

In $x^2 - 5x - 66 = 0$, $a = 1$, $b = -5$, and $c = -66$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-66)}}{2(1)}$$
$$= \frac{5 \pm 17}{2}$$

Write as two equations.

$$x = \frac{5 + 17}{2} \text{ or } x = \frac{5 - 17}{2}$$
$$= 11 \qquad = -6$$

The solution set is $\{-6, 11\}$.

Study Guide and Review

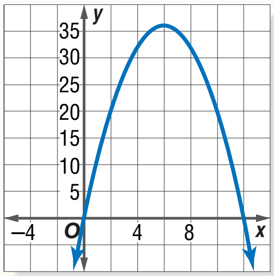
5-7 Analyzing Graphs of Quadratic Functions (pp. 286–292)

Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function.

48. $y = -6(x + 2)^2 + 3$ **49.** $y = -\frac{1}{3}x^2 + 8x$

50. $y = (x - 2)^2 - 2$ **51.** $y = 2x^2 + 8x + 10$

52. NUMBER THEORY The graph shows the product of two numbers with a sum of 12. Find an equation that models this product and use it to determine the two numbers that would give a maximum product.



Example 9 **Write the quadratic function $y = 3x^2 + 42x + 142$ in vertex form. Then identify the vertex, axis of symmetry, and the direction of opening.**

$y = 3x^2 + 42x + 142$ Original equation

$y = 3(x^2 + 14x) + 142$ Group and factor.

$y = 3(x^2 + 14x + 49) + 142 - 3(49)$ Complete the square.

$y = 3(x + 7)^2 - 5$ Write $x^2 + 14x + 49$ as a perfect square.

So, $a = 3$, $h = -7$, and $k = -5$. The vertex is at $(-7, -5)$, and the axis of symmetry is $x = -7$. Since a is positive, the graph opens up.

5-8 Graphing and Solving Quadratic Inequalities (pp. 294–301)

Graph each inequality.

53. $y > x^2 - 5x + 15$ **54.** $y \geq -x^2 + 7x - 11$

Solve each inequality using a graph, a table, or algebraically.

55. $6x^2 + 5x > 4$ **56.** $8x + x^2 \geq -16$

57. $4x^2 - 9 \leq -4x$ **58.** $3x^2 - 5 > 6x$

59. GAS MILEAGE The gas mileage y in miles per gallon for a particular vehicle is given by the equation $y = 10 + 0.9x - 0.01x^2$, where x is the speed of the vehicle between 10 and 75 miles per hour. Find the range of speeds that would give a gas mileage of at least 25 miles per gallon.

Example 10 **Solve $x^2 + 3x - 10 < 0$.**

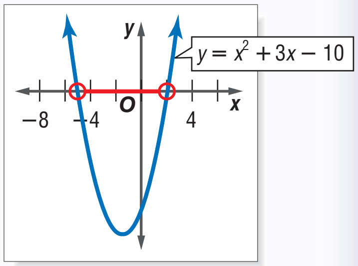
Find the roots of the related equation.

$0 = x^2 + 3x - 10$ Related equation

$0 = (x + 5)(x - 2)$ Factor.

$x + 5 = 0$ or $x - 2 = 0$ Zero Product Property

$x = -5$ $\quad x = 2$ Solve each equation.



The graph opens up since $a > 0$. The graph lies below the x-axis between $x = -5$ and $x = 2$. The solution set is $\{x \mid -5 < x < 2\}$.

CHAPTER 5

Practice Test

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = x^2 - 2x + 5$

2. $f(x) = -3x^2 + 8x$

3. $f(x) = -2x^2 - 7x - 1$

Determine whether each function has a maximum or a minimum value. State the maximum or minimum value of each function.

4. $f(x) = x^2 + 6x + 9$

5. $f(x) = 3x^2 - 12x - 24$

6. $f(x) = -x^2 + 4x$

7. Write a quadratic equation with roots -4 and 5 in standard form.

Solve each equation using the method of your choice. Find exact solutions.

8. $x^2 + x - 42 = 0$

9. $-1.6x^2 - 3.2x + 18 = 0$

10. $15x^2 + 16x - 7 = 0$

11. $x^2 + 8x - 48 = 0$

12. $x^2 + 12x + 11 = 0$

13. $x^2 - 9x - \frac{19}{4} = 0$

14. $3x^2 + 7x - 31 = 0$

15. $10x^2 + 3x = 1$

16. $-11x^2 - 174x + 221 = 0$

17. BALLOONING At a hot-air balloon festival, you throw a weighted marker straight down from an altitude of 250 feet toward a bull's-eye below. The initial velocity of the marker when it leaves your hand is 28 feet per second. Find out how long it will take the marker to hit the target by solving the equation $-16t^2 - 28t + 250 = 0$.

Simplify.

18. $(5 - 2i) - (8 - 11i)$

19. $(14 - 5i)^2$

Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

20. $y = (x + 2)^2 - 3$

21. $y = x^2 + 10x + 27$

22. $y = -9x^2 + 54x - 8$

Graph each inequality.

23. $y \leq x^2 + 6x - 7$

24. $y > -2x^2 + 9$

25. $y \geq -\frac{1}{2}x^2 - 3x + 1$

Solve each inequality using a graph, a table, or algebraically.

26. $(x - 5)(x + 7) < 0$

27. $3x^2 \geq 16$

28. $-5x^2 + x + 2 < 0$

29. PETS A rectangular turtle pen is 6 feet long by 4 feet wide. The pen is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new pen?

30. MULTIPLE CHOICE Which of the following is the sum of both solutions of the equation $x^2 + 8x - 48 = 0$?

A -16

B -8

C -4

D 12

CHAPTER 5

Standardized Test Practice

Cumulative, Chapters 1–5

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the effect on the graph of the equation $y = x^2 + 4$ when it is changed to $y = x^2 - 3$?

A The slope of the graph changes.

B The graph widens.

C The graph is the same shape, and the vertex of the graph is moved down.

D The graph is the same shape, and the vertex of the graph is shifted to the left.

2. What is the solution set for the equation $3(2x + 1)^2 = 27$?

F $\{-5, 4\}$

G $\{-2, 1\}$

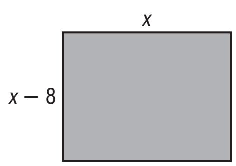
H $\{2, -1\}$

J $\{-3, 3\}$

TEST-TAKING TIP

Question 2 To solve equations or inequalities, you can replace the variables in the question with the values given in each answer choice. The answer choice that results in true statements is the correct answer choice.

3. For what value of x would the rectangle below have an area of 48 square units?



A 4

B 6

C 8

D 12

4. Which shows the functions correctly listed in order from widest to narrowest graph?

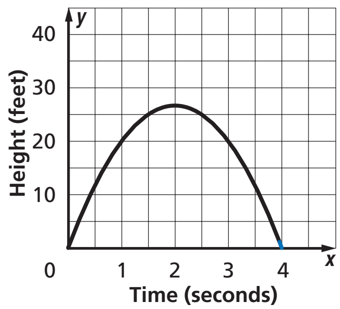
F $y = 8x^2, y = 2x^2, y = \frac{1}{2}x^2, y = -\frac{4}{5}x^2$

G $y = -\frac{4}{5}x^2, y = \frac{1}{2}x^2, y = 2x^2, y = 8x^2$

H $y = \frac{1}{2}x^2, y = -\frac{4}{5}x^2, y = 2x^2, y = 8x^2$

J $y = 8x^2, y = 2x^2, y = -\frac{4}{5}x^2, y = \frac{1}{2}x^2$

5. The graph below shows the height of an object from the time it is propelled from Earth.



For how long is the object above a height of 20 feet?

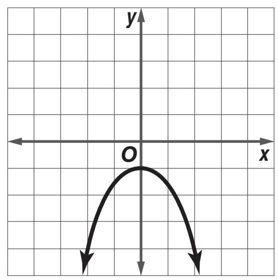
A 0.5 second

B 1 second

C 2 seconds

D 4 seconds

6. Which equation is the parent function of the graph represented below?



F $y = x^2$

G $y = |x|$

H $y = x$

J $y = \sqrt{x}$

Preparing for Standardized Tests

For test-taking strategies and more practice, see pages 941–956.

7. An object is shot straight upward into the air with an initial speed of 800 feet per second. The height h that the object will be after t seconds is given by the equation $h = -16t^2 + 800t$. When will the object reach a height of 10,000 feet?

A 10 seconds

B 25 seconds

C 100 seconds

D 625 seconds

8. What are the roots of the quadratic equation $3x^2 + x = 4$?

F $-1, \frac{4}{3}$

G $-\frac{4}{3}, 1$

H $-2, \frac{2}{3}$

J $-\frac{2}{3}, 2$

9. Which equation will produce the narrowest parabola when graphed?

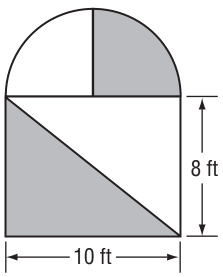
A $y = 3x^2$

B $y = \frac{3}{4}x^2$

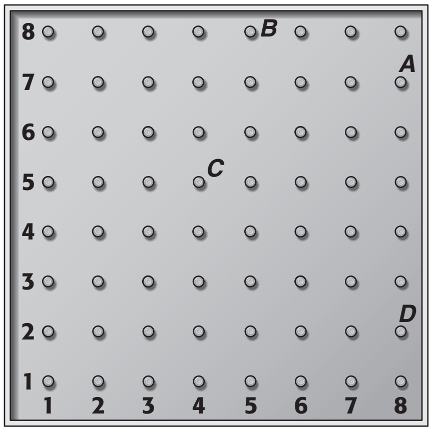
C $y = -\frac{3}{4}x^2$

D $y = -6x^2$

10. **GRIDDABLE** To the nearest tenth, what is the area in square feet of the shaded region below?



11. Mary was given this geoboard to model the slope $-\frac{3}{4}$.



If the peg in the upper right-hand corner represents the origin on a coordinate plane, where could Mary place a rubber band to represent the given slope?

F from peg A to peg B

G from peg A to peg C

H from peg B to peg D

J from peg C to peg D

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. Scott launches a model rocket from ground level. The rocket's height h in meters is given by the equation $h = -4.9t^2 + 56t$, where t is the time in seconds after the launch.

a. What is the maximum height the rocket will reach? Round to the nearest tenth of a meter. Show each step and explain your method.

b. How long after it is launched will the rocket reach its maximum height? Round to the nearest tenth of a second.

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	5-7	5-5	5-3	5-7	5-7	5-1	5-3	5-3	2-3	1-4	2-3	5-7