



# CHAPTER 10

## Conic Sections

### BIG Ideas

- Use the Midpoint and Distance Formulas.
- Write and graph equations of parabolas, circles, ellipses, and hyperbolas.
- Identify conic sections.
- Solve systems of quadratic equations and inequalities.

### Key Vocabulary

circle (p. 574)

conic section (p. 567)

ellipse (p. 581)

hyperbola (p. 590)

parabola (p. 567)

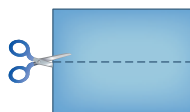
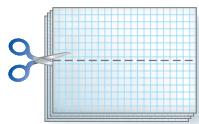
### Real-World Link

**The Ellipse** The Ellipse, which is formally known as President's Park South, is located to the south of the White House. The city planner for Washington, D.C., had intended for The Ellipse to be the backyard for the White House.

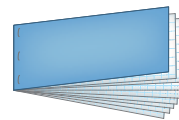
### FOLDABLES<sup>®</sup> Study Organizer

**Conic Sections** Make this Foldable to help you organize your notes. Begin with four sheets of grid paper and one sheet of construction paper.

- 1 Cut** each sheet of grid paper in half lengthwise. Cut the sheet of construction paper in half lengthwise to form a front and back cover for the booklet of grid paper.



- 2 Staple** all the sheets together to form a long, thin notepad of grid paper.



# GET READY for Chapter 10

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Option 2



Take the Online Readiness Quiz at [algebra2.com](http://algebra2.com).

## Option 1

Take the Quick Check below. Refer to the Quick Review for help.

### QUICK Check

Solve each equation by completing the square. (Lesson 5-5)

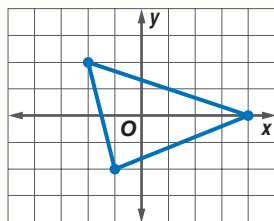
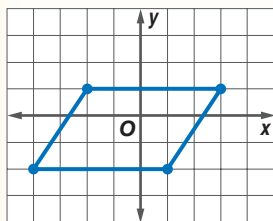
1.  $x^2 + 10x + 24 = 0$     2.  $x^2 - 2x + 2 = 0$

3.  $2x^2 + 5x - 12 = 0$     4.  $x^2 + 8x = -15$

5. **FRAMING** Julio is framing a picture in a 12-inch by 12-inch square frame. The frame is twice as wide at the top and bottom as it is at the sides. If the area of the picture is 54 square inches, what are the dimensions? (Lesson 5-5)

A translation is given for each figure.

- Write the vertex matrix for the figure.
  - Write the translation matrix.
  - Find the coordinates in matrix form of the vertices of the translated figure. (Lesson 4-4)
6. translated 4 units left and 2 units up    7. translated 5 units right and 3 units down



8. **ARCHITECTURE** The Connors plot their deck plans on a grid with each unit equal to 1 foot. They place the corners of a hot tub at (2, 5), (14, 5), (14, 17), and (2, 17). Changes to the plan now require that the hot tub's perimeter be three-fourths that of the original. Determine possible new coordinates for the hot tub. (Lesson 4-4)

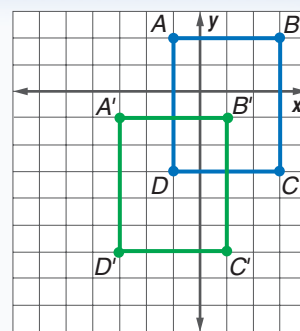
### QUICK Review

**Example 1** Solve  $x^2 - x - 156 = 0$  by completing the square.

$$\begin{aligned} x^2 + (-x) &= 156 \\ x^2 + (-x) + \frac{1}{4} &= 156 + \frac{1}{4} \\ \left(x - \frac{1}{2}\right)^2 &= \frac{625}{4} \\ x - \frac{1}{2} &= \pm \frac{25}{2} \\ x &= \frac{1}{2} \pm \frac{25}{2} \rightarrow x = -12 \text{ or } 13 \end{aligned}$$

**Example 2**

Translate the blue figure 2 units left and 3 units down.



- Write the vertex matrix for the given figure.
- Write the translation matrix.
- Find the coordinates in matrix form of the vertices of the translated figure.

a.  $\begin{bmatrix} -1 & 3 & 3 & -1 \\ 2 & 2 & -3 & -3 \end{bmatrix}$

- b. To translate the figure, add the translation matrix  $\begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix}$  to the vertex matrix.

c.  $\begin{bmatrix} -1 & 3 & 3 & -1 \\ 2 & 2 & -3 & -3 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix}$   
Original Vertex Matrix Translation Matrix  
 $= \begin{bmatrix} -3 & 1 & 1 & -3 \\ -1 & -1 & -6 & -6 \end{bmatrix}$



# Midpoint and Distance Formulas

## Main Ideas

- Find the midpoint of a segment on the coordinate plane.
- Find the distance between two points on the coordinate plane.

## GET READY for the Lesson

A square grid is superimposed on a map of eastern Nebraska where emergency medical assistance by helicopter is available from both Lincoln and Omaha. You can use the formulas in this lesson to determine whether the site of an emergency is closer to Lincoln or to Omaha.



Each side = 10 miles

**The Midpoint Formula** Recall that point  $M$  is the midpoint of segment  $PQ$  if  $M$  is between  $P$  and  $Q$  and  $PM = MQ$ . There is a formula for the coordinates of the midpoint of a segment in terms of the coordinates of the endpoints.

## Study Tip

### Midpoints

The coordinates of the midpoint are the means of the coordinates of the endpoints.

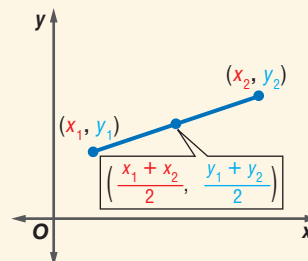
## KEY CONCEPT

### Midpoint Formula

**Words** If a line segment has endpoints at  $(x_1, y_1)$  and  $(x_2, y_2)$ , then the midpoint of the segment has coordinates  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ .

**Symbols** midpoint =  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

### Model

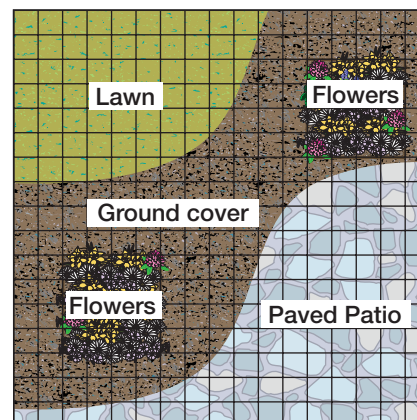


You will show that this formula is correct in Exercise 38.

## EXAMPLE Find a Midpoint

**LANDSCAPING** A landscape design includes two square flower beds and a sprinkler halfway between them. Find the coordinates of the sprinkler if the origin is at the lower left corner of the grid.

The centers of the flower beds are at  $(4, 5)$  and  $(14, 13)$ . The sprinkler will be at the midpoint of the segment joining these points.



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 14}{2}, \frac{5 + 13}{2}\right)$$

$$= \left(\frac{18}{2}, \frac{18}{2}\right) \text{ or } (9, 9)$$

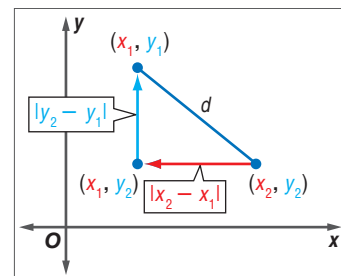
The sprinkler will have coordinates (9, 9).

### CHECK Your Progress

1. The landscape architect decides to place a bench in the middle of the lawn area. Find the coordinates of the bench using the endpoints (0, 17) and (7, 11).

**The Distance Formula** Recall that the distance between two points on a number line whose coordinates are  $a$  and  $b$  is  $|a - b|$  or  $|b - a|$ . You can use this fact and the Pythagorean Theorem to derive a formula for the distance between two points on a coordinate plane.

Suppose  $(x_1, y_1)$  and  $(x_2, y_2)$  name two points. Draw a right triangle with vertices at these points and the point  $(x_1, y_2)$ . The lengths of the legs of the right triangle are  $|x_2 - x_1|$  and  $|y_2 - y_1|$ . Let  $d$  represent the distance between  $(x_1, y_1)$  and  $(x_2, y_2)$ .



$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Substitute.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$|x_2 - x_1|^2 = (x_2 - x_1)^2; |y_2 - y_1|^2 = (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the nonnegative square root of each side.

### Study Tip

#### Distance

In mathematics, just as in life, distances are always nonnegative.

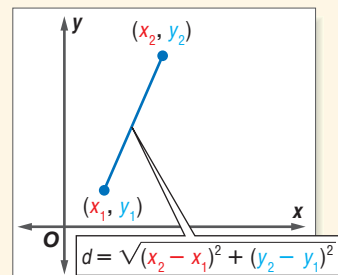
### KEY CONCEPT

#### Distance Formula

**Words** The distance between two points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

**Model**



### EXAMPLE Find the Distance Between Two Points

- 2 Find the distance between  $A(-3, 6)$  and  $B(4, -4)$ .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance Formula

$$= \sqrt{[4 - (-3)]^2 + (-4 - 6)^2}$$

Let  $(x_1, y_1) = (-3, 6)$  and  $(x_2, y_2) = (4, -4)$ .

$$= \sqrt{7^2 + (-10)^2}$$

Subtract.

$$= \sqrt{49 + 100} \text{ or } \sqrt{149} \text{ units}$$

### CHECK Your Progress

2. Find the distance between  $R(-6, 5)$  and  $S(-3, -2)$ .





## STANDARDIZED TEST EXAMPLE

A coordinate grid is placed over a California map. Bakersfield is located at  $(3, -7)$ , and Fresno is located at  $(-7, 9)$ . If Tulare is halfway between Bakersfield and Fresno, which is the closest to the distance in coordinate units from Bakersfield to Tulare?

A 6.25

B 9.5

C 12.5

D 19

### Read the Test Item

The question asks us to find the distance between one city and the midpoint. Find the midpoint and then use the Distance Formula.

### Solve the Test Item

Use the Midpoint Formula to find the coordinates of Tulare.

$$\begin{aligned}\text{midpoint} &= \left( \frac{3 + (-7)}{2}, \frac{(-7) + 9}{2} \right) && \text{Midpoint Formula} \\ &= (-2, 1) && \text{Simplify.}\end{aligned}$$

Use the Distance Formula to find the distance between Bakersfield  $(3, -7)$  and Tulare  $(-2, 1)$ .

$$\begin{aligned}\text{distance} &= \sqrt{(-2 - 3)^2 + (1 - (-7))^2} && \text{Distance Formula} \\ &= \sqrt{(-5)^2 + 8^2} && \text{Subtract.} \\ &= \sqrt{89} \text{ or about } 9.4 && \text{Simplify.}\end{aligned}$$

The answer is B.

### Test-Taking Tip

In order to check your answer, find the distance between Tulare and Fresno. Since Tulare is at the midpoint, these distances should be equal.

## CHECK Your Progress

3. The coordinates for points  $A$  and  $B$  are  $(-4, -5)$  and  $(10, -7)$ , respectively. Find the distance between the midpoint of  $A$  and  $B$  and point  $B$ .

F  $\sqrt{10}$  units

H  $\sqrt{50}$  units

G  $5\sqrt{10}$  units

J  $10\sqrt{5}$  units

Online Personal Tutor at [algebra2.com](http://algebra2.com)

## CHECK Your Understanding

**Example 1**  
(pp. 562–563)

Find the midpoint of the line segment with endpoints at the given coordinates.

1.  $(-5, 6), (1, 7)$

2.  $(8, 9), (-3, -4.5)$

3.  $(13, -4), (10, 14.6)$

4.  $(-12, -2), (-3.5, -7)$

**Example 2**  
(p. 563)

Find the distance between each pair of points with the given coordinates.

5.  $(2, -4), (10, -10)$

6.  $(7, 8), (-4, 9)$

7.  $(0.5, 1.4), (1.1, 2.9)$

8.  $(-4.3, 2.6), (6.5, -3.4)$

**Example 3**  
(p. 564)

9. **STANDARDIZED TEST PRACTICE** The map of a mall is overlaid with a numeric grid. The kiosk for the cell phone store is halfway between Terry's Ice Cream and the See Clearly eyeglass store. If the ice cream store is at  $(2, 4)$  and the eyeglass store is at  $(78, 46)$ , find the distance the kiosk is from the eyeglass store.

# Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–15	1
16–21	2, 3

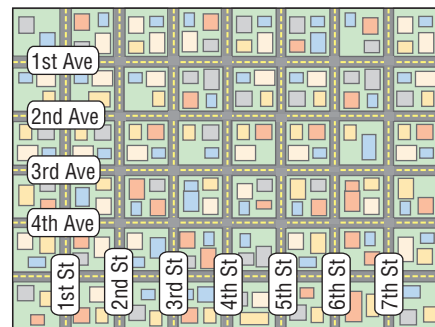


Find the midpoint of the line segment with endpoints at the given coordinates.

10.  $(8, 3), (16, 7)$       11.  $(-5, 3), (-3, -7)$   
 12.  $(6, -5), (-2, -7)$       13.  $(5, 9), (12, 18)$

14. **GEOMETRY** Triangle  $MNP$  has vertices  $M(3, 5)$ ,  $N(-2, 8)$ , and  $P(7, -4)$ . Find the coordinates of the midpoint of each side.

15. **REAL ESTATE** In John's town, the numbered streets and avenues form a grid. He belongs to a gym at the corner of 12th Street and 15th Avenue, and the deli where he works is at the corner of 4th Street and 5th Avenue. He wants to rent an apartment halfway between the two. In what area should he look?



Find the distance between each pair of points with the given coordinates.

16.  $(-4, 9), (1, -3)$       17.  $(1, -14), (-6, 10)$       18.  $(-4, -10), (-3, -11)$   
 19.  $(9, -2), (12, -14)$       20.  $(0.23, 0.4), (0.68, -0.2)$       21.  $(2.3, -1.2), (-4.5, 3.7)$

22. **GEOMETRY** Quadrilateral  $RSTV$  has vertices  $R(-4, 6)$ ,  $S(4, 5)$ ,  $T(6, 3)$ , and  $V(5, -8)$ . Find the perimeter of the quadrilateral.

23. **GEOMETRY** Triangle  $BCD$  has vertices  $B(4, 9)$ ,  $C(8, -9)$ , and  $D(-6, 5)$ . Find the length of median  $\overline{BP}$ . (*Hint: A median connects a vertex of a triangle to the midpoint of the opposite side.*)

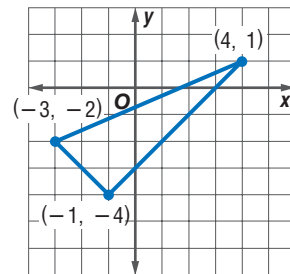
Find the midpoint of the line segment with endpoints at the given coordinates. Then find the distance between the points.

24.  $(-3, -\frac{2}{11}), (5, \frac{9}{11})$       25.  $(0, \frac{1}{5}), (\frac{3}{5}, -\frac{3}{5})$   
 26.  $(2\sqrt{3}, -5), (-3\sqrt{3}, 9)$       27.  $(\frac{2\sqrt{3}}{3}, \frac{\sqrt{5}}{4}), (-\frac{2\sqrt{3}}{3}, \frac{\sqrt{5}}{2})$

28. **GEOMETRY** Find the perimeter and area of the triangle at the right.

29. **GEOMETRY** A circle has a radius with endpoints at  $(2, 5)$  and  $(-1, -4)$ . Find the circumference and area of the circle.

30. **GEOMETRY** Circle  $Q$  has a diameter  $\overline{AB}$ . If  $A$  is at  $(-3, -5)$  and the center of the circle is at  $(2, 3)$ , find the coordinates of  $B$ .



**GEOGRAPHY** For Exercises 31 and 32, use the following information.

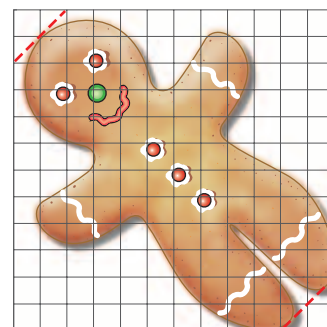
The U.S. Geological Survey (USGS) has determined the official center of the United States.

31. Approximate the center of the United States. Describe your method.  
 32. **RESEARCH** Use the Internet or other reference to look up the USGS geographical center of the United States. How does the location given by USGS compare to the result of your method?



**TRAVEL** For Exercises 33 and 34, use the figure at the right, where a grid is superimposed on a map of a portion of the state of Alabama.

33. How far is it from Birmingham to Montgomery if each unit on the grid represents 40 miles?
34. How long would it take a plane to fly from Huntsville to Montgomery if its average speed is 180 miles per hour?



**EXTRA PRACTICE**  
See pages 911, 935.  
**Math online**  
Self-Check Quiz at  
[algebra2.com](http://algebra2.com)

**H.O.T. Problems**

35. **WOODWORKING** A stage crew is making the set for a children's play. They want to make some gingerbread shapes out of leftover squares of wood with sides measuring 1 foot. They can make taller shapes by cutting them out of the wood diagonally. To the nearest inch, how tall is the gingerbread shape in the drawing?
36. **OPEN ENDED** Find two points that are  $\sqrt{29}$  units apart.
37. **REASONING** Identify all of the points that are equidistant from the endpoints of a given segment.
38. **CHALLENGE** Verify the Midpoint Formula. (*Hint: You must show that the formula gives the coordinates of a point on the line through the given endpoints and that the point is equidistant from the endpoints.*)
39. **Writing in Math** Explain how to use the Distance Formula to approximate the distance between two cities on a map.

**STANDARDIZED TEST PRACTICE**

40. **ACT/SAT** Point  $D(5, -1)$  is the midpoint of segment  $\overline{CE}$ . If point  $C$  has coordinates  $(3, 2)$ , what are the coordinates of point  $E$ ?
- A  $(8, 1)$   
B  $(7, -4)$   
C  $(2, -3)$   
D  $(4, \frac{1}{2})$
41. **REVIEW** If  $\log_{10} x = -3$ , what is the value of  $x$ ?
- F  $x = 1000$       H  $x = \sqrt{\frac{1}{100}}$   
G  $x = \frac{1}{1000}$       J  $x = -\sqrt{\frac{1}{100}}$

**Spiral Review**

42. **COMPUTERS** Suppose a computer that costs \$3000 new is only worth \$600 after 3 years. What is the average annual rate of depreciation? (*Lesson 9-6*)

Solve each equation. Round to the nearest ten-thousandth. (*Lesson 9-5*)

43.  $3e^x - 2 = 0$       44.  $e^{3x} = 4$       45.  $\ln(x + 2) = 5$

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Write in the form  $y = a(x - h)^2 + k$ . (*Lesson 5-5*)

46.  $y = x^2 + 6x + 9$       47.  $y = 2x^2 + 20x + 50$       48.  $y = -3x^2 - 18x - 10$



## Main Ideas

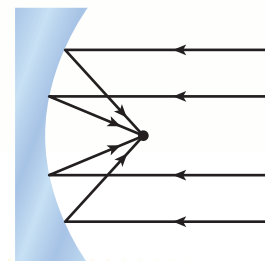
- Write equations of parabolas in standard form.
- Graph parabolas.

## New Vocabulary

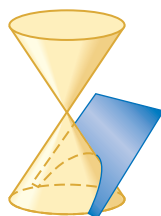
parabola  
conic section  
focus  
directrix  
latus rectum

## GET READY for the Lesson

A mirror or other reflective object in the shape of a parabola reflects all parallel incoming rays to the same point. Or, if that point is the source of rays, the reflected rays are all parallel.



**Equations of Parabolas** In Chapter 5, you learned that the graph of an equation of the form  $y = ax^2 + bx + c$  is a **parabola**. A parabola can also be obtained by slicing a double cone on a slant as shown below on the left. Any figure that can be obtained by slicing a double cone is called a **conic section**. Other conic sections are also shown below.



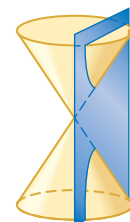
parabola



circle



ellipse



hyperbola

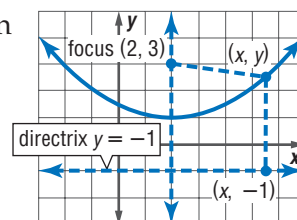
## Study Tip

## Focus of a Parabola

The focus is the special point referred to at the beginning of the lesson.

A parabola can also be defined as the set of all points in a plane that are the same distance from a given point called the **focus** and a given line called the **directrix**.

The parabola at the right has its focus at  $(2, 3)$ , and the equation of its directrix is  $y = -1$ . You can use the Distance Formula to find an equation of this parabola.



Let  $(x, y)$  be any point on this parabola. The distance from this point to the focus must be the same as the distance from this point to the directrix. The distance from a point to a line is measured along the perpendicular from the point to the line.

$$\text{distance from } (x, y) \text{ to } (2, 3) = \text{distance from } (x, y) \text{ to } (x, -1)$$

$$\sqrt{(x - 2)^2 + (y - 3)^2} = \sqrt{(x - x)^2 + [y - (-1)]^2}$$

$$(x - 2)^2 + (y - 3)^2 = 0^2 + (y + 1)^2 \quad \text{Square each side.}$$

$$(x - 2)^2 + y^2 - 6y + 9 = y^2 + 2y + 1 \quad \text{Square } y - 3 \text{ and } y + 1.$$

$$(x - 2)^2 + 8 = 8y \quad \text{Isolate the } y\text{-terms.}$$

$$\frac{1}{8}(x - 2)^2 + 1 = y \quad \text{Divide each side by 8.}$$

An equation of the parabola with focus at  $(2, 3)$  and directrix with equation  $y = -1$  is  $y = \frac{1}{8}(x - 2)^2 + 1$ . The equation of the *axis of symmetry* for this parabola is  $x = 2$ . The axis of symmetry intersects the parabola at a point called the *vertex*. The vertex is the point where the graph turns. The vertex of this parabola is at  $(2, 1)$ . Since  $\frac{1}{8}$  is positive, the parabola opens upward. Any equation of the form  $y = ax^2 + bx + c$  can be written in standard form.

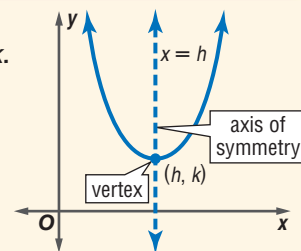


## KEY CONCEPT

## Equation of a Parabola

The standard form of the equation of a parabola with vertex  $(h, k)$  and axis of symmetry  $x = h$  is  $y = a(x - h)^2 + k$ .

- If  $a > 0$ ,  $k$  is the minimum value of the related function and the parabola opens upward.
- If  $a < 0$ ,  $k$  is the maximum value of the related function and the parabola opens downward.



## EXAMPLE

### Analyze the Equation of a Parabola

- 1 Write  $y = 3x^2 + 24x + 50$  in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

$$y = 3x^2 + 24x + 50$$

Original equation

$$= 3(x^2 + 8x) + 50$$

Factor 3 from the  $x$ -terms.

$$= 3(x^2 + 8x + \blacksquare) + 50 - 3(\blacksquare)$$

Complete the square on the right side.

$$= 3(x^2 + 8x + 16) + 50 - 3(16)$$

The 16 added when you complete the square is multiplied by 3.

$$= 3(x + 4)^2 + 2$$

$$= 3[x - (-4)]^2 + 2$$

$$(h, k) = (-4, 2)$$

The vertex of this parabola is located at  $(-4, 2)$ , and the equation of the axis of symmetry is  $x = -4$ . The parabola opens upward.



## CHECK Your Progress

1. Write  $y = 4x^2 + 16x + 34$  in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

## Study Tip

### Look Back

To review **completing the square**, see Lesson 5-5.

## Study Tip

### Translations

If  $h$  is positive, translate the graph  $h$  units to the right. If  $h$  is negative, translate the graph  $h$  units to the left. Similarly, if  $k$  is positive, translate the graph  $k$  units up. If  $k$  is negative, translate the graph  $k$  units down.

Notice that each side of the graph is the reflection of the other side about the  $y$ -axis.

## Graph Parabolas

You can use symmetry and translations to graph parabolas. The equation  $y = a(x - h)^2 + k$  can be obtained from  $y = ax^2$  by replacing  $x$  with  $x - h$  and  $y$  with  $y - k$ . Therefore, the graph of  $y = a(x - h)^2 + k$  is the graph of the parent function  $y = ax^2$  translated  $h$  units to the right or left and  $k$  units up or down.

## EXAMPLE

### Graph Parabolas

- 2 Graph each equation.

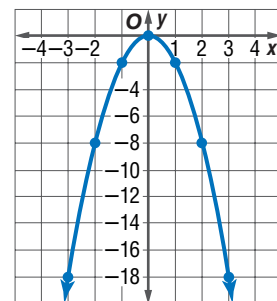
a.  $y = -2x^2$

For this equation,  $h = 0$  and  $k = 0$ .

The vertex is at the origin. Since the equation of the axis of symmetry is  $x = 0$ , substitute some small positive integers for  $x$  and find the corresponding  $y$ -values.

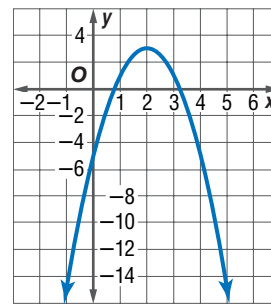
Since the graph is symmetric about the  $y$ -axis, the points at  $(-1, -2)$ ,  $(-2, -8)$ , and  $(-3, -18)$  are also on the parabola. Use all of these points to draw the graph.

$x$	$y$
1	-2
2	-8
3	-18



b.  $y = -2(x - 2)^2 + 3$

The equation is of the form  $y = a(x - h)^2 + k$ , where  $h = 2$  and  $k = 3$ . The graph of this equation is the graph of  $y = -2x^2$  in part a translated 2 units to the right and up 3 units. The vertex is now at  $(2, 3)$ .



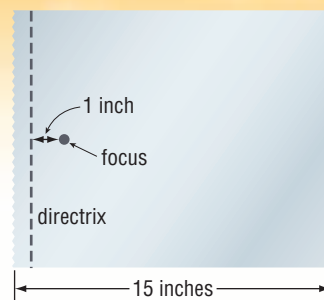
2A.  $y = 3x^2$

2B.  $y = 3(x - 1)^2 - 4$

## ALGEBRA LAB

### Parabolas

**Step 1** Start with a sheet of wax paper that is about 15 inches long and 12 inches wide. Make a line that is perpendicular to the sides of the sheet by folding the sheet near one end. Open up the paper again. This line is the directrix. Mark a point about midway between the sides of the sheet so that the distance from the directrix is about 1 inch. This is the focus.



Put the focus on top of any point on the directrix and crease the paper. Make about 20 more creases by placing the focus on top of other points on the directrix. The lines form the outline of a parabola.

**Step 2** Start with a new sheet of wax paper. Form another outline of a parabola with a focus that is about 3 inches from the directrix.

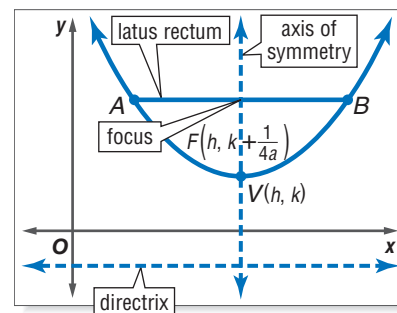
**Step 3** On a new sheet of wax paper, form a third outline of a parabola with a focus that is about 5 inches from the directrix.

### ANALYZE THE RESULTS

Compare the shapes of the three parabolas. How does the distance between the focus and the directrix affect the shape of a parabola?

The shape of a parabola and the distance between the focus and directrix depend on the value of  $a$  in the equation. The line segment through the focus of a parabola and perpendicular to the axis of symmetry is called the **latus rectum**. The endpoints of the latus rectum lie on the parabola.

In the figure, the latus rectum is  $\overline{AB}$ . The length of the latus rectum of the parabola with equation  $y = a(x - h)^2 + k$  is  $\left|\frac{1}{a}\right|$  units. The endpoints of the latus rectum are  $\left|\frac{1}{2a}\right|$  units from the focus.



Equations of parabolas with vertical axes of symmetry have the parent function  $y = x^2$  and are of the form  $y = a(x - h)^2 + k$ . These are functions. Equations of parabolas with horizontal axes of symmetry are of the form  $x = a(y - k)^2 + h$  and are not functions. The parent graph for these equations is  $x = y^2$ .





KEY CONCEPT		Information About Parabolas
Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Vertex	$(h, k)$	$(h, k)$
Axis of Symmetry	$x = h$	$y = k$
Focus	$(h, k + \frac{1}{4a})$	$(h + \frac{1}{4a}, k)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$ , downward if $a < 0$	right if $a > 0$ , left if $a < 0$
Length of Latus Rectum	$ \frac{1}{a} $ units	$ \frac{1}{a} $ units

### EXAMPLE Graph an Equation Not in Standard Form

#### 3 Graph $4x - y^2 = 2y + 13$ .

First, write the equation in the form  $x = a(y - k)^2 + h$ .

$$4x - y^2 = 2y + 13$$

There is a  $y^2$  term, so isolate the  $y$  and  $y^2$  terms.

$$4x = y^2 + 2y + 13$$

Add  $y^2$  to each side.

$$4x = (y^2 + 2y + \blacksquare) + 13 - \blacksquare$$

Complete the square.

$$4x = (y^2 + 2y + 1) + 13 - 1$$

Add and subtract 1, since  $(\frac{2}{2})^2 = 1$ .

$$4x = (y + 1)^2 + 12$$

Write  $y^2 + 2y + 1$  as a square.

$$x = \frac{1}{4}(y + 1)^2 + 3$$

$(h, k) = (3, -1)$

Then use the following information to draw the graph based on the parent graph,  $x = y^2$ .

vertex:  $(3, -1)$

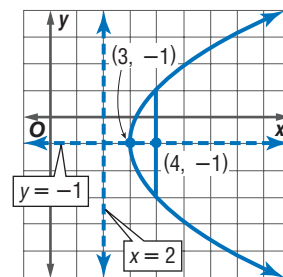
axis of symmetry:  $y = -1$

focus:  $(3 + \frac{1}{4(\frac{1}{4})}, -1)$  or  $(4, -1)$

directrix:  $x = 3 - \frac{1}{4(\frac{1}{4})}$  or 2

direction of opening: right, since  $a > 0$

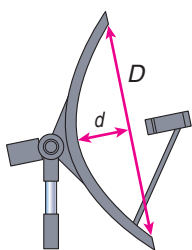
length of latus rectum:  $|\frac{1}{(\frac{1}{4})}|$  or 4 units



The graph is wider than the graph of  $x = y^2$  since  $a < 1$  and shifted 3 units right and 1 unit down.

### Study Tip

When graphing these functions, it may be helpful to sketch the graph of the parent function.



### Real-World Link

The important characteristics of a satellite dish are the diameter  $D$ , depth  $d$ , and the ratio  $\frac{f}{D}$ , where  $f$  is the distance between the focus and the vertex. A typical dish has the values  $D = 60$  cm,  $d = 6.25$  cm, and  $\frac{f}{D} = 0.6$ .

Source: 2000networks.com

### CHECK Your Progress Graph each equation.

3A.  $3x - y^2 = 4y + 25$

3B.  $y = x^2 + 6x - 4$

### EXAMPLE Write and Graph an Equation for a Parabola

#### 4 SATELLITE TV Use the information at the left about satellite dishes.

- a. Write an equation that models a cross section of a satellite dish. Assume that the focus is at the origin and the parabola opens to the right.

First, solve for  $f$ . Since  $\frac{f}{D} = 0.6$ , and  $D = 60$ ,  $f = 0.6(60)$  or 36.

The focus is at  $(0, 0)$ , and the parabola opens to the right. So the vertex must be at  $(-36, 0)$ . Thus,  $h = -36$  and  $k = 0$ . Now find  $a$ .

$$-36 + \frac{1}{4a} = 0 \quad h = -36; \text{ The } x\text{-coordinate of the focus is } 0.$$

$$\frac{1}{4a} = 36 \quad \text{Add 36 to each side.}$$

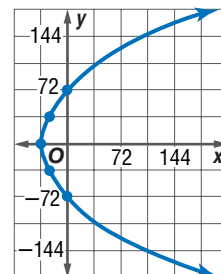
$$1 = 144a \quad \text{Multiply each side by } 4a.$$

$$\frac{1}{144} = a \quad \text{Divide each side by 144.}$$

An equation of the parabola is  $x = \frac{1}{144}y^2 - 36$ .

**b. Graph the equation.**

The length of the latus rectum is  $\left| \frac{1}{\frac{1}{144}} \right|$  or 144 units, so the graph must pass through  $(0, 72)$  and  $(0, -72)$ . According to the diameter and depth of the dish, the graph must pass through  $(-29.75, 30)$  and  $(-29.75, -30)$ . Use these points and the information from part a to draw the graph.



**CHECK Your Progress**

4. Write and graph an equation for a satellite dish with diameter  $D$  of 34 inches and ratio  $\frac{f}{D}$  of 0.6.

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**CHECK Your Understanding**

**Example 1**  
(p. 568)

1. Write  $y = 2x^2 - 12x + 6$  in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

**Examples 2, 3**  
(pp. 568–570)

**Graph each equation.**

2.  $y = (x - 3)^2 - 4$

3.  $y = 2(x + 7)^2 + 3$

4.  $y = -3x^2 - 8x - 6$

5.  $x = \frac{2}{3}y^2 - 6y + 12$

**Example 4**  
(pp. 570–571)

6. **COMMUNICATION** A microphone is placed at the focus of a parabolic reflector to collect sound for the television broadcast of a football game. Write an equation for the cross section, assuming that the focus is at the origin, the focus is 6 inches from the vertex, and the parabola opens to the right.

**Exercises**

HOMEWORK	HELP
For Exercises	See Examples
7–10	1
11–14	2
15–19	3
20–23	4

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

7.  $y = x^2 - 6x + 11$

8.  $x = y^2 + 14y + 20$

9.  $y = \frac{1}{2}x^2 + 12x - 8$

10.  $x = 3y^2 + 5y - 9$

**Graph each equation.**

11.  $y = -\frac{1}{6}x^2$

12.  $x = \frac{1}{2}y^2$

13.  $y = \frac{1}{3}(x + 6)^2 + 3$

14.  $y = -\frac{1}{2}(x - 1)^2 + 4$

15.  $4(x - 2) = (y + 3)^2$

16.  $(y - 8)^2 = -4(x - 4)$

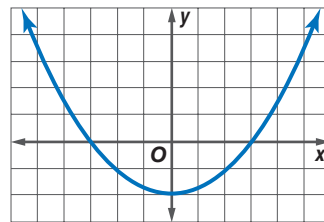
17.  $y = x^2 - 12x + 20$

18.  $x = y^2 - 14y + 25$

19.  $x = 5y^2 + 25y + 60$



20. Write an equation for the graph at the right.



21. **MANUFACTURING** The reflective surface in a flashlight has a parabolic shape with a cross section that can be modeled by  $y = \frac{1}{3}x^2$ , where  $x$  and  $y$  are in centimeters. How far from the vertex should the filament of the light bulb be located?

22. **BRIDGES** The Bayonne Bridge connects Staten Island, New York, to New Jersey. It has an arch in the shape of a parabola. Write an equation of a parabola to model the arch, assuming that the origin is at the surface of the water, beneath the vertex of the arch.



23. **FOOTBALL** When a ball is thrown or kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 25 feet, and hits the ground 100 feet from where it was kicked. Assuming that the ball was kicked at the origin, write an equation of the parabola that models the flight of the ball.

For Exercises 24–27, use the equation  $x = 3y^2 + 4y + 1$ .

24. Draw the graph. Find the  $x$ -intercept(s) and  $y$ -intercept(s).

25. What is the equation of the axis of symmetry?

26. What are the coordinates of the vertex?

27. How does the graph compare to the graph of the parent function  $x = y^2$ ?

Write an equation for each parabola described below. Then draw the graph.

28. vertex (0, 1), focus (0, 5)

29. vertex (8, 6), focus (2, 6)

30. focus (−4, −2), directrix  $x = -8$

31. vertex (1, 7), directrix  $y = 3$

32. vertex (−7, 4), axis of symmetry  $x = -7$ , measure of latus rectum 6,  $a < 0$

33. vertex (4, 3), axis of symmetry  $y = 3$ , measure of latus rectum 4,  $a > 0$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

34.  $y = 3x^2 - 24x + 50$

35.  $y = -2x^2 + 5x - 10$

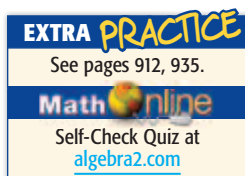
36.  $x = -4y^2 + 6y + 2$

37.  $x = 5y^2 - 10y + 9$

38.  $y = \frac{1}{2}x^2 - 3x + \frac{19}{2}$

39.  $x = -\frac{1}{3}y^2 - 12y + 15$

40. **UMBRELLAS** A beach umbrella has an arch in the shape of a parabola that opens downward. The umbrella spans 9 feet across and  $1\frac{1}{2}$  feet high. Write an equation of a parabola to model the arch, assuming that the origin is at the point where the pole and umbrella meet, beneath the vertex of the arch.





**H.O.T. Problems**

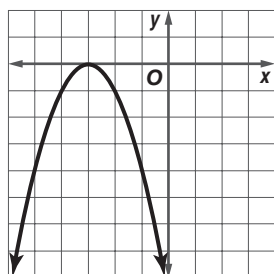
- 41. REASONING** How do you change the equation of the parent function  $y = x^2$  to shift the graph to the right?
- 42. OPEN ENDED** Write an equation for a parabola that opens to the left. Use the parent graph to sketch the graph of your equation.
- 43. FIND THE ERROR** Yasu is finding the standard form of the equation  $y = x^2 + 6x + 4$ . What mistake did she make in her work?
- $$y = x^2 + 6x + 4$$

$$y = x^2 + 6x + 9 + 4$$

$$y = (x + 3)^2 + 4$$
- 44. CHALLENGE** The parabola with equation  $y = (x - 4)^2 + 3$  has its vertex at (4, 3) and passes through (5, 4). Find an equation of a different parabola with its vertex at (4, 3) and that passes through (5, 4).
- 45. Writing in Math** Use the information on page 567 to explain how parabolas can be used in manufacturing. Include why a car headlight with a parabolic reflector is better than one with an unreflected light bulb.

**STANDARDIZED TEST PRACTICE**

- 46. ACT/SAT** Which is the parent function of the graph shown below?



- A  $y = -x$                       C  $y = -|x|$   
 B  $y = -\sqrt{x}$                   D  $y = -x^2$

- 47. REVIEW**  $\log_9 30 =$

- F  $\log_{10} 9 + \log_{10} 30$   
 G  $\log_{10} 9 - \log_{10} 30$   
 H  $(\log_{10} 9)(\log_{10} 30)$   
 J  $\frac{\log_{10} 30}{\log_{10} 9}$

**Spiral Review**

Find the distance between each pair of points with the given coordinates.

(Lesson 10-1)

48. (7, 3), (-5, 8)                      49. (4, -1), (-2, 7)                      50. (-3, 1), (0, 6)

- 51. RADIOACTIVITY** The decay of Radon-222 can be modeled by the equation  $y = ae^{-0.1813t}$ , where  $t$  is measured in days. What is the half-life of Radon-222? (Lesson 9-6)

- 52. HEALTH** Alisa's heart rate is usually 120 beats per minute when she runs. If she runs for 2 hours every day, about how many times will her heart beat during the amount of time she exercises in two weeks? Express in scientific notation. (Lesson 6-1)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Simplify each radical expression. (Lessons 7-1 and 7-2)

53.  $\sqrt{16}$                       54.  $\sqrt{25}$                       55.  $\sqrt{81}$                       56.  $\sqrt{144}$   
 57.  $\sqrt{12}$                       58.  $\sqrt{18}$                       59.  $\sqrt{48}$                       60.  $\sqrt{72}$

**Main Ideas**

- Write equations of circles.
- Graph circles.

**New Vocabulary**

circle  
center

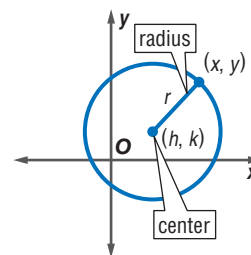
**GET READY for the Lesson**

Radar equipment can be used to detect and locate objects that are too far away to be seen by the human eye. The radar systems at major airports can typically detect and track aircraft up to 45 to 70 miles in any direction from the airport. The boundary of the region that a radar system can monitor can be modeled by a circle.



**Equations of Circles** A **circle** is the set of all points in a plane that are equidistant from a given point in the plane, called the **center**. Any segment whose endpoints are the center and a point on the circle is a **radius** of the circle.

Assume that  $(x, y)$  are the coordinates of a point on the circle at the right. The center is at  $(h, k)$ , and the radius is  $r$ . You can find an equation of the circle by using the Distance Formula.



$$\begin{aligned}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} &= d && \text{Distance Formula} \\ \sqrt{(x - h)^2 + (y - k)^2} &= r && (x_1, y_1) = (h, k), \\ &&& (x_2, y_2) = (x, y), d = r \\ (x - h)^2 + (y - k)^2 &= r^2 && \text{Square each side.}\end{aligned}$$

This is the standard form of the equation of a circle.

**KEY CONCEPT****Equation of a Circle**

The equation of a circle with center  $(h, k)$  and radius  $r$  units is  
 $(x - h)^2 + (y - k)^2 = r^2$ .

You can use the standard form of the equation of a circle to write an equation for a circle given its center and the radius or diameter. Recall that a segment that passes through the center of a circle whose endpoints are on the circle is a diameter.



### Real-World Link

WiFi technology uses radio waves to transmit data. It allows high-speed access to the Internet without the use of cables.

Source: wifiphone.org



## Real-World EXAMPLE

### Write an Equation Given the Radius

**1 DELIVERY** An appliance store offers free delivery within 35 miles of the store. The Jackson store is located 100 miles north and 45 miles east of the corporate office. Write an equation to represent the delivery boundary of the Jackson store if the origin of the coordinate system is the corporate office.

**Words** Since the corporate office is at  $(0, 0)$ , the Jackson store is at  $(45, 100)$ . The boundary of the delivery region is the circle centered at  $(45, 100)$  with radius 35 miles.

**Variables**  $(x - h)^2 + (y - k)^2 = r^2$  Equation of a circle

**Equation**  $[x - (-45)]^2 + (y - 100)^2 = 35^2$   $(h, k) = (45, 100), r = 35$   
 $(x - 45)^2 + (y - 100)^2 = 1225$  Simplify.



### CHECK Your Progress

1. **WIFI** A certain wireless transmitter has a range of thirty miles in any direction. If a WiFi phone is 4 miles south and 3 miles west of the headquarters building, write an equation to represent the area that the phone can communicate via the WiFi system.

## EXAMPLE

### Write an Equation Given a Diameter

- 2 Write an equation for a circle if the endpoints of a diameter are at  $(5, 4)$  and  $(-2, -6)$ .

$$\begin{aligned} (h, k) &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) && \text{Midpoint Formula} \\ &= \left( \frac{5 + (-2)}{2}, \frac{4 + (-6)}{2} \right) && (x_1, y_1) = (5, 4), (x_2, y_2) = (-2, -6) \\ &= \left( \frac{3}{2}, \frac{-2}{2} \right) && \text{Add.} \\ &= \left( \frac{3}{2}, -1 \right) && \text{Simplify.} \end{aligned}$$

Now find the radius.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{\left( \frac{3}{2} - 5 \right)^2 + (-1 - 4)^2} && (x_1, y_1) = (5, 4), (x_2, y_2) = \left( \frac{3}{2}, -1 \right) \\ &= \sqrt{\left( -\frac{7}{2} \right)^2 + (-5)^2} && \text{Subtract.} \\ &= \sqrt{\frac{149}{4}} && \text{Simplify.} \end{aligned}$$

The radius of the circle is  $\sqrt{\frac{149}{4}}$  units, so  $r^2 = \frac{149}{4}$ .

Substitute  $h$ ,  $k$ , and  $r^2$  into the standard form of the equation of a circle.

An equation of the circle is  $\left( x - \frac{3}{2} \right)^2 + (y + 1)^2 = \frac{149}{4}$ .



### CHECK Your Progress

2. Write an equation for a circle if the endpoints of a diameter are at  $(3, -3)$  and  $(1, 5)$ .







**Graph Circles** You can use symmetry to help you graph circles.

### EXAMPLE Graph an Equation in Standard Form

- 3** Find the center and radius of the circle with equation  $x^2 + y^2 = 25$ . Then graph the circle.

The center of the circle is at  $(0, 0)$ , and the radius is 5.

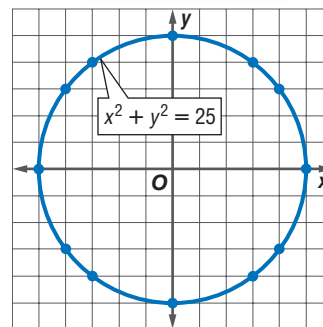
The table lists some integer values for  $x$  and  $y$  that satisfy the equation.

$x$	$y$
0	5
3	4
4	3
5	0

Since the circle is centered at the origin, it is symmetric about the  $y$ -axis. Therefore, the points at  $(-3, 4)$ ,  $(-4, 3)$ , and  $(-5, 0)$  lie on the graph.

The circle is also symmetric about the  $x$ -axis, so the points at  $(-4, -3)$ ,  $(-3, -4)$ ,  $(0, -5)$ ,  $(3, -4)$ , and  $(4, -3)$  lie on the graph.

Graph all of these points and draw the circle that passes through them.



#### Cross-Curricular Project



**Math online** The epicenter of an earthquake can be located by using the equation of a circle. Visit [algebra2.com](http://algebra2.com) to continue work on your project.

#### CHECK Your Progress

- 3.** Find the center and radius of the circle with equation  $x^2 + y^2 = 81$ . Then graph the circle.

Circles with centers that are not at  $(0, 0)$  can be graphed using translations. The equation  $(x - h)^2 + (y - k)^2 = r^2$  is obtained from the equation  $x^2 + y^2 = r^2$  by replacing  $x$  with  $x - h$  and  $y$  with  $y - k$ . So, the graph of  $(x - h)^2 + (y - k)^2 = r^2$  is the graph of  $x^2 + y^2 = r^2$  translated  $h$  units to the right or left and  $k$  units up or down.

### EXAMPLE Graph an Equation Not in Standard Form

- 4** Find the center and radius of the circle with equation  $x^2 + y^2 - 4x + 8y - 5 = 0$ . Then graph the circle.

Complete the squares.

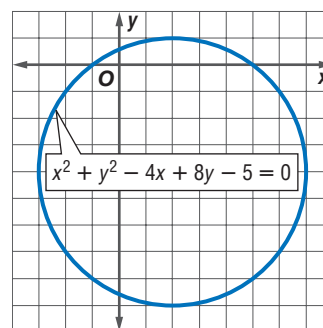
$$x^2 + y^2 - 4x + 8y - 5 = 0$$

$$x^2 - 4x + \blacksquare + y^2 + 8y + \blacksquare = 5 + \blacksquare + \blacksquare$$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = 5 + 4 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 25$$

The center of the circle is at  $(2, -4)$ , and the radius is 5. In the equation from Example 3,  $x$  has been replaced by  $x - 2$ , and  $y$  has been replaced by  $y + 4$ . The graph is the graph from Example 3 translated 2 units to the right and down 4 units.



#### CHECK Your Progress

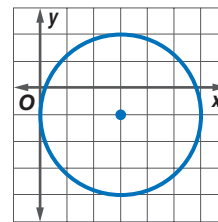
- 4.** Find the center and radius of the circle with equation  $x^2 + y^2 + 4x - 10y - 7 = 0$ . Then graph the circle.



**Personal Tutor at algebra2.com**

**Example 1**  
(p. 575)

1. Write an equation for the graph at the right.



**AEROSPACE** For Exercises 2 and 3, use the following information.

In order for a satellite to remain in a circular orbit above the same spot on Earth, the satellite must be 35,800 kilometers above the equator.

2. Write an equation for the orbit of the satellite. Use the center of Earth as the origin and 6400 kilometers for the radius of Earth.  
3. Draw a labeled sketch of Earth and the orbit to scale.

**Example 2**  
(p. 575)

Write an equation for the circle that satisfies each set of conditions.

4. center  $(-1, -5)$ , radius 2 units  
5. endpoints of a diameter at  $(-4, 1)$  and  $(4, -5)$   
6. endpoints of a diameter at  $(2, -2)$  and  $(-2, -6)$

**Examples 3, 4**  
(p. 576)

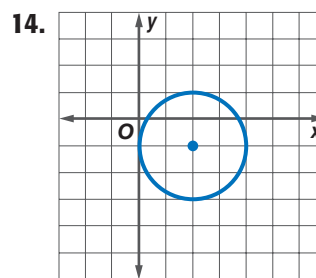
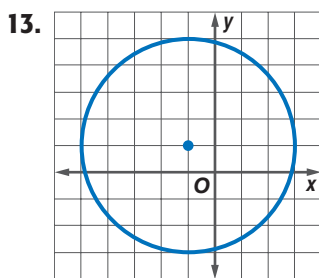
Find the center and radius of the circle with the given equation. Then graph the circle.

7.  $(x - 4)^2 + (y - 1)^2 = 9$     8.  $x^2 + (y - 14)^2 = 34$     9.  $(x - 4)^2 + y^2 = \frac{16}{25}$   
10.  $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{8}{9}$     11.  $x^2 + y^2 + 8x - 6y = 0$     12.  $x^2 + y^2 + 4x - 8 = 0$

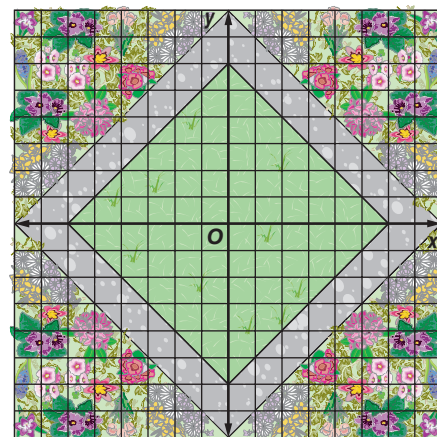
**Exercises**

HOMEWORK HELP	
For Exercises	See Examples
13–17	1
18, 19	2
20–25	3
26–31	4

Write an equation for each graph.



15. **LANDSCAPING** The design of a garden is shown at the right. A pond is to be built in the center region. What is the equation of the largest circular pond centered at the origin that would fit within the walkways?



Write an equation for the circle that satisfies each set of conditions.

16. center  $(0, 3)$ , radius 7 units
17. center  $(-8, 7)$ , radius  $\frac{1}{2}$  unit
18. endpoints of a diameter at  $(-5, 2)$  and  $(3, 6)$
19. endpoints of a diameter at  $(11, 18)$  and  $(-13, -19)$

Find the center and radius of the circle with the given equation. Then graph the circle.

20.  $x^2 + (y + 2)^2 = 4$
21.  $x^2 + y^2 = 144$
22.  $(x - 3)^2 + (y - 1)^2 = 25$
23.  $(x + 3)^2 + (y + 7)^2 = 81$
24.  $(x - 3)^2 + y^2 = 16$
25.  $(x - 3)^2 + (y + 7)^2 = 50$
26.  $x^2 + y^2 + 6y = -50 - 14x$
27.  $x^2 + y^2 - 6y - 16 = 0$
28.  $x^2 + y^2 + 2x - 10 = 0$
29.  $x^2 + y^2 - 18x - 18y + 53 = 0$
30.  $x^2 + y^2 + 9x - 8y + 4 = 0$
31.  $x^2 + y^2 - 3x + 8y = 20$

Write an equation for the circle that satisfies each set of conditions.

32. center  $(8, -9)$ , passes through  $(21, 22)$
33. center  $(-\sqrt{13}, 42)$ , passes through the origin
34. center at  $(-8, -7)$ , tangent to  $y$ -axis
35. center at  $(4, 2)$ , tangent to  $x$ -axis
36. center in the first quadrant; tangent to  $x = -3$ ,  $x = 5$ , and the  $x$ -axis
37. center in the second quadrant; tangent to  $y = -1$ ,  $y = 9$ , and the  $y$ -axis

38. **EARTHQUAKES** The Rose Bowl is located about 35 miles west and about 40 miles north of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 55 miles from the stadium. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.

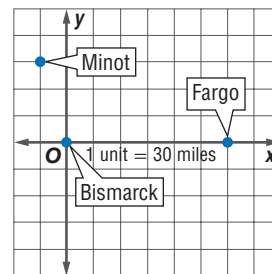


### Real-World Link

Southern California has about 10,000 earthquakes per year. Most are too small to be felt.

Source: earthquake.usgs.gov

39. **RADIO** The diagram at the right shows the relative locations of some cities in North Dakota. The  $x$ -axis represents Interstate 94. While driving west on the highway, Doralina is listening to a radio station broadcasting from Minot. She estimates the range of the signal to be 120 miles. How far west of Bismarck will she be able to pick up the signal?



For Exercises 40–43, use the following information.

Since a circle is not the graph of a function, you cannot enter its equation directly into a graphing calculator. Instead, you must solve the equation for  $y$ . The result will contain a  $\pm$  symbol, so you will have two functions.

40. Solve  $(x + 3)^2 + y^2 = 16$  for  $y$ .
41. What two functions should you enter to graph the given equation?
42. Graph  $(x + 3)^2 + y^2 = 16$  on a graphing calculator.
43. Solve  $(x + 3)^2 + y^2 = 16$  for  $x$ . What parts of the circle do the two expressions for  $x$  represent?



**EXTRA PRACTICE**  
See pages 912, 935.  
**Math online**  
Self-Check Quiz at [algebra2.com](http://algebra2.com)

- 44. OPEN ENDED** Write an equation for a circle with center at  $(6, -2)$ .
- 45. REASONING** Write  $x^2 + y^2 + 6x - 2y - 54 = 0$  in standard form by completing the square. Describe the transformation that can be applied to the graph of  $x^2 + y^2 = 64$  to obtain the graph of the given equation.
- 46. FIND THE ERROR** Juwan says that the circle with equation  $(x - 4)^2 + y^2 = 36$  has radius 36 units. Lucy says that the radius is 6 units. Who is correct? Explain your reasoning.
- 47. CHALLENGE** A circle has its center on the line with equation  $y = 2x$ . It passes through  $(1, -3)$  and has a radius of  $\sqrt{5}$  units. Write an equation of the circle.
- 48. Writing in Math** Use the information about radar equipment on page 574 to explain why circles are important in air traffic control. Include an equation of the circle that determines the boundary of the region where planes can be detected if the range of the radar is 50 miles and the radar is at the origin.

**STANDARDIZED TEST PRACTICE**

- 49. ACT/SAT** What is the center of the circle with equation  $x^2 + y^2 - 10x + 6y + 27 = 0$ ?
- A  $(-10, 6)$   
 B  $(1, 1)$   
 C  $(10, -6)$   
 D  $(5, -3)$
- 50. REVIEW** If the surface area of a cube is increased by a factor of 9, how is the length of the side of the cube changed?
- F It is 2 times the original length.  
 G It is 3 times the original length.  
 H It is 4 times the original length.  
 J It is 5 times the original length.

**Spiral Review**

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. (Lesson 10-2)

**51.**  $x = -3y^2 + 1$       **52.**  $y + 2 = -(x - 3)^2$       **53.**  $y = x^2 + 4x$

Find the midpoint of the line segment with endpoints having the given coordinates. (Lesson 10-1)

**54.**  $(5, -7), (3, -1)$       **55.**  $(2, -9), (-4, 5)$       **56.**  $(8, 0), (-5, 12)$

Find all of the rational zeros for each function. (Lesson 6-8)

**57.**  $f(x) = x^3 + 5x^2 + 2x - 8$       **58.**  $g(x) = 2x^3 - 9x^2 + 7x + 6$

- 59. PHOTOGRAPHY** The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture? (Lesson 3-2)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. Assume that all variables are positive. (Lesson 5-5)

**60.**  $c^2 = 13^2 - 5^2$       **61.**  $c^2 = 10^2 - 8^2$       **62.**  $(\sqrt{7})^2 = a^2 - 3^2$       **63.**  $4^2 = 6^2 - b^2$



Algebra Lab  
Investigating Ellipses

## ACTIVITY

Follow the steps below to construct another type of conic section.

- Step 1** Place two thumbtacks in a piece of cardboard, about 1 foot apart.
- Step 2** Tie a knot in a piece of string and loop it around the thumbtacks.
- Step 3** Place your pencil in the string. Keep the string tight and draw a curve.
- Step 4** Continue drawing until you return to your starting point.

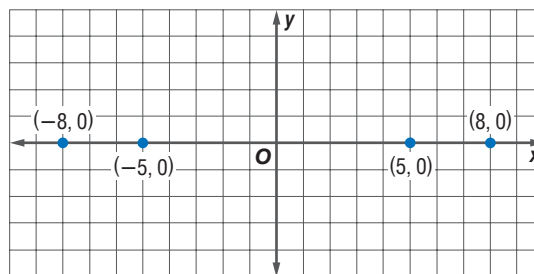
The curve you have drawn is called an **ellipse**. The points where the thumbtacks are located are called the **foci** of the ellipse. *Foci* is the plural of *focus*.



## MODEL AND ANALYZE

Place a large piece of grid paper on a piece of cardboard.

1. Place the thumbtacks at  $(8, 0)$  and  $(-8, 0)$ . Choose a string long enough to loop around both thumbtacks. Draw an ellipse.
2. Repeat Exercise 1, but place the thumbtacks at  $(5, 0)$  and  $(-5, 0)$ . Use the same loop of string and draw an ellipse. How does this ellipse compare to the one in Exercise 1?



Place the thumbtacks at each set of points and draw an ellipse. You may change the length of the loop of string if you like.

3.  $(12, 0), (-12, 0)$
4.  $(2, 0), (-2, 0)$
5.  $(14, 4), (-10, 4)$

## ANALYZE THE RESULTS

In Exercises 6–10, describe what happens to the shape of an ellipse when each change is made.

6. The thumbtacks are moved closer together.
7. The thumbtacks are moved farther apart.
8. The length of the loop of string is increased.
9. The thumbtacks are arranged vertically.
10. One thumbtack is removed, and the string is looped around the remaining thumbtack.

**Main Ideas**

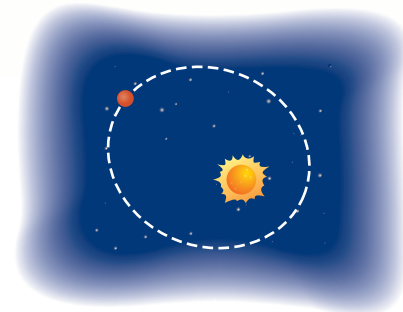
- Write equations of ellipses.
- Graph ellipses.

**New Vocabulary**

ellipse  
foci  
major axis  
minor axis  
center

**GET READY for the Lesson**

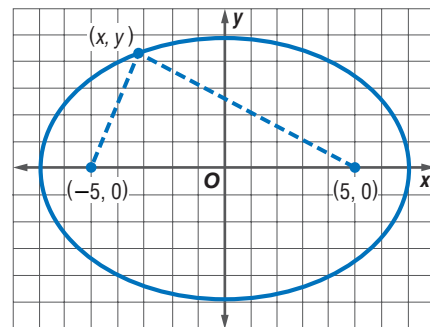
Fascination with the sky has caused people to wonder, observe, and make conjectures about the planets since the beginning of history. Since the early 1600s, the orbits of the planets have been known to be ellipses with the Sun at a focus.



**Equations of Ellipses** As you discovered in the Algebra Lab on page 580, an **ellipse** is the set of all points in a plane such that the sum of the distances from two fixed points is constant. The two fixed points are called the **foci** of the ellipse.

The ellipse at the right has foci at  $(5, 0)$  and  $(-5, 0)$ . The distances from either of the  $x$ -intercepts to the foci are 2 units and 12 units, so the sum of the distances from any point with coordinates  $(x, y)$  on the ellipse to the foci is 14 units.

You can use the Distance Formula and the definition of an ellipse to find an equation of this ellipse.

**Study Tip****Ellipses**

In an ellipse, the constant sum that is the distance from two fixed points must be greater than the distance between the foci.

$$\begin{array}{l} \text{The distance between } (x, y) \text{ and } (-5, 0) \\ \text{the distance between } (x, y) \text{ and } (5, 0) \end{array} + = 14.$$

$$\sqrt{(x+5)^2 + y^2} + \sqrt{(x-5)^2 + y^2} = 14$$

$$\sqrt{(x+5)^2 + y^2} = 14 - \sqrt{(x-5)^2 + y^2}$$

Isolate the radicals.

$$(x+5)^2 + y^2 = 196 - 28\sqrt{(x-5)^2 + y^2} + (x-5)^2 + y^2$$

$$x^2 + 10x + 25 + y^2 = 196 - 28\sqrt{(x-5)^2 + y^2} + x^2 - 10x + 25 + y^2$$

$$20x - 196 = -28\sqrt{(x-5)^2 + y^2}$$

Simplify.

$$5x - 49 = -7\sqrt{(x-5)^2 + y^2}$$

Divide each side by 4.

$$25x^2 - 490x + 2401 = 49[(x-5)^2 + y^2]$$

Square each side.

$$25x^2 - 490x + 2401 = 49x^2 - 490x + 1225 + 49y^2$$

Distributive Property

$$-24x^2 - 49y^2 = -1176$$

Simplify.

$$\frac{x^2}{49} + \frac{y^2}{24} = 1$$

Divide each side by  $-1176$ .

An equation for this ellipse is  $\frac{x^2}{49} + \frac{y^2}{24} = 1$ .



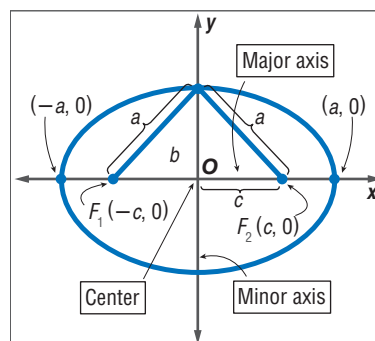
## Study Tip

### Vertices of Ellipses

The endpoints of each axis are called the **vertices** of the ellipse.

Every ellipse has two axes of symmetry. The points at which the ellipse intersects its axes of symmetry determine two segments with endpoints on the ellipse called the **major axis** and the **minor axis**. The axes intersect at the **center** of the ellipse. The foci of an ellipse always lie on the major axis.

Study the ellipse at the right. The sum of the distances from the foci to any point on the ellipse is the same as the length of the major axis, or  $2a$  units. The distance from the center to either focus is  $c$  units. By the Pythagorean Theorem,  $a$ ,  $b$ , and  $c$  are related by the equation  $c^2 = a^2 - b^2$ . Notice that the  $x$ - and  $y$ -intercepts,  $(\pm a, 0)$  and  $(0, \pm b)$ , satisfy the quadratic equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . This is the standard form of the equation of an ellipse with its center at the origin and a horizontal major axis.



## Study Tip

### Equations of Ellipses

In either case,  $a^2 \geq b^2$  and  $c^2 = a^2 - b^2$ . You can determine if the foci are on the  $x$ -axis or the  $y$ -axis by looking at the equation. If the  $x^2$  term has the greater denominator, the foci are on the  $x$ -axis. If the  $y^2$  term has the greater denominator, the foci are on the  $y$ -axis.

### KEY CONCEPT

### Equations of Ellipses with Centers at the Origin

Standard Form of Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

### EXAMPLE

### Write an Equation for a Graph

#### 1 Write an equation for the ellipse.

To write the equation for the ellipse, we need to find the values of  $a$  and  $b$  for the ellipse. We know that the length of the major axis of any ellipse is  $2a$  units. In this ellipse, the length of the major axis is the distance between the points at  $(0, 6)$  and  $(0, -6)$ . This distance is 12 units.

$$2a = 12 \quad \text{Length of major axis} = 12$$

$$a = 6 \quad \text{Divide each side by 2.}$$

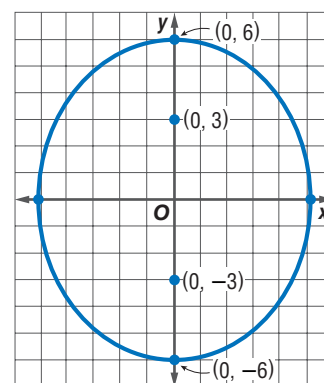
The foci are located at  $(0, 3)$  and  $(0, -3)$ , so  $c = 3$ . We can use the relationship between  $a$ ,  $b$ , and  $c$  to determine the value of  $b$ .

$$c^2 = a^2 - b^2 \quad \text{Equation relating } a, b, \text{ and } c$$

$$9 = 36 - b^2 \quad c = 3 \text{ and } a = 6$$

$$b^2 = 27 \quad \text{Solve for } b^2.$$

Since the major axis is vertical, substitute 36 for  $a^2$  and 27 for  $b^2$  in the form  $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ . An equation of the ellipse is  $\frac{y^2}{36} + \frac{x^2}{27} = 1$ .





### Real-World Link

The whispering gallery at Chicago's Museum of Science and Industry has a parabolic dish at each focus to help collect sound.

Source: msichicago.org



### CHECK Your Progress

- Write an equation for the ellipse with endpoints of the major axis at  $(-5, 0)$  and  $(5, 0)$  and endpoints of the minor axis at  $(0, -2)$  and  $(0, 2)$ .

### EXAMPLE

### Write an Equation Given the Lengths of the Axes



**MUSEUMS** In an ellipse, sound or light coming from one focus is reflected to the other focus. In a whispering gallery, a person can hear another person whisper from across the room if the two people are standing at the foci. The whispering gallery at the Museum of Science and Industry in Chicago has an elliptical cross section that is 13 feet 6 inches by 47 feet 4 inches.

- Write an equation to model this ellipse. Assume that the center is at the origin and the major axis is horizontal.

The length of the major axis is  $47\frac{1}{3}$  or  $\frac{142}{3}$  feet.

$$2a = \frac{142}{3} \quad \text{Length of major axis} = \frac{142}{3}$$

$$a = \frac{71}{3} \quad \text{Divide each side by 2.}$$

The length of the minor axis is  $13\frac{1}{2}$  or  $\frac{27}{2}$  feet.

$$2b = \frac{27}{2} \quad \text{Length of minor axis} = \frac{27}{2}$$

$$b = \frac{27}{4} \quad \text{Divide each side by 2.}$$

Substitute  $a = \frac{71}{3}$  and  $b = \frac{27}{4}$  into the form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . An equation of the ellipse is  $\frac{x^2}{\left(\frac{71}{3}\right)^2} + \frac{y^2}{\left(\frac{27}{4}\right)^2} = 1$ .

- How far apart are the points at which two people should stand to hear each other whisper?

People should stand at the two foci of the ellipse. The distance between the foci is  $2c$  units.

$$c^2 = a^2 - b^2$$

Equation relating  $a$ ,  $b$ , and  $c$

$$c = \sqrt{a^2 - b^2}$$

Take the square root of each side.

$$2c = 2\sqrt{a^2 - b^2}$$

Multiply each side by 2.

$$2c = 2\sqrt{\left(\frac{71}{3}\right)^2 - \left(\frac{27}{4}\right)^2}$$

Substitute  $a = \frac{71}{3}$  and  $b = \frac{27}{4}$ .

$$2c \approx 45.37$$

Use a calculator.

The points where two people should stand to hear each other whisper are about 45.37 feet or about 45 feet 4 inches apart.



### CHECK Your Progress

**BILLIARDS** Elliptipool is an elliptical pool table with only one pocket that is located on one of the foci. If the ball is placed on the other focus and shot off any edge, it will drop into the pocket located on the other focus. The pool table has axes that are 4 feet 6 inches and 5 feet.

- Write an equation to model this ellipse. Assume that the center is at the origin and the major axis is horizontal.
- How far apart are the two foci?



Personal Tutor at [algebra2.com](http://algebra2.com)





**Graph Ellipses** As with circles, you can use completing the square, symmetry, and transformations to help graph ellipses. An ellipse with its center at the origin is represented by an equation of the form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  or  $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ .

## Study Tip

### Graphing Calculator

You can graph an ellipse on a graphing calculator by first solving for  $y$ . Then graph the two equations that result on the same screen.

## EXAMPLE Graph an Equation in Standard Form

- 1** Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation  $\frac{x^2}{16} + \frac{y^2}{4} = 1$ . Then graph the ellipse.

The center of this ellipse is at  $(0, 0)$ . Since  $a^2 = 16$ ,  $a = 4$ . Since  $b^2 = 4$ ,  $b = 2$ .

The length of the major axis is  $2(4)$  or 8 units, and the length of the minor axis is  $2(2)$  or 4 units. Since the  $x^2$  term has the greater denominator, the major axis is horizontal.

$$c^2 = a^2 - b^2 \quad \text{Equation relating } a, b, \text{ and } c$$

$$c^2 = 4^2 - 2^2 \text{ or } 12 \quad a = 4, b = 2$$

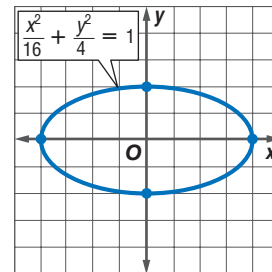
$$c = \sqrt{12} \text{ or } 2\sqrt{3} \quad \text{Take the square root of each side.}$$

The foci are at  $(2\sqrt{3}, 0)$  and  $(-2\sqrt{3}, 0)$ .

You can use a calculator to find some approximate nonnegative values for  $x$  and  $y$  that satisfy the equation. Since the ellipse is centered at the origin, it is symmetric about the  $y$ -axis. Therefore, the points at  $(-4, 0)$ ,  $(-3, 1.3)$ ,  $(-2, 1.7)$ , and  $(-1, 1.9)$  lie on the graph.

The ellipse is also symmetric about the  $x$ -axis, so the points at  $(-3, -1.3)$ ,  $(-2, -1.7)$ ,  $(-1, -1.9)$ ,  $(0, -2)$ ,  $(1, -1.9)$ ,  $(2, -1.7)$ , and  $(3, -1.3)$  lie on the graph.

Graph the intercepts,  $(-4, 0)$ ,  $(4, 0)$ ,  $(0, 2)$ , and  $(0, -2)$ , and draw the ellipse that passes through them and the other points.



## CHECK Your Progress

- 3.** Find the coordinates of the foci and the lengths of the major and minor axes of the ellipse with equation  $\frac{x^2}{49} + \frac{y^2}{36} = 1$ . Then graph the ellipse.

Suppose an ellipse is translated  $h$  units right and  $k$  units up, moving the center to the point  $(h, k)$ . Such a move would be equivalent to replacing  $x$  with  $x - h$  and replacing  $y$  with  $y - k$ .

## KEY CONCEPT

### Equations of Ellipses with Centers at $(h, k)$

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$

## EXAMPLE Graph an Equation Not in Standard Form

### Study Tip

#### Look Back

To review **grouping** and **factoring** common factors of each variable separately, see Lessons 5-3 and 5-5.

- 4** Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation  $x^2 + 4y^2 + 4x - 24y + 24 = 0$ . Then graph the ellipse.

Complete the square for each variable to write this equation in standard form.

$$x^2 + 4y^2 + 4x - 24y + 24 = 0$$

$$(x^2 + 4x + \blacksquare) + 4(y^2 - 6y + \blacksquare) = -24 + \blacksquare + 4(\blacksquare)$$

$$(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = -24 + 4 + 4(9)$$

$$(x + 2)^2 + 4(y - 3)^2 = 16$$

$$\frac{(x + 2)^2}{16} + \frac{(y - 3)^2}{4} = 1$$

Original equation

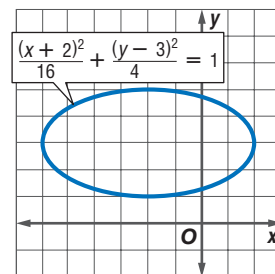
Complete the squares.

$$\left(\frac{4}{2}\right)^2 = 4, \left(\frac{-6}{2}\right)^2 = 9$$

Write the trinomials as perfect squares.

Divide each side by 16.

The graph of this ellipse is the graph from Example 3 translated 2 units to the left and up 3 units. The center is at  $(-2, 3)$  and the foci are at  $(-2 + 2\sqrt{3}, 0)$  and  $(-2 - 2\sqrt{3}, 0)$ . The length of the major axis is still 8 units, and the length of the minor axis is still 4 units.



### CHECK Your Progress

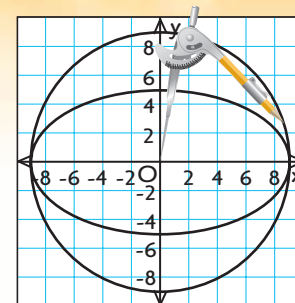
- 4.** Find the coordinates and foci and the lengths of the major and minor axes of the ellipse with equation  $x^2 + 6y^2 + 8x - 12y + 16 = 0$ . Then graph the ellipse.

You can use a circle to locate the foci on the graph of a given ellipse.

## ALGEBRA LAB

### Locating Foci

- Step 1** Graph an ellipse so that its center is at the origin. Let the endpoints of the major axis be at  $(-9, 0)$  and  $(9, 0)$ , and let the endpoints of the minor axis be at  $(0, -5)$  and  $(0, 5)$ .
- Step 2** Use a compass to draw a circle with center at  $(0, 0)$  and radius 9 units.
- Step 3** Draw the line with equation  $y = 5$  and mark the points at which the line intersects the circle.
- Step 4** Draw perpendicular lines from the points of intersection to the  $x$ -axis. The foci of the ellipse are located at the points where the perpendicular lines intersect the  $x$ -axis.

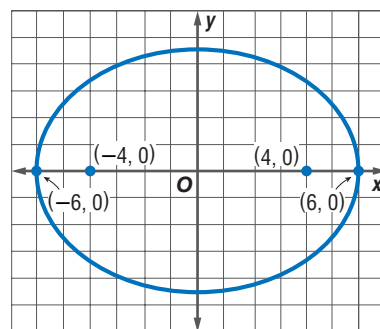


### MAKE A CONJECTURE

Draw another ellipse and locate its foci. Why does this method work?

**Example 1**  
(pp. 582–583)

1. Write an equation for the ellipse shown at the right.



Write an equation for the ellipse that satisfies each set of conditions.

2. endpoints of major axis at  $(2, 2)$  and  $(2, -10)$ , endpoints of minor axis at  $(0, -4)$  and  $(4, -4)$   
 3. endpoints of major axis at  $(0, 10)$  and  $(0, -10)$ , foci at  $(0, 8)$  and  $(0, -8)$

**Example 2**  
(p. 583)

4. **ASTRONOMY** At its closest point, Earth is 0.99 astronomical units from the center of the Sun. At its farthest point, Earth is 1.021 astronomical units from the center of the Sun. Write an equation for the orbit of Earth, assuming that the center of the orbit is the origin and the Sun lies on the  $x$ -axis.

**Examples 3, 4**  
(pp. 584–585)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

5.  $\frac{y^2}{18} + \frac{x^2}{9} = 1$

6.  $\frac{(x-1)^2}{20} + \frac{(y+2)^2}{4} = 1$

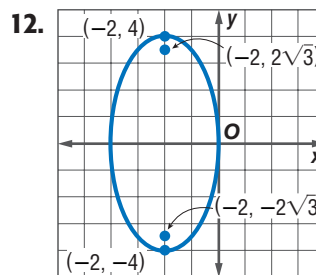
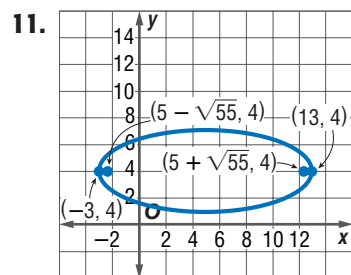
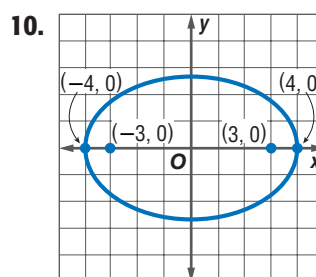
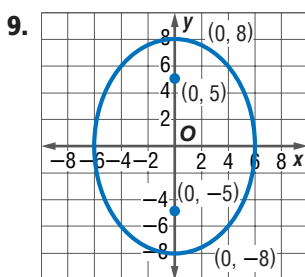
7.  $4x^2 + 8y^2 = 32$

8.  $x^2 + 25y^2 - 8x + 100y + 91 = 0$

**Exercises**

For Exercises	See Examples
9–15	1
16, 17	2
18–21	3
22–25	4

Write an equation for each ellipse.



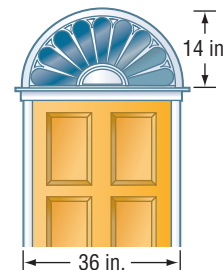
Write an equation for the ellipse that satisfies each set of conditions.

13. endpoints of major axis at  $(-11, 5)$  and  $(7, 5)$ , endpoints of minor axis at  $(-2, 9)$  and  $(-2, 1)$   
 14. endpoints of major axis at  $(2, 12)$  and  $(2, -4)$ , endpoints of minor axis at  $(4, 4)$  and  $(0, 4)$   
 15. major axis 20 units long and parallel to  $y$ -axis, minor axis 6 units long, center at  $(4, 2)$



- 16. ASTRONOMY** At its closest point, Venus is 0.719 astronomical units from the Sun. At its farthest point, Venus is 0.728 astronomical units from the Sun. Write an equation for the orbit of Venus. Assume that the center of the orbit is the origin, the Sun lies on the  $x$ -axis, and the radius of the Sun is 400,000 miles.

- 17. INTERIOR DESIGN** The rounded top of the window is the top half of an ellipse. Write an equation for the ellipse if the origin is at the midpoint of the bottom edge of the window.



Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

18.  $\frac{y^2}{10} + \frac{x^2}{5} = 1$

19.  $\frac{x^2}{25} + \frac{y^2}{9} = 1$

20.  $\frac{(x+8)^2}{144} + \frac{(y-2)^2}{81} = 1$

21.  $\frac{(y+11)^2}{144} + \frac{(x-5)^2}{121} = 1$

22.  $3x^2 + 9y^2 = 27$

23.  $27x^2 + 9y^2 = 81$

24.  $7x^2 + 3y^2 - 28x - 12y = -19$

25.  $16x^2 + 25y^2 + 32x - 150y = 159$

Write an equation for the ellipse that satisfies each set of conditions.

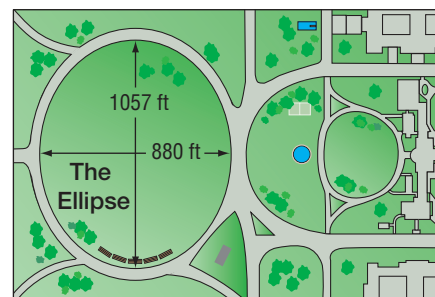
26. major axis 16 units long and parallel to  $x$ -axis, minor axis 9 units long, center at  $(5, 4)$   
 27. endpoints of major axis at  $(10, 2)$  and  $(-8, 2)$ , foci at  $(6, 2)$  and  $(-4, 2)$   
 28. endpoints of minor axis at  $(0, 5)$  and  $(0, -5)$ , foci at  $(12, 0)$  and  $(-12, 0)$   
 29. Write the equation  $10x^2 + 2y^2 = 40$  in standard form.  
 30. What is the standard form of the equation  $x^2 + 6y^2 - 2x + 12y - 23 = 0$ ?



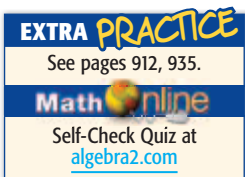
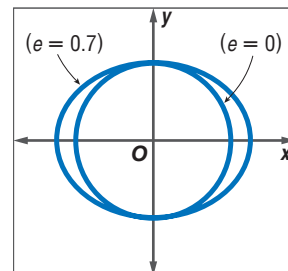
#### Real-World Link

The Ellipse, also known as President's Park South, has an area of about 16 acres.

- 31. WHITE HOUSE** There is an open area south of the White House known as The Ellipse. Write an equation to model The Ellipse. Assume that the origin is at the center of The Ellipse.



- 32. ASTRONOMY** In an ellipse, the ratio  $\frac{c}{a}$  is called the eccentricity and is denoted by the letter  $e$ . Eccentricity measures the elongation of an ellipse. The closer  $e$  is to 0, the more an ellipse looks like a circle. Pluto has the most eccentric orbit in our solar system with  $e \approx 0.25$ . Find an equation to model the orbit of Pluto, given that the length of the major axis is about 7.34 billion miles. Assume that the major axis is horizontal and that the center of the orbit is the origin.



#### H.O.T. Problems

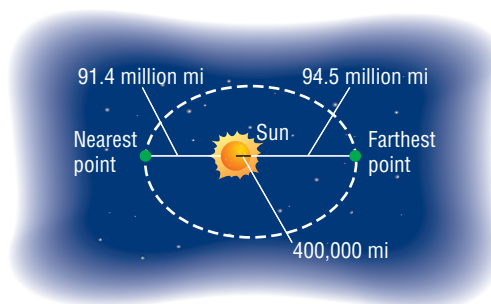
- 33. REASONING** Explain why a circle is a special case of an ellipse.  
**34. OPEN ENDED** Write an equation for an ellipse with its center at  $(2, -5)$  and a horizontal major axis.





**35. CHALLENGE** Find an equation for the ellipse with foci at  $(\sqrt{3}, 0)$  and  $(-\sqrt{3}, 0)$  that passes through  $(0, 3)$ .

**36. Writing in Math** Use the information about the solar system on page 581 and the figure at the right to explain why ellipses are important in the study of the solar system. Explain why an equation that is an accurate model of the path of a planet might be useful.



### STANDARDIZED TEST PRACTICE

**37. ACT/SAT** Winona is making an elliptical target for throwing darts. She wants the target to be 27 inches wide and 15 inches high. Which equation should Winona use to draw the target?

- A  $\frac{y^2}{13.5} + \frac{x^2}{7.5} = 1$   
 B  $\frac{y^2}{182.25} + \frac{x^2}{56.25} = 1$   
 C  $\frac{y^2}{56.25} + \frac{x^2}{182.25} = 1$   
 D  $\frac{y^2}{7.5} + \frac{x^2}{13.5} = 1$

**38. REVIEW** What is the standard form of the equation of the conic given below?

$$2x^2 - 4y^2 - 8x - 24y - 16 = 0$$

- F  $\frac{(x-4)^2}{11} - \frac{(y+3)^2}{3} = 1$   
 G  $\frac{(y-3)^2}{3} - \frac{(x-2)^2}{6} = 1$   
 H  $\frac{(y+3)^2}{4} - \frac{(x+2)^2}{5} = 1$   
 J  $\frac{(x-4)^2}{11} + \frac{(y+3)^2}{3} = 1$

### Spiral Review

Write an equation for the circle that satisfies each set of conditions. (Lesson 10-3)

39. center  $(3, -2)$ , radius 5 units  
 40. endpoints of a diameter at  $(5, -9)$  and  $(3, 11)$   
 41. Write an equation of a parabola with vertex  $(3, 1)$  and focus  $(3, 1\frac{1}{2})$ . Then draw the graph. (Lesson 10-2)

**MARRIAGE** For Exercises 42–44, use the table below that shows the number of married Americans over the last few decades. (Lesson 2-5)

Year	1980	1990	1995	1999	2000	2010
People (millions)	104.6	112.6	116.7	118.9	120.2	?

Source: U.S. Census Bureau

42. Draw a scatter plot in which  $x$  is the number of years since 1980.  
 43. Find a prediction equation.  
 44. Predict the number of married Americans in 2010.

### GET READY for the Next Lesson

**PREREQUISITE SKILL** Graph the line with the given equation. (Lessons 2-1, 2-2, and 2-3)

45.  $y = 2x$   
 46.  $y = -2x$   
 47.  $y = -\frac{1}{2}x$   
 48.  $y = \frac{1}{2}x$   
 49.  $y + 2 = 2(x - 1)$   
 50.  $y + 2 = -2(x - 1)$

Find the distance between each pair of points with the given coordinates. (Lesson 10-1)

1.  $(9, 5), (4, -7)$       2.  $(0, -5), (10, -3)$

The coordinates of the endpoints of a segment are given. Find the coordinates of the midpoint of each segment. (Lesson 10-1)

3.  $(1, 5), (-4, -3)$       4.  $(-3, 8), (-11, -6)$

**DISTANCE** For Exercises 5 and 6, use the following information. (Lesson 10-1)

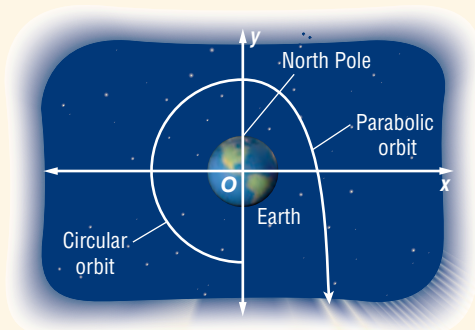
Jessica lives at the corner of 5<sup>th</sup> Avenue and 12<sup>th</sup> street. Julie lives at the corner of 15<sup>th</sup> Avenue and 4<sup>th</sup> street.

5. How many blocks apart do the two girls live?
6. If they want to meet for lunch halfway between their houses, where would they meet?

Write each equation in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola. (Lesson 10-2)

7.  $y = x^2 - 6x + 4$   
8.  $x = y^2 + 2y - 3$

9. **SPACE SCIENCE** A spacecraft is in a circular orbit 93 miles above Earth. Once it attains the velocity needed to escape Earth's gravity, the spacecraft will follow a parabolic path with the center of Earth as the focus. Suppose the spacecraft reaches escape velocity above the North Pole. Write an equation to model the parabolic path of the spacecraft, assuming that the center of Earth is at the origin and the radius of Earth is 3977 miles. (Lesson 10-2)



Identify the coordinates of the vertex and focus, the equation of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. (Lesson 10-2)

10.  $y^2 = 6x$

11.  $y = x^2 + 8x + 20$

12. Find the center and radius of the circle with equation  $x^2 + (y - 4)^2 = 49$ . Then graph the circle. (Lesson 10-3)

13. **SPRINKLERS** A sprinkler waters a circular section of lawn about 20 feet in diameter. The homeowner decides that placing the sprinkler at  $(7, 5)$  will maximize the area of grass being watered. Write an equation to represent the boundary the sprinkler waters. (Lesson 10-3)

14. Write an equation for the circle that has center at  $(-1, 0)$  and passes through  $(2, -6)$ . (Lesson 10-4)

15. **MULTIPLE CHOICE** What is the radius of the circle with equation  $x^2 + y^2 + 8x + 8y + 28 = 0$ ? (Lesson 10-3)

- A 2  
B 4  
C 8  
D 28

16. Write an equation of the ellipse with foci at  $(3, 8)$  and  $(3, -6)$  and endpoints of the major axis at  $(3, -8)$  and  $(3, 10)$ . (Lesson 10-4)

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with the given equation. Then graph the ellipse. (Lesson 10-4)

17.  $\frac{(x - 4)^2}{9} + \frac{(y + 2)^2}{1} = 1$

18.  $16x^2 + 5y^2 + 32x - 10y - 59 = 0$

**Main Ideas**

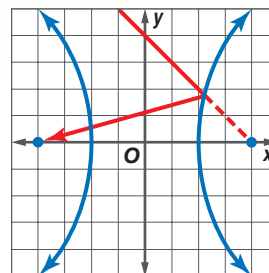
- Write equations of hyperbolas.
- Graph hyperbolas.

**New Vocabulary**

hyperbola  
foci  
center  
vertex  
asymptote  
transverse axis  
conjugate axis

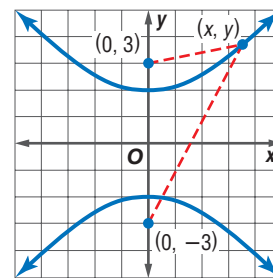
**GET READY for the Lesson**

A hyperbola is a conic section with the property that rays directed toward one focus are reflected toward the other focus. Notice that, unlike the other conic sections, a hyperbola has two branches.



**Equations of Hyperbolas** A **hyperbola** is the set of all points in a plane such that the absolute value of the difference of the distances from two fixed points, called the **foci**, is constant.

The hyperbola at the right has foci at  $(0, 3)$  and  $(0, -3)$ . The distances from either of the  $y$ -intercepts to the foci are 1 unit and 5 units, so the difference of the distances from any point with coordinates  $(x, y)$  on the hyperbola to the foci is 4 or  $-4$  units, depending on the order in which you subtract.



You can use the Distance Formula and the definition of a hyperbola to find an equation of this hyperbola.

$$\begin{array}{l} \text{The distance between} \quad \text{the distance between} \\ (x, y) \text{ and } (0, 3) \quad - \quad (x, y) \text{ and } (0, -3) = \pm 4. \end{array}$$

$$\sqrt{x^2 + (y - 3)^2} - \sqrt{x^2 + (y + 3)^2} = \pm 4$$

$$\sqrt{x^2 + (y - 3)^2} = \pm 4 + \sqrt{x^2 + (y + 3)^2} \quad \text{Isolate the radicals.}$$

$$x^2 + (y - 3)^2 = 16 \pm 8\sqrt{x^2 + (y + 3)^2} + x^2 + (y + 3)^2$$

$$x^2 + y^2 - 6y + 9 = 16 \pm 8\sqrt{x^2 + (y + 3)^2} + x^2 + y^2 + 6y + 9$$

$$-12y - 16 = \pm 8\sqrt{x^2 + (y + 3)^2} \quad \text{Simplify.}$$

$$3y + 4 = \pm 2\sqrt{x^2 + (y + 3)^2} \quad \text{Divide each side by } -4.$$

$$9y^2 + 24y + 16 = 4[x^2 + (y + 3)^2] \quad \text{Square each side.}$$

$$9y^2 + 24y + 16 = 4x^2 + 4y^2 + 24y + 36 \quad \text{Distributive Property}$$

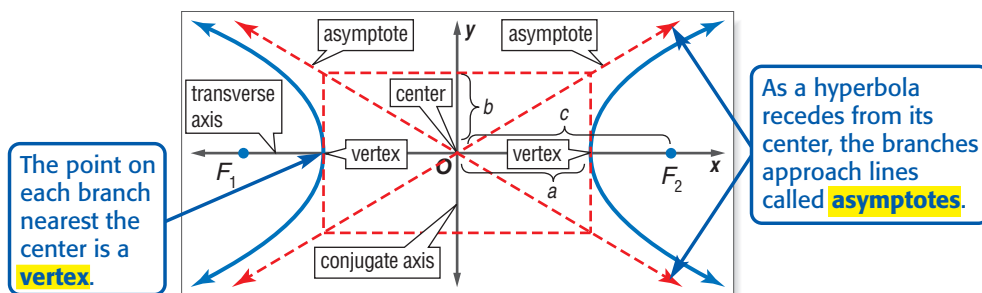
$$5y^2 - 4x^2 = 20 \quad \text{Simplify.}$$

$$\frac{y^2}{4} - \frac{x^2}{5} = 1 \quad \text{Divide each side by 20.}$$

An equation of this hyperbola is  $\frac{y^2}{4} - \frac{x^2}{5} = 1$ .



The diagram below shows the parts of a hyperbola.



The distance from the **center** to a vertex of a hyperbola is  $a$  units. The distance from the center to a focus is  $c$  units. There are two axes of symmetry. The **transverse axis** is a segment of length  $2a$  whose endpoints are the vertices of the hyperbola. The **conjugate axis** is a segment of length  $2b$  units that is perpendicular to the transverse axis at the center. The values of  $a$ ,  $b$ , and  $c$  are related differently for a hyperbola than for an ellipse. For a hyperbola,  $c^2 = a^2 + b^2$ .

### Reading Math

**Standard Form** In the standard form of a hyperbola, the squared terms are subtracted ( $-$ ). For an ellipse, they are added ( $+$ ).

KEY CONCEPT Equations of Hyperbolas with Centers at the Origin		
Standard Form of Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Length of Transverse Axis	$2a$ units	$2a$ units
Length of Conjugate Axis	$2b$ units	$2b$ units
Equations of Asymptotes	$y = \pm \frac{b}{a}x$	$y = \pm \frac{a}{b}x$

### EXAMPLE Write an Equation for a Graph

**1** Write an equation for the hyperbola shown at the right.

The center is the midpoint of the segment connecting the vertices, or  $(0, 0)$ .

The value of  $a$  is the distance from the center to a vertex, or 3 units. The value of  $c$  is the distance from the center to a focus, or 4 units.

$$c^2 = a^2 + b^2 \quad \text{Equation relating } a, b, \text{ and } c \text{ for a hyperbola}$$

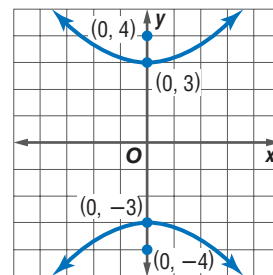
$$4^2 = 3^2 + b^2 \quad c = 4, a = 3$$

$$16 = 9 + b^2 \quad \text{Evaluate the squares.}$$

$$7 = b^2 \quad \text{Solve for } b^2.$$

Since the transverse axis is vertical, an equation of the hyperbola is

$$\frac{y^2}{9} - \frac{x^2}{7} = 1.$$





## CHECK Your Progress

- Write an equation for the hyperbola with vertices at  $(0, 4)$  and  $(0, -4)$  and foci at  $(0, 5)$  and  $(0, -5)$ .



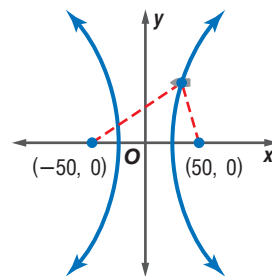
## Real-World EXAMPLE

### Write an Equation Given the Foci



**NAVIGATION** The LORAN navigational system is based on hyperbolas. Two stations send out signals at the same time. A ship notes the difference in the times at which it receives the signals. The ship is on a hyperbola with the stations at the foci. Suppose a ship determines that the difference of its distances from two stations is 50 nautical miles. Write an equation for a hyperbola on which the ship lies if the stations are at  $(-50, 0)$  and  $(50, 0)$ .

First, draw a figure. By studying either of the  $x$ -intercepts, you can see that the difference of the distances from any point on the hyperbola to the stations at the foci is the same as the length of the transverse axis, or  $2a$ . Therefore,  $2a = 50$ , or  $a = 25$ . According to the coordinates of the foci,  $c = 50$ .



Use the values of  $a$  and  $c$  to determine the value of  $b$  for this hyperbola.

$$c^2 = a^2 + b^2 \quad \text{Equation relating } a, b, \text{ and } c \text{ for a hyperbola}$$

$$50^2 = 25^2 + b^2 \quad c = 50, a = 25$$

$$2500 = 625 + b^2 \quad \text{Evaluate the squares.}$$

$$1875 = b^2 \quad \text{Solve for } b^2.$$

Since the transverse axis is horizontal, the equation is of the form  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .

Substitute the values for  $a^2$  and  $b^2$ . An equation of the hyperbola is

$$\frac{x^2}{625} - \frac{y^2}{1875} = 1.$$



## CHECK Your Progress

- Two microphones are set up underwater 3000 feet apart to observe dolphins. Sound travels under water at 5000 feet per second. One microphone picked up the sound of a dolphin 0.25 second before the other microphone picks up the same sound. Find the equation of the hyperbola that describes the possible locations of the dolphin.



Personal Tutor at [algebra2.com](http://algebra2.com)



### Real-World Link

LORAN stands for *Long Range Navigation*. The LORAN system is generally accurate to within 0.25 nautical mile.

Source: U.S. Coast Guard



**Graph Hyperbolas** It is easier to graph a hyperbola if the asymptotes are drawn first. To graph the asymptotes, use the values of  $a$  and  $b$  to draw a rectangle with dimensions  $2a$  and  $2b$ . The diagonals of the rectangle should intersect at the center of the hyperbola. The asymptotes will contain the diagonals of the rectangle.

## EXAMPLE Graph an Equation in Standard Form

- 3** Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation  $\frac{x^2}{9} - \frac{y^2}{4} = 1$ . Then graph the hyperbola.

The center of this hyperbola is at the origin. According to the equation,  $a^2 = 9$  and  $b^2 = 4$ , so  $a = 3$  and  $b = 2$ . The coordinates of the vertices are  $(3, 0)$  and  $(-3, 0)$ .

$$c^2 = a^2 + b^2 \quad \text{Equation relating } a, b, \text{ and } c \text{ for a hyperbola}$$

$$c^2 = 3^2 + 2^2 \quad a = 3, b = 2$$

$$c^2 = 13 \quad \text{Simplify.}$$

$$c = \sqrt{13} \quad \text{Take the square root of each side.}$$

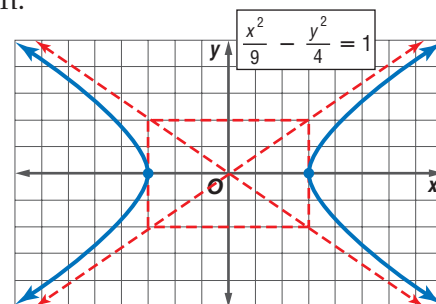
The foci are at  $(\sqrt{13}, 0)$  and  $(-\sqrt{13}, 0)$ .

The equations of the asymptotes are  $y = \pm \frac{b}{a}x$  or  $y = \pm \frac{2}{3}x$ .

You can use a calculator to find some approximate nonnegative values for  $x$  and  $y$  that satisfy the equation. Since the hyperbola is centered at the origin, it is symmetric about the  $y$ -axis. Therefore, the points at  $(-8, 4.9)$ ,  $(-7, 4.2)$ ,  $(-6, 3.5)$ ,  $(-5, 2.7)$ ,  $(-4, 1.8)$ , and  $(-3, 0)$  lie on the graph.

The hyperbola is also symmetric about the  $x$ -axis, so the points at  $(-8, -4.9)$ ,  $(-7, -4.2)$ ,  $(-6, -3.5)$ ,  $(-5, -2.7)$ ,  $(-4, -1.8)$ ,  $(4, -1.8)$ ,  $(5, -2.7)$ ,  $(6, -3.5)$ ,  $(7, -4.2)$ , and  $(8, -4.9)$  also lie on the graph.

Draw a 6-unit by 4-unit rectangle. The asymptotes contain the diagonals of the rectangle. Graph the vertices, which, in this case, are the  $x$ -intercepts. Use the asymptotes as a guide to draw the hyperbola that passes through the vertices and the other points. The graph does not intersect the asymptotes.



### Study Tip

#### Graphing Calculator

You can graph a hyperbola on a graphing calculator. Similar to an ellipse, first solve the equation for  $y$ . Then graph the two equations that result on the same screen.

### CHECK Your Progress

- 3.** Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation  $\frac{x^2}{49} - \frac{y^2}{25} = 1$ . Then graph the hyperbola.

So far, you have studied hyperbolas that are centered at the origin. A hyperbola may be translated so that its center is at  $(h, k)$ . This corresponds to replacing  $x$  by  $x - h$  and  $y$  by  $y - k$  in both the equation of the hyperbola and the equations of the asymptotes.

KEY CONCEPT		
	Equations of Hyperbolas with Centers at $(h, k)$	
Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Equations of Asymptotes	$y - k = \pm \frac{b}{a}(x - h)$	$y - k = \pm \frac{a}{b}(x - h)$



When graphing a hyperbola given an equation that is not in standard form, begin by rewriting the equation in standard form.

### EXAMPLE Graph an Equation Not in Standard Form

- 4** Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation  $4x^2 - 9y^2 - 32x - 18y + 19 = 0$ . Then graph the hyperbola.

Complete the square for each variable to write this equation in standard form.

$$4x^2 - 9y^2 - 32x - 18y + 19 = 0$$

Original equation

$$4(x^2 - 8x + \blacksquare) - 9(y^2 + 2y + \blacksquare) = -19 + 4(\blacksquare) - 9(\blacksquare)$$

Complete the squares.

$$4(x^2 - 8x + 16) - 9(y^2 + 2y + 1) = -19 + 4(16) - 9(1)$$

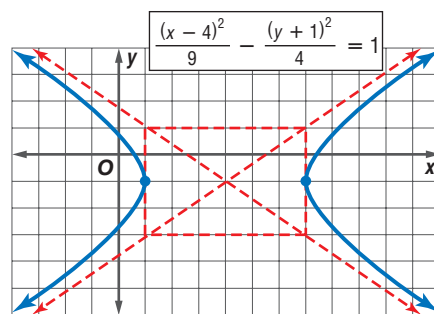
Write the trinomials as perfect squares.

$$4(x - 4)^2 - 9(y + 1)^2 = 36$$

Divide each side by 36.

$$\frac{(x - 4)^2}{9} - \frac{(y + 1)^2}{4} = 1$$

The graph of this hyperbola is the graph from Example 3 translated 4 units to the right and down 1 unit. The vertices are at  $(7, -1)$  and  $(1, -1)$ , and the foci are at  $(4 + \sqrt{13}, -1)$  and  $(4 - \sqrt{13}, -1)$ . The equations of the asymptotes are  $y + 1 = \pm \frac{2}{3}(x - 4)$ .



### CHECK Your Progress

- 4.** Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation  $9x^2 - 25y^2 - 36x - 50y - 214 = 0$ . Then graph the hyperbola.

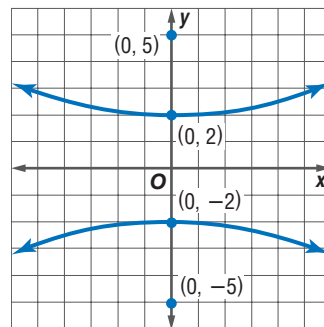
### CHECK Your Understanding

**Example 1**  
(pp. 591–592)

- 1.** Write an equation for the hyperbola shown at right.

**Example 2**  
(p. 592)

- 2.** A hyperbola has foci at  $(4, 0)$  and  $(-4, 0)$ . The value of  $a$  is 1. Write an equation for the hyperbola.



- 3. ASTRONOMY** Comets or other objects that pass by Earth or the Sun only once and never return may follow hyperbolic paths. Suppose a comet's path can be modeled by a branch of the hyperbola with equation  $\frac{y^2}{225} - \frac{x^2}{400} = 1$ . Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola. Then graph the hyperbola.



Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

4.  $\frac{y^2}{18} - \frac{x^2}{20} = 1$

5.  $\frac{(y+6)^2}{20} - \frac{(x-1)^2}{25} = 1$

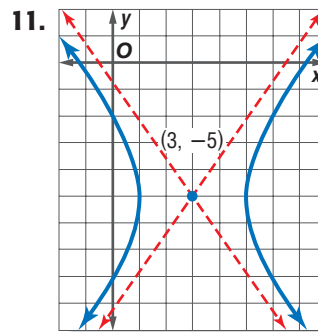
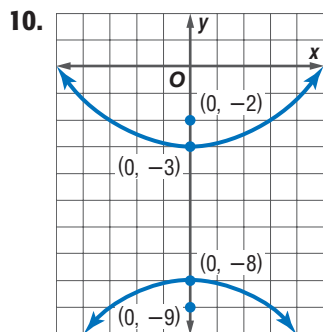
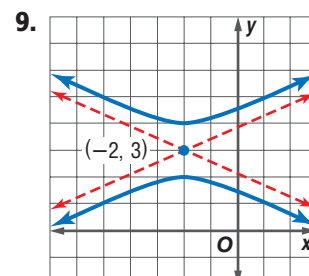
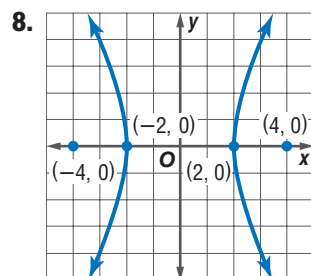
6.  $x^2 - 36y^2 = 36$

7.  $5x^2 - 4y^2 - 40x - 16y - 36 = 0$

## Exercises

For Exercises	See Examples
8–11	1
12–15	2
16–21	3
22–25	4

Write an equation for each hyperbola.



Write an equation for the hyperbola that satisfies each set of conditions.

12. vertices  $(-5, 0)$  and  $(5, 0)$ , conjugate axis of length 12 units

13. vertices  $(0, -4)$  and  $(0, 4)$ , conjugate axis of length 14 units

14. vertices  $(9, -3)$  and  $(-5, -3)$ , foci  $(2 \pm \sqrt{53}, -3)$

15. vertices  $(-4, 1)$  and  $(-4, 9)$ , foci  $(-4, 5 \pm \sqrt{97})$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

16.  $\frac{x^2}{81} - \frac{y^2}{49} = 1$

17.  $\frac{y^2}{36} - \frac{x^2}{4} = 1$

18.  $\frac{y^2}{16} - \frac{x^2}{25} = 1$

19.  $\frac{x^2}{9} - \frac{y^2}{25} = 1$

20.  $\frac{(y-4)^2}{16} - \frac{(x+2)^2}{9} = 1$

21.  $\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$

22.  $x^2 - 2y^2 = 2$

23.  $x^2 - y^2 = 4$

24.  $y^2 = 36 + 4x^2$

25.  $6y^2 = 2x^2 + 12$

26.  $\frac{(x+1)^2}{4} - \frac{(y+3)^2}{9} = 1$

27.  $\frac{(x+6)^2}{36} - \frac{(y+3)^2}{9} = 1$

28.  $y^2 - 3x^2 + 6y + 6x - 18 = 0$

29.  $4x^2 - 25y^2 - 8x - 96 = 0$

30. Find an equation for a hyperbola centered at the origin with a horizontal transverse axis of length 8 units and a conjugate axis of length 6 units.



31. What is an equation for the hyperbola centered at the origin with a vertical transverse axis of length 12 units and a conjugate axis of length 4 units?

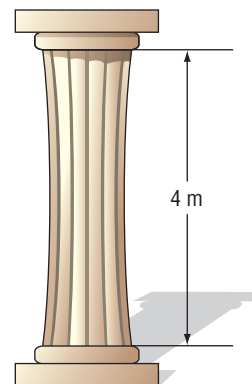


### Real-World Link

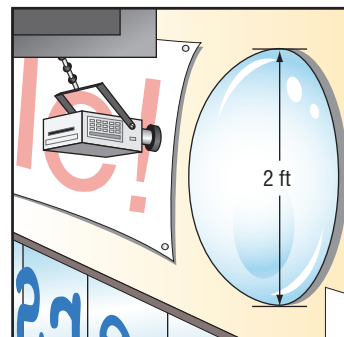
The Parthenon, originally constructed in 447 B.C., has 46 columns around its exterior perimeter.

Source: [www.mlahanas.de/Greeks/Arts/Parthenon.htm](http://www.mlahanas.de/Greeks/Arts/Parthenon.htm)

32. **STRUCTURAL DESIGN** An architect's design for a building includes some large pillars with cross sections in the shape of hyperbolas. The curves can be modeled by the equation  $\frac{x^2}{0.25} - \frac{y^2}{9} = 1$ , where the units are in meters. If the pillars are 4 meters tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to the nearest centimeter.



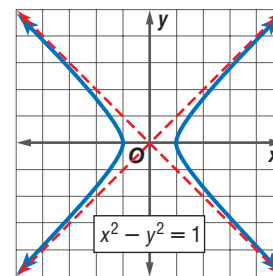
33. **PHOTOGRAPHY** A curved mirror is placed in a store for a wide-angle view of the room. The equation  $\frac{x^2}{1} - \frac{y^2}{3} = 1$  models the curvature of the mirror. A small security camera is placed 3 feet from the vertex of the mirror so that a diameter of 2 feet of the mirror is visible. If the back of the room lies on  $x = -18$ , what width of the back of the room is visible to the camera?



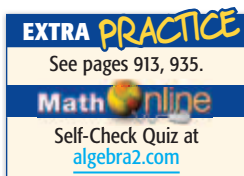
### NONRECTANGULAR HYPERBOLA For Exercises 34–37, use the following information.

A hyperbola with asymptotes that are not perpendicular is called a **nonrectangular hyperbola**. Most of the hyperbolas you have studied so far are nonrectangular.

A **rectangular hyperbola** is a hyperbola with perpendicular asymptotes. For example, the graph of  $x^2 - y^2 = 1$  is a rectangular hyperbola. The graphs of equations of the form  $xy = c$ , where  $c$  is a constant, are rectangular hyperbolas with the coordinate axes as their asymptotes.



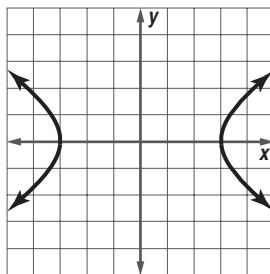
34. Plot some points and use them to graph the equation. Be sure to consider negative values for the variables.
35. Find the coordinates of the vertices of the graph of  $xy = 2$ .
36. Graph  $xy = -2$ .
37. Describe the transformations that can be applied to the graph of  $xy = 2$  to obtain the graph of  $xy = -2$ .
38. **OPEN ENDED** Find and graph a counterexample to the following statement.  
If the equation of a hyperbola is  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then  $a^2 \geq b^2$ .
39. **REASONING** Describe how the graph of  $y^2 - \frac{x^2}{k^2} = 1$  changes as  $|k|$  increases.
40. **CHALLENGE** A hyperbola with a horizontal transverse axis contains the point at  $(4, 3)$ . The equations of the asymptotes are  $y - x = 1$  and  $y + x = 5$ . Write the equation for the hyperbola.
41. **Writing in Math** Explain how hyperbolas and parabolas are different. Include differences in the graphs of hyperbolas and parabolas and differences in the reflective properties of hyperbolas and parabolas.



### H.O.T. Problems

42. **ACT/SAT** The foci of the graph are at  $(\sqrt{13}, 0)$  and  $(-\sqrt{13}, 0)$ . Which equation does the graph represent?

- A  $\frac{x^2}{9} - \frac{y^2}{4} = 1$   
 B  $\frac{x^2}{3} - \frac{y^2}{2} = 1$   
 C  $\frac{x^2}{3} - \frac{y^2}{\sqrt{13}} = 1$   
 D  $\frac{x^2}{9} - \frac{y^2}{13} = 1$



43. **REVIEW** To begin a game, Nate must randomly draw a red, blue, green, or yellow game piece, and a tile from a group of 26 tiles labeled with all the letters of the alphabet. What is the probability that Nate will draw the green game piece and a tile with a letter from his name?

- F  $\frac{1}{26}$                       H  $\frac{3}{52}$   
 G  $\frac{1}{13}$                       J  $\frac{1}{2}$

## Spiral Review

Write an equation for the ellipse that satisfies each set of conditions. (Lesson 10-4)

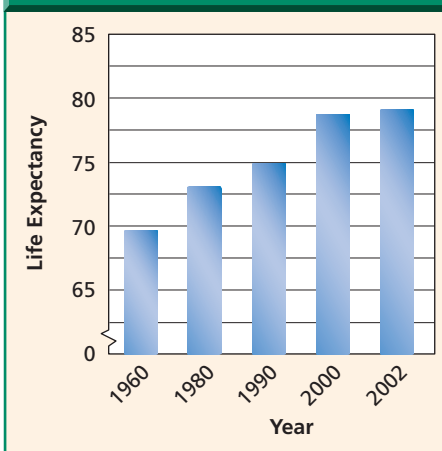
44. endpoints of major axis at  $(1, 2)$  and  $(9, 2)$ , endpoints of minor axis at  $(5, 1)$  and  $(5, 3)$   
 45. major axis 8 units long and parallel to  $y$ -axis, minor axis 6 units long, center at  $(-3, 1)$   
 46. foci at  $(5, 4)$  and  $(-3, 4)$ , major axis 10 units long  
 47. Find the center and radius of the circle with equation  $x^2 + y^2 - 10x + 2y + 22 = 0$ . Then graph the circle. (Lesson 10-3)

Solve each equation by factoring.

(Lesson 5-2)

48.  $x^2 + 6x + 8 = 0$   
 49.  $2q^2 + 11q = 21$   
 50. **LIFE EXPECTANCY** Refer to the graph at the right. What was the average rate of change of life expectancy from 1960 to 2002? (Lesson 2-3)  
 51. Solve  $|2x + 1| = 9$ . (Lesson 1-4)  
 52. Simplify  $7x + 8y + 9y - 5x$ . (Lesson 1-2)

**Life Expectancy, (selected years)**



Source: National Center for Health Statistics

## GET READY for the Next Lesson

**PREREQUISITE SKILL** Each equation is of the form  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ . Identify the values of  $A$ ,  $B$ , and  $C$ . (Lesson 6-1)

53.  $2x^2 + 3xy - 5y^2 = 0$                       54.  $-3x^2 + xy + 2y^2 + 4x - 7y = 0$   
 55.  $x^2 - 4x + 4y + 2 = 0$                       56.  $-xy - 2x - 3y + 6 = 0$

## Main Ideas

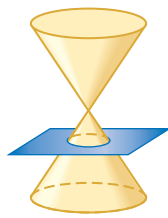
- Write equations of conic sections in standard form.
- Identify conic sections from their equations.

## GET READY for the Lesson

Recall that parabolas, circles, ellipses, and hyperbolas are called *conic sections* because they are the cross sections formed when a double cone is sliced by a plane.



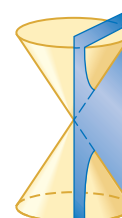
parabola



circle



ellipse



hyperbola

**Standard Form** The equation of any conic section can be written in the form of a general second-degree equation in two variables  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ , where  $A$ ,  $B$ , and  $C$  are not all zero. If you are given an equation in this general form, you may be able to complete the square to write the equation in one of the standard forms you have learned.

## Reading Math

**Ellipses** In this lesson, the word *ellipse* means an ellipse that is not a circle.

## CONCEPT SUMMARY

## Standard Form of Conic Section

Conic Section	Standard Form of Equation
Parabola	$y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$
Circle	$(x - h)^2 + (y - k)^2 = r^2$
Ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ or $\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1, a \neq b$
Hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ or $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$

## EXAMPLE

## Rewrite an Equation of a Conic Section

- 1 Write the equation  $x^2 + 4y^2 - 6x - 7 = 0$  in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

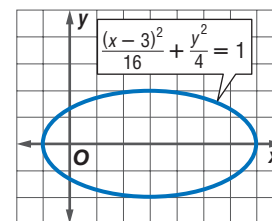
$$x^2 + 4y^2 - 6x - 7 = 0 \quad \text{Original equation}$$

$$x^2 - 6x + \blacksquare + 4y^2 = 7 + \blacksquare \quad \text{Isolate terms.}$$

$$x^2 - 6x + 9 + 4y^2 = 7 + 9 \quad \text{Complete the square.}$$

$$(x - 3)^2 + 4y^2 = 16 \quad x^2 - 6x + 9 = (x - 3)^2$$

$$\frac{(x - 3)^2}{16} + \frac{y^2}{4} = 1 \quad \text{Divide each side by 16.}$$



The graph of the equation is an ellipse with its center at  $(3, 0)$ .

## CHECK Your Progress

1. Write the equation  $x^2 + y^2 - 4x - 6y - 3 = 0$  in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

**Identify Conic Sections** Instead of writing the equation in standard form, you can determine what type of conic section an equation of the form  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ , where  $B = 0$ , represents by looking at  $A$  and  $C$ .

### CONCEPT SUMMARY

### Identifying Conic Sections

Conic Section	Relationship of $A$ and $C$
Parabola	$A = 0$ or $C = 0$ , but not both.
Circle	$A = C$
Ellipse	$A$ and $C$ have the same sign and $A \neq C$ .
Hyperbola	$A$ and $C$ have opposite signs.

### EXAMPLE Analyze an Equation of a Conic Section

- 2 Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

a.  $y^2 - 2x^2 - 4x - 4y - 4 = 0$

$A = -2$  and  $C = 1$ . Since  $A$  and  $C$  have opposite signs, the graph is a hyperbola.

b.  $4x^2 + 4y^2 + 20x - 12y + 30 = 0$

$A = 4$  and  $C = 4$ . Since  $A = C$ , the graph is a circle.

c.  $y^2 - 3x + 6y + 12 = 0$

$C = 1$ . Since there is no  $x^2$  term,  $A = 0$ . The graph is a parabola.

## CHECK Your Progress

2A.  $3x^2 + 3y^2 - 6x + 9y - 15 = 0$

2B.  $4x^2 + 3y^2 + 12x - 9y + 14 = 0$

2C.  $y^2 = 3x$

 **Online** Personal Tutor at [algebra2.com](http://algebra2.com)

## CHECK Your Understanding

**Example 1**  
(pp. 598–599)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

1.  $y = x^2 + 3x + 1$

2.  $y^2 - 2x^2 - 16 = 0$

3.  $x^2 + y^2 = x + 2$

4.  $x^2 + 4y^2 + 2x - 24y + 33 = 0$

**Example 2**  
(p. 599)

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

5.  $y^2 - x - 10y + 34 = 0$

6.  $3x^2 + 2y^2 + 12x - 28y + 104 = 0$



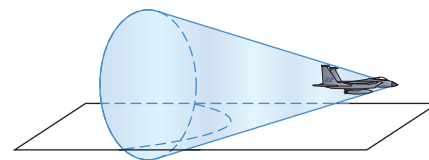
Extra Examples at [algebra2.com](http://algebra2.com)





**AVIATION** For Exercises 7 and 8, use the following information.

When an airplane flies faster than the speed of sound, it produces a shock wave in the shape of a cone. Suppose the shock wave generated by a jet intersects the ground in a curve that can be modeled by the equation  $x^2 - 14x + 4 = 9y^2 - 36y$ .



7. Identify the shape of the curve.
8. Graph the equation.

## Exercises

### HOMEWORK HELP

For Exercises	See Examples
9–18	1
19–25	2

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

9.  $6x^2 + 6y^2 = 162$
10.  $4x^2 + 2y^2 = 8$
11.  $x^2 = 8y$
12.  $4y^2 - x^2 + 4 = 0$
13.  $(x - 1)^2 - 9(y - 4)^2 = 36$
14.  $y + 4 = (x - 2)^2$
15.  $(y - 4)^2 = 9(x - 4)$
16.  $x^2 + y^2 + 4x - 6y = -4$
17.  $x^2 + y^2 + 6y + 13 = 40$
18.  $x^2 - y^2 + 8x = 16$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

19.  $x^2 + y^2 - 8x - 6y + 5 = 0$
20.  $3x^2 - 2y^2 + 32y - 134 = 0$
21.  $y^2 + 18y - 2x = -84$
22.  $7x^2 - 28x + 4y^2 + 8y = -4$

For Exercises 23–25, match each equation below with the situation that it could represent.

- a.  $9x^2 + 4y^2 - 36 = 0$
- b.  $0.004x^2 - x + y - 3 = 0$
- c.  $x^2 + y^2 - 20x + 30y - 75 = 0$

23. **SPORTS** the flight of a baseball
24. **PHOTOGRAPHY** the oval opening in a picture frame
25. **GEOGRAPHY** the set of all points that are 20 miles from a landmark

**AVIATION** For Exercises 26–28, use the following information.

A military jet performs for an air show. The path of the plane during one trick can be modeled by a conic section with equation  $24x^2 + 1000y - 31,680x - 45,600 = 0$ , where distances are represented in feet.

26. Identify the shape of the curved path of the jet. Write the equation in standard form.
27. If the jet begins its path upward or ascent at  $(0, 0)$ , what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?
28. What is the maximum height of the jet?



### Real-World Career

#### Pilot

While flying the plane, a pilot must also be constantly scanning flight instruments, monitoring the engine, and communicating with the air traffic controller.



For more information, go to [algebra2.com](http://algebra2.com).



**LIGHT** For Exercises 29 and 30, use the following information.

A lamp standing near a wall throws an arc of light in the shape of a conic section. Suppose the edge of the light can be represented by the equation  $3y^2 - 2y - 4x^2 + 2x - 8 = 0$ .

29. Identify the shape of the edge of the light.
30. Graph the equation.

**WATER** For Exercises 31 and 32, use the following information.

If two stones are thrown into a lake at different points, the points of intersection of the resulting ripples will follow a conic section. Suppose the conic section has the equation  $x^2 - 2y^2 - 2x - 5 = 0$ .

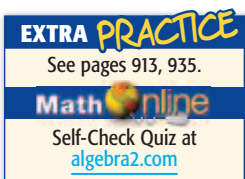
31. Identify the shape of the curve.
32. Graph the equation.

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

33.  $x^2 + 2y^2 = 2x + 8$
34.  $x^2 - 8y + y^2 + 11 = 0$
35.  $9y^2 + 18y = 25x^2 + 216$
36.  $3x^2 + 4y^2 + 8y = 8$
37.  $x^2 + 4y^2 - 11 = 2(4y - x)$
38.  $y + x^2 = -(8x + 23)$
39.  $6x^2 - 24x - 5y^2 - 10y - 11 = 0$
40.  $25y^2 + 9x^2 - 50y - 54x = 119$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

41.  $5x^2 + 6x - 4y = x^2 - y^2 - 2x$
42.  $2x^2 + 12x + 18 - y^2 = 3(2 - y^2) + 4y$
43. Identify the shape of the graph of the equation  $2x^2 + 3x - 4y + 2 = 0$ .
44. What type of conic section is represented by the equation  $y^2 - 6y = x^2 - 8$ ?



### H.O.T. Problems

45. **OPEN ENDED** Write an equation of the form  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ , where  $A = 2$ , that represents a circle.
46. **REASONING** Explain why the graph of the equation  $x^2 + y^2 - 4x + 2y + 5 = 0$  is a single point.

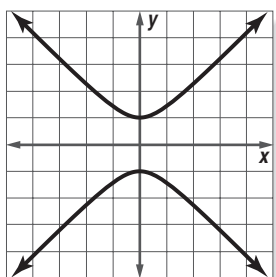
**CHALLENGE** For Exercises 47 and 48, use the following information.

The graph of an equation of the form  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$  is a special case of a hyperbola.

47. Identify the graph of such an equation.
48. Explain how to obtain such a set of points by slicing a double cone with a plane.
49. **REASONING** Refer to Exercise 32 on page 587. Eccentricity can be studied for conic sections other than ellipses. The expression for the eccentricity of a hyperbola is  $\frac{c}{a}$ , just as for an ellipse. The eccentricity of a parabola is 1. Find inequalities for the eccentricities of noncircular ellipses and hyperbolas, respectively.
50. **Writing in Math** Use the information about conic sections on page 598 to explain how you can use a flashlight to make conic sections. Explain how you could point the flashlight at a ceiling or wall to make a circle and how you could point the flashlight to make a branch of a hyperbola.



51. **ACT/SAT** What is the equation of the graph?



- A  $y = x^2 + 1$       B  $y - x = 1$   
 C  $y^2 - x^2 = 1$       D  $x^2 + y^2 = 1$

52. **REVIEW** The graph of  $\left(\frac{x}{4}\right)^2 - \left(\frac{y}{5}\right)^2 = 1$  is a hyperbola. Which set of equations represents the asymptotes of the hyperbola's graph?

- F  $y = \frac{4}{5}x, y = -\frac{4}{5}x$   
 G  $y = \frac{1}{4}x, y = -\frac{1}{4}x$   
 H  $y = \frac{5}{4}x, y = -\frac{5}{4}x$   
 J  $y = \frac{1}{5}x, y = -\frac{1}{5}x$

## Spiral Review

Write an equation of the hyperbola that satisfies each set of conditions.

(Lesson 10-5)

53. vertices (5, 10) and (5, -2), conjugate axis of length 8 units  
 54. vertices (6, -6) and (0, -6), foci  $(3 \pm \sqrt{13}, -6)$   
 55. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation  $4x^2 + 9y^2 - 24x + 72y + 144 = 0$ . Then graph the ellipse. (Lesson 10-4)

Simplify. Assume that no variable equals 0. (Lesson 6-1)

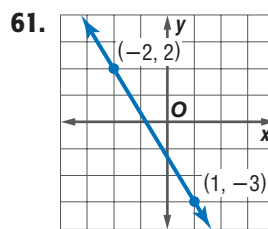
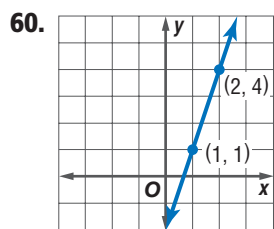
56.  $(x^3)^4$

57.  $(m^5n^{-3})^2m^2n^7$

58.  $\frac{x^2y^{-3}}{x^{-5}y}$

59. **HEALTH** The prediction equation  $y = 205 - 0.5x$  relates a person's maximum heart rate for exercise  $y$  and age  $x$ . Use the equation to find the maximum heart rate for an 18-year old. (Lesson 2-5)

Write an equation in slope-intercept form for each graph. (Lesson 2-4)



## GET READY for the Next Lesson

**PREREQUISITE SKILL** Solve each system of equations. (Lesson 3-2)

62.  $y = x + 4$   
 $2x + y = 10$

63.  $4x + y = 14$   
 $4x - y = 10$

64.  $x + 5y = 10$   
 $3x - 2y = -4$

**Main Ideas**

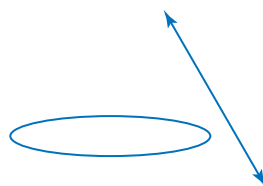
- Solve systems of quadratic equations algebraically and graphically.
- Solve systems of quadratic inequalities graphically.

**GET READY for the Lesson**

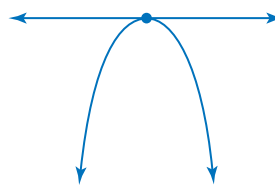
Suppose you are playing a computer game in which an enemy space station is located at the origin in a coordinate system. The space station is surrounded by a circular force field of radius 50 units. If the spaceship you control is flying toward the center along the line with equation  $y = 3x$ , the point where the ship hits the force field is a solution of a system of equations.



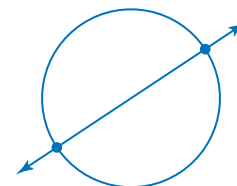
**Systems of Quadratic Equations** If the graphs of a system of equations are a conic section and a line, the system may have zero, one, or two solutions. Some of the possible situations are shown below.



no solutions



one solution



two solutions

**Study Tip****Look Back**

To review **solving systems of linear equations**, see Lesson 3-2.

You have solved systems of linear equations graphically and algebraically. You can use similar methods to solve systems involving quadratic equations.

**EXAMPLE Linear-Quadratic System**

**1** Solve the system of equations.

$$x^2 - 4y^2 = 9$$

$$4y - x = 3$$

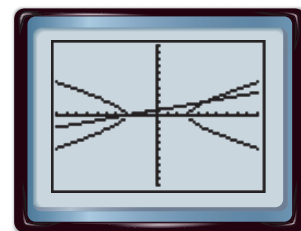
You can use a graphing calculator to help visualize the relationships of the graphs of the equations and predict the number of solutions.

Solve each equation for  $y$  to obtain

$$y = \pm \frac{\sqrt{x^2 - 9}}{2} \text{ and } y = \frac{1}{4}x + \frac{3}{4}. \text{ Enter}$$

$$y = \frac{\sqrt{x^2 - 9}}{2}, y = -\frac{\sqrt{x^2 - 9}}{2}, \text{ and } y = \frac{1}{4}x + \frac{3}{4}$$

on the Y= screen. The graph indicates that the hyperbola and line intersect in two points. So the system has two solutions.



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1  
(continued on the next page)



Use substitution to solve the system. First rewrite  $4y - x = 3$  as  $x = 4y - 3$ .

$$x^2 - 4y^2 = 9 \quad \text{First equation in the system}$$

$$(4y - 3)^2 - 4y^2 = 9 \quad \text{Substitute } 4y - 3 \text{ for } x.$$

$$12y^2 - 24y = 0 \quad \text{Simplify.}$$

$$y^2 - 2y = 0 \quad \text{Divide each side by 12.}$$

$$y(y - 2) = 0 \quad \text{Factor.}$$

$$y = 0 \quad \text{or} \quad y - 2 = 0 \quad \text{Zero Product Property}$$

$$y = 2 \quad \text{Solve for } y.$$

Now solve for  $x$ .

$$x = 4y - 3 \quad \text{Equation for } x \text{ in terms of } y \quad x = 4y - 3$$

$$= 4(0) - 3 \quad \text{Substitute the } y\text{-values.} \quad = 4(2) - 3$$

$$= -3 \quad \text{Simplify.} \quad = 5$$

The solutions of the system are  $(-3, 0)$  and  $(5, 2)$ . Based on the graph, these solutions are reasonable.



**Check Your Progress** Solve each system of equations.

**1A.**  $y = x - 1$

$$x^2 + y^2 = 25$$

**1B.**  $x + y = 1$

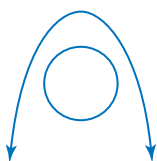
$$y = x^2 - 5$$



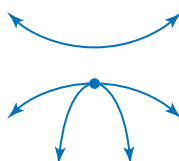
If the graphs of a system of equations are two conic sections, the system may have zero, one, two, three, or four solutions. Here are possible situations.

**Concepts  
in Motion**

Animation  
[algebra2.com](http://algebra2.com)



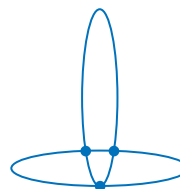
no solutions



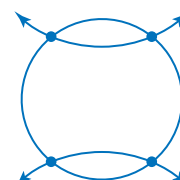
one solution



two solutions



three solutions



four solutions

## EXAMPLE

### Quadratic-Quadratic System



**2** Solve the system of equations.

$$y^2 = 13 - x^2$$

$$x^2 + 4y^2 = 25$$

A graph of the system indicates that the circle and ellipse intersect in four points. So, this system has four solutions. Use the elimination method to solve.

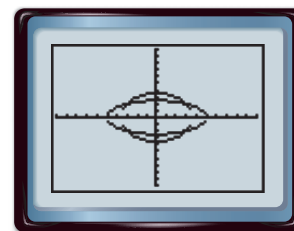
$$-x^2 - y^2 = -13 \quad \text{Rewrite the first original equation.}$$

$$(+)\ x^2 + 4y^2 = 25 \quad \text{Second original equation}$$

$$3y^2 = 12 \quad \text{Add.}$$

$$y^2 = 4 \quad \text{Divide each side by 3.}$$

$$y = \pm 2 \quad \text{Take the square root of each side.}$$



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

## Study Tip

### Graphing Calculators

If you use ZSquare on the ZOOM menu, the graph of the first equation will look like a circle.

Substitute 2 and  $-2$  for  $y$  in either of the original equations and solve for  $x$ .

$x^2 + 4y^2 = 25$	Second original equation	$x^2 + 4y^2 = 25$
$x^2 + 4(2)^2 = 25$	Substitute for $y$ .	$x^2 + 4(-2)^2 = 25$
$x^2 = 9$	Subtract 16 from each side.	$x^2 = 9$
$x = \pm 3$	Take the square root of each side.	$x = \pm 3$

The solutions are  $(3, 2)$ ,  $(-3, 2)$ ,  $(-3, -2)$ , and  $(3, -2)$ .



### Check Your Progress

Solve each system of equations.

2A.  $x^2 + y^2 = 36$

$x^2 + 9y^2 = 36$

2B.  $x^2 - y^2 = 8$

$x^2 + y^2 = 120$



Personal Tutor at [algebra2.com](http://algebra2.com)

A graphing calculator can be used to approximate solutions of a system of quadratic equations.

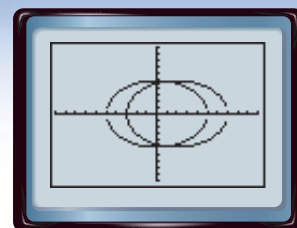
## GRAPHING CALCULATOR LAB

### Quadratic Systems

#### THINK AND DISCUSS

The calculator screen shows the graphs of two circles.

1. Write the system of equations represented.
2. Enter the equations into a TI-83/84 Plus and use the intersect feature on the CALC menu to solve the system. Round to the nearest hundredth.
3. Solve the system algebraically.
4. Can you always find the exact solution of a system using a graphing calculator? Explain.



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

**Systems of Quadratic Inequalities** Systems of quadratic inequalities are solved by graphing. As with linear inequalities, examine the inequality symbol to determine whether to include the boundary.

### Study Tip

#### Graphing Quadratic Inequalities

If you are unsure about which region to shade, you can test one or more points, as you did with linear inequalities.

### EXAMPLE

#### System of Quadratic Inequalities



Solve the system of inequalities by graphing.

$y \leq x^2 - 2$

$x^2 + y^2 < 16$

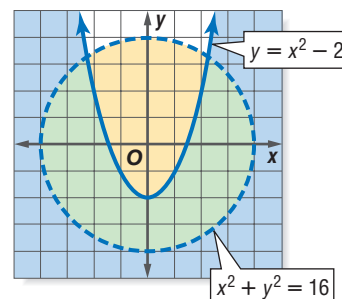
The intersection of the graphs, shaded green, represents the solution of the system.

**CHECK**  $(0, -3)$  is in the shaded area. Use this point to check your solution.

$y \leq x^2 - 2$        $x^2 + y^2 < 16$

$-3 \leq (0)^2 - 2$        $0^2 + (-3)^2 < 16$

$-3 \leq -2$  true       $9 < 16$  true



Extra Examples at [algebra2.com](http://algebra2.com)

3. Solve by graphing.  $x^2 + y^2 \leq 49$  and  $y \geq x^2 + 1$

CHECK Your Understanding

**Example 1**  
(pp. 603–604)

Find the exact solution(s) of each system of equations.

1.  $y = 5$

2.  $y - x = 1$

$y^2 = x^2 + 9$

$x^2 + y^2 = 25$

**Example 2**  
(pp. 604–605)

3.  $3x = 8y^2$

4.  $5x^2 + y^2 = 30$

$8y^2 - 2x^2 = 16$

$9x^2 - y^2 = -16$

5. **CELL PHONES** A person using a cell phone can be located in respect to three cellular towers. In a coordinate system where a unit represents one mile, the caller is determined to be 50 miles from a tower at the origin, 40 miles from a tower at  $(0, 30)$ , and 13 miles from a tower at  $(35, 18)$ . Where is the caller?

**Example 3**  
(pp. 605–606)

Solve each system of inequalities by graphing.

6.  $x + y < 4$

7.  $x^2 + y^2 < 25$

$9x^2 - 4y^2 \geq 36$

$4x^2 - 9y^2 < 36$

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–13	1
14–19	2
20–25	3

Find the exact solution(s) of each system of equations.

8.  $y = x + 2$

9.  $y = x + 3$

10.  $x^2 + y^2 = 36$

$y = x^2$

$y = 2x^2$

$y = x + 2$

11.  $y^2 + x^2 = 9$

12.  $\frac{x^2}{30} + \frac{y^2}{6} = 1$

13.  $\frac{x^2}{36} - \frac{y^2}{4} = 1$

$y = 7 - x$

$x = y$

$x = y$

14.  $4x + y^2 = 20$

15.  $y + x^2 = 3$

16.  $x^2 + y^2 = 64$

$4x^2 + y^2 = 100$

$x^2 + 4y^2 = 36$

$x^2 + 64y^2 = 64$

17.  $y^2 + x^2 = 25$

18.  $y^2 = x^2 - 25$

19.  $y^2 = x^2 - 7$

$y^2 + 9x^2 = 25$

$x^2 - y^2 = 7$

$x^2 + y^2 = 25$

Solve each system of inequalities by graphing.

20.  $x + 2y > 1$

21.  $x + y \leq 2$

22.  $x^2 + y^2 \geq 4$

$x^2 + y^2 \leq 25$

$4x^2 - y^2 \geq 4$

$4y^2 + 9x^2 \leq 36$

23.  $x^2 + y^2 < 36$

24.  $y^2 < x$

25.  $x^2 \leq y$

$4x^2 + 9y^2 > 36$

$x^2 - 4y^2 < 16$

$y^2 - x^2 \geq 4$

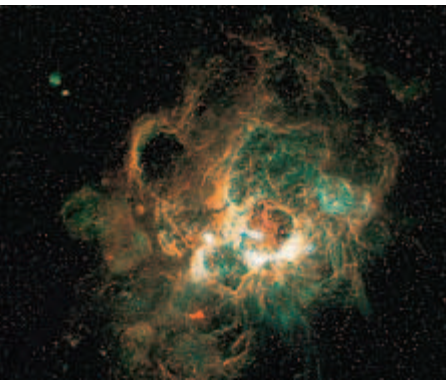
26. Graph each system of equations. Use the graph to solve the system.

a.  $4x - 3y = 0$

b.  $y = 5 - x^2$

$x^2 + y^2 = 25$

$y = 2x^2 + 2$



### Real-World Link

The astronomical unit (AU) is the mean distance between Earth and the Sun. One AU is about 93 million miles or 150 million kilometers

Source: infoplease.com

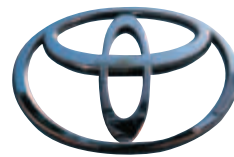
### ASTRONOMY For Exercises 27 and 28, use the following information.

The orbit of Pluto can be modeled by the equation  $\frac{x^2}{39.5^2} + \frac{y^2}{38.3^2} = 1$ , where the units are astronomical units. Suppose a comet is following a path modeled by the equation  $x = y^2 + 20$ .

27. Find the point(s) of intersection of the orbits of Pluto and the comet. Round to the nearest tenth.
28. Will the comet necessarily hit Pluto? Explain.
29. Where do the graphs of  $y = 2x + 1$  and  $2x^2 + y^2 = 11$  intersect?
30. What are the coordinates of the points that lie on the graphs of both  $x^2 + y^2 = 25$  and  $2x^2 + 3y^2 = 66$ ?

31. **ROCKETS** Two rockets are launched at the same time, but from different heights. The height  $y$  in feet of one rocket after  $t$  seconds is given by  $y = -16t^2 + 150t + 5$ . The height of the other rocket is given by  $y = -16t^2 + 160t$ . After how many seconds are the rockets at the same height?

32. **ADVERTISING** The corporate logo for an automobile manufacturer is shown at the right. Write a system of three equations to model this logo.



### SATELLITES For Exercises 33–35, use the following information.

Two satellites are placed in orbit about Earth. The equations of the two orbits are  $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$  and  $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$ , where distances are in kilometers and Earth is the center of each curve.

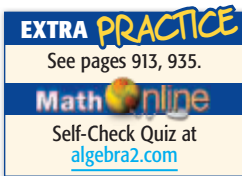
33. Solve each equation for  $y$ .
34. Use a graphing calculator to estimate the intersection points of the two orbits.
35. Compare the orbits of the two satellites.

Write a system of equations that satisfies each condition. Use a graphing calculator to verify that you are correct.

36. two parabolas that intersect in two points
37. a hyperbola and a circle that intersect in three points
38. a circle and an ellipse that do not intersect
39. a circle and an ellipse that intersect in four points
40. a hyperbola and an ellipse that intersect in two points
41. two circles that intersect in three points



### Graphing Calculator



### H.O.T. Problems

42. **REASONING** Sketch a parabola and an ellipse that intersect in exactly three points.
43. **OPEN ENDED** Write a system of quadratic equations for which  $(2, 6)$  is a solution.

**CHALLENGE** For Exercises 44–48, find all values of  $k$  for which the system of equations has the given number of solutions. If no values of  $k$  meet the condition, write *none*.

$$x^2 + y^2 = k^2 \quad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

- |                     |                    |                   |
|---------------------|--------------------|-------------------|
| 44. no solutions    | 45. one solution   | 46. two solutions |
| 47. three solutions | 48. four solutions |                   |



49. **Which One Doesn't Belong?** Which system of equations is NOT like the others? Explain your reasoning.

$$\begin{aligned}x^2 + y^2 &= 16 \\ x + y &= 3\end{aligned}$$

$$\begin{aligned}x^2 + y^2 &= 25 \\ \frac{x^2}{25} - \frac{y^2}{16} &= 1\end{aligned}$$

$$\begin{aligned}x^2 + y^2 &= 20 \\ \frac{x^2}{25} + \frac{y^2}{16} &= 1\end{aligned}$$

$$\begin{aligned}y - 2x &= -5 \\ y^2 + x &= 9\end{aligned}$$

50. **Writing in Math** Use the information on page 603 to explain how systems of equations apply to video games. Include a linear-quadratic system of equations that applies to this situation and the coordinates of the point at which the spaceship will hit the force field, assuming that the spaceship moves from the bottom of the screen toward the center.

### STANDARDIZED TEST PRACTICE

51. **ACT/SAT** How many solutions does the system of equations  $\frac{x^2}{5^2} - \frac{y^2}{3^2} = 1$  and  $(x - 3)^2 + y^2 = 9$  have?

A 0  
B 1  
C 2  
D 4

52. **REVIEW** Given: Two angles are supplementary. One angle is  $25^\circ$  more than the measure of the other angle. Conclusion: The measures of the angles are  $65^\circ$  and  $90^\circ$ . This conclusion—

F is contradicted by the first statement given.  
G is verified by the first statement given.  
H invalidates itself because a  $90^\circ$  angle cannot be supplementary to another.  
J verifies itself because  $90^\circ$  is  $25^\circ$  more than  $65^\circ$ .

### Spiral Review

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation. (Lesson 10-6)

53.  $x^2 + y^2 + 4x + 2y - 6 = 0$

54.  $9x^2 + 4y^2 - 24y = 0$

55. Find the coordinates of the vertices and foci and the equations of the asymptotes of the hyperbola with the equation  $6y^2 - 2x^2 = 24$ . Then graph the hyperbola. (Lesson 10-5)

### Cross-Curricular Project

#### Algebra and Earth Science

**Earthquake Extravaganza** It is time to complete your project. Use the information and data you have gathered about earthquakes to prepare a research report or Web page. Be sure to include graphs, tables, diagrams, and any calculations you need for the earthquake you chose.



Cross-Curricular Project at [algebra2.com](http://algebra2.com)



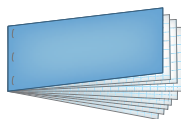
Study Guide  
and Review

Download Vocabulary  
Review from [algebra2.com](http://algebra2.com)

**FOLDABLES™**  
Study Organizer

GET READY to Study

Be sure the following  
Key Concepts are noted  
in your Foldable.



## Key Concepts

### Midpoint and Distance

**Formulas** (Lesson 10-1)

$$\bullet M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\bullet d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Circles

 (Lesson 10-2)

- The equation of a circle with center  $(h, k)$  and radius  $r$  can be written in the form  $(x - h)^2 + (y - k)^2 = r^2$ .

### Parabolas

 (Lesson 10-3)

<b>Standard Form</b>	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
<b>Axis of Symmetry</b>	$x = h$	$y = k$

### Ellipses

 (Lesson 10-4)

<b>Standard Form of Equation</b>	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$
<b>Direction of Major Axis</b>	horizontal	vertical

### Hyperbolas

 (Lesson 10-5)

<b>Standard Form</b>	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$
<b>Transverse Axis</b>	horizontal	vertical

### Solving Quadratic Systems

 (Lesson 10-7)

- Systems of quadratic equations can be solved using substitution and elimination.
- A system of quadratic equations can have zero, one, two, three, or four solutions.

## Key Vocabulary

asymptote (p. 591)	foci of a hyperbola (p. 590)
center of a circle (p. 574)	foci of an ellipse (p. 581)
center of a hyperbola (p. 591)	focus of a parabola (p. 567)
center of an ellipse (p. 582)	hyperbola (p. 590)
circle (p. 574)	latus rectum (p. 569)
conic section (p. 567)	major axis (p. 582)
conjugate axis (p. 591)	minor axis (p. 582)
directrix (p. 567)	parabola (p. 567)
ellipse (p. 581)	transverse axis (p. 591)
	vertex of a hyperbola (p. 591)

## Vocabulary Check

Tell whether each statement is *true* or *false*. If the statement is false, correct it to make it true.

- An ellipse is the set of all points in a plane such that the sum of the distances from two given points in the plane, called the foci, is constant.
- The major axis is the longer of the two axes of symmetry of an ellipse.
- A parabola is the set of all points that are the same distance from a given point called the directrix and a given line called the focus.
- The radius is the distance from the center of a circle to any point on the circle.
- The conjugate axis of a hyperbola is a line segment parallel to the transverse axis.
- A conic section is formed by slicing a double cone by a plane.
- The set of all points in a plane that are equidistant from a given point in a plane, called the center, forms a circle.

## Lesson-by-Lesson Review

**10-1** Midpoint and Distance Formulas (pp. 562–566)

Find the midpoint of the line segment with endpoints at the given coordinates.

8.  $(1, 2), (4, 6)$       9.  $(-8, 0), (-2, 3)$   
 10.  $\left(\frac{3}{5}, -\frac{7}{4}\right), \left(\frac{1}{4}, -\frac{2}{5}\right)$       11.  $(13, 24), (19, 28)$

Find the distance between each pair of points with the given coordinates.

12.  $(-2, 10), (-2, 13)$       13.  $(8, 5), (-9, 4)$   
 14.  $(7, -3), (1, 2)$       15.  $\left(\frac{5}{4}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{2}\right)$

**HIKING** For Exercises 16 and 17, use the following information.

Marc wants to hike from his camp to a waterfall. The waterfall is 5 miles south and 8 miles east of his campsite.

16. How far away is the waterfall?  
 17. Marc wants to stop for lunch halfway to the waterfall. If the camp is at the origin, where should he stop?

**Example 1** Find the midpoint of a segment whose endpoints are at  $(-5, 9)$  and  $(11, -1)$ .

Let  $(x_1, y_1) = (-5, 9)$  and  $(x_2, y_2) = (11, -1)$ .

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 11}{2}, \frac{9 + (-1)}{2}\right) \\ = \left(\frac{6}{2}, \frac{8}{2}\right) \text{ or } (3, 4) \quad \text{Simplify.}$$

**Example 2** Find the distance between  $P(6, -4)$  and  $Q(-3, 8)$ . Let  $(x_1, y_1) = (6, -4)$  and  $(x_2, y_2) = (-3, 8)$ .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula} \\ = \sqrt{(-3 - 6)^2 + [8 - (-4)]^2} \\ = \sqrt{81 + 144} \quad \text{Subtract.} \\ = \sqrt{225} \text{ or } 15 \text{ units} \quad \text{Simplify.}$$

**10-2** Parabolas (pp. 567–573)

Identify the coordinates of the vertex and focus, the equation of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

18.  $(x - 1)^2 = 12(y - 1)$   
 19.  $y + 6 = 16(x - 3)^2$   
 20.  $x^2 - 8x + 8y + 32 = 0$   
 21.  $x = 16y^2$   
 22. Write an equation for a parabola with vertex  $(0, 1)$  and focus  $(0, -1)$ . Then graph the parabola.

(continued on the next page)

**Example 3** Graph  $4y - x^2 = 14x - 27$ .

Write the equation in the form  $y = a(x - h)^2 + k$  by completing the square.

$$4y = x^2 + 14x - 27 \quad \text{Isolate the terms with } x.$$

$$4y = (x^2 + 14x + \blacksquare) - 27 - \blacksquare$$

$$4y = (x^2 + 14x + 49) - 27 - 49$$

$$4y = (x + 7)^2 - 76 \quad x^2 + 14x + 49 = (x + 7)^2$$

$$y = \frac{1}{4}(x + 7)^2 - 19 \quad \text{Divide each side by 4.}$$

## 10-2

### Parabolas (pp. 567-573)

- 23. SPORTS** When a golf ball is hit, the path it travels is shaped like a parabola. Suppose a golf ball is hit from ground level, reaches a maximum height of 100 feet, and lands 400 feet away. Assuming the ball was hit at the origin, write an equation of the parabola that models the flight of the ball.

vertex:  $(-7, -19)$

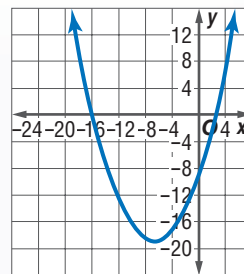
axis of symmetry:  
 $x = -7$

direction of opening:  
upward since  $a > 0$

focus:  $\left(-7, -19 + \frac{1}{4\left(\frac{1}{4}\right)}\right)$

or  $(-7, -18)$

directrix:  $y = -19 - \frac{1}{4\left(\frac{1}{4}\right)}$  or  $y = -20$



## 10-3

### Circles (pp. 574-579)

Write an equation for the circle that satisfies each set of conditions.

- 24.** center  $(2, -3)$ , radius 5 units
- 25.** center  $(-4, 0)$ , radius  $\frac{3}{4}$  units
- 26.** endpoints of a diameter at  $(9, 4)$  and  $(-3, -2)$
- 27.** center at  $(-1, 2)$ , tangent to  $x$ -axis

Find the center and radius of the circle with the given equation. Then graph the circle.

- 28.**  $x^2 + y^2 = 169$
- 29.**  $(x + 5)^2 + (y - 11)^2 = 49$
- 30.**  $x^2 + y^2 - 6x + 16y - 152 = 0$
- 31.**  $x^2 + y^2 + 6x - 2y - 15 = 0$
- 32. WEATHER** On average the circular eye of a tornado is about 200 feet in diameter. Suppose a satellite photo showed the center of its eye at the point  $(72, 39)$ . Write an equation to represent the possible boundary of a tornado's eye.

**Example 4** Graph  $x^2 + y^2 + 8x - 24y + 16 = 0$ .

First write the equation in the form  $(x - h)^2 + (y - k)^2 = r^2$ .

$$x^2 + y^2 + 8x - 24y + 16 = 0$$

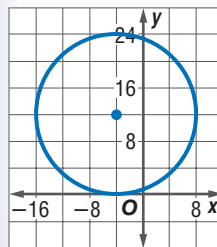
$$x^2 + 8x + \blacksquare + y^2 - 24y + \blacksquare = -16 + \blacksquare + \blacksquare$$

$$x^2 + 8x + 16 + y^2 - 24y + 144 = -16 + 16 + 144$$

$$(x + 4)^2 + (y - 12)^2 = 144$$

The center of the circle is at  $(-4, 12)$  and the radius is 12.

Now draw the graph.



## 10-4

## Ellipses (pp. 581-588)

33. Write an equation for the ellipse with endpoints of the major axis at (4, 1) and (-6, 1) and endpoints of the minor axis at (-1, 3) and (-1, -1).

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

34.  $\frac{x^2}{16} + \frac{y^2}{25} = 1$

35.  $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{9} = 1$

36.  $x^2 + 4y^2 - 2x + 16y + 13 = 0$

37. The Oval Office in the White House is an ellipse. The major axis is 10.9 meters and the minor axis is 8.8 meters. Write an equation to model the Oval Office. Assume that the origin is at the center of the Oval Office.

**Example 5** Graph  $x^2 + 3y^2 - 16x + 24y + 31 = 0$ .

First write the equation in standard form by completing the squares.

$$x^2 + 3y^2 - 16x + 24y + 31 = 0$$

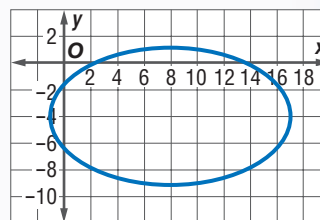
$$x^2 - 16x + \blacksquare + 3(y^2 + 8y + \blacksquare) = -31 + \blacksquare + 3(\blacksquare)$$

$$x^2 - 16x + 64 + 3(y^2 + 8y + 16) = -31 + 64 + 3(16)$$

$$(x - 8)^2 + 3(y + 4)^2 = 81$$

$$\frac{(x - 8)^2}{81} + \frac{(y + 4)^2}{27} = 1$$

The center of the ellipse is at (8, -4). The length of the major axis is 18, and the length of the minor axis is  $6\sqrt{3}$ .



## 10-5

## Hyperbolas (pp. 590-597)

38. Write an equation for a hyperbola that has vertices at (2, 5) and (2, 1) and a conjugate axis of length 6 units.

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

39.  $\frac{y^2}{4} - \frac{x^2}{9} = 1$

40.  $16x^2 - 25y^2 - 64x - 336 = 0$

41.  $\frac{(x-2)^2}{1} - \frac{(y+1)^2}{9} = 1$

42.  $9y^2 - 16x^2 = 144$

(continued on the next page)

**Example 6** Graph  $9x^2 - 4y^2 + 18x + 32y - 91 = 0$ .

Complete the square for each variable to write this equation in standard form.

$$9x^2 - 4y^2 + 18x + 32y - 91 = 0$$

$$9(x^2 + 2x + \blacksquare) - 4(y^2 - 8y + \blacksquare) = 91 + 9(\blacksquare) - 4(\blacksquare)$$

$$9(x^2 + 2x + 1) - 4(y^2 - 8y + 16) = 91 + 9(1) - 4(16)$$

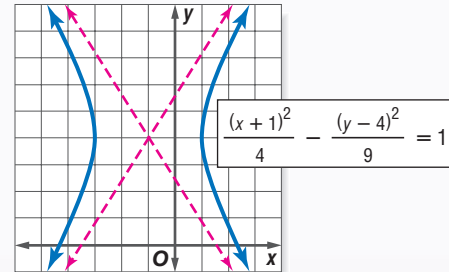
$$9(x + 1)^2 - 4(y - 4)^2 = 36$$

$$\frac{(x + 1)^2}{4} - \frac{(y - 4)^2}{9} = 1$$

### 10-5 Hyperbolas (pp. 590–597)

- 43. MIRRORS** A hyperbolic mirror is a mirror in the shape of one branch of a hyperbola. Such a mirror reflects light rays directed at one focus toward the other focus. Suppose a hyperbolic mirror is modeled by the upper branch of the hyperbola with equation  $\frac{y^2}{9} - \frac{x^2}{16} = 1$ . A light source is located at  $(-10, 0)$ . Where should the light from the source hit the mirror so that the light will be reflected to  $(0, -5)$ ?

The center is at  $(-1, 4)$ . The vertices are at  $(-3, 4)$  and  $(1, 4)$  and the foci are at  $(-1 \pm \sqrt{13}, 4)$ . The equations of the asymptotes are  $y - 4 = \pm \frac{3}{2}(x + 1)$ .



### 10-6 Conic Sections (pp. 598–602)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

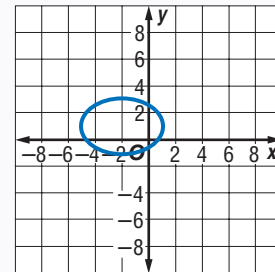
44.  $-4x^2 + y^2 + 8x - 8 = 0$   
45.  $x^2 + 4x - y = 0$   
46.  $x^2 + y^2 - 4x - 6y + 4 = 0$   
47.  $9x^2 + 4y^2 = 36$

Without writing the equation in standard form, state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

48.  $7x^2 + 9y^2 = 63$   
49.  $5y^2 + 2y + 4x - 13x^2 = 81$   
50.  $x^2 - 8x + 16 = 6y$   
51.  $x^2 + 4x + y^2 - 285 = 0$   
**52. ASTRONOMY** A satellite travels in a hyperbolic orbit. It reaches a vertex of its orbit at  $(9, 0)$  and then travels along a path that gets closer and closer to the line  $y = \frac{2}{9}x$ . Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at  $(0, 0)$ .

**Example 7** Without writing the equation in standard form, state whether the graph of  $4x^2 + 9y^2 + 16x - 18y - 11 = 0$  is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

In this equation,  $A = 4$  and  $C = 9$ . Since  $A$  and  $C$  are both positive and  $A \neq C$ , the graph is an ellipse.





## 10-7

## Solving Quadratic Systems (pp. 603-608)

Find the exact solution(s) of each system of equations.

53.  $x^2 + y^2 - 18x + 24y + 200 = 0$   
 $4x + 3y = 0$

54.  $4x^2 + y^2 = 16$   
 $x^2 + 2y^2 = 4$

Solve each system of inequalities by graphing.

55.  $y < x$   
 $y > x^2 - 4$

56.  $x^2 + y^2 \leq 9$   
 $x^2 + 4y^2 \leq 16$

57. **ARCHITECTURE** An architect is building the front entrance of a building in the shape of a parabola with the equation  $y = -\frac{1}{10}(x - 10)^2 + 20$ . While the entrance is being built the construction team puts in two support beams with equations  $y = -x + 10$  and  $y = x - 10$ . Where do the support beams meet the parabola?

**Example 8** Solve the system of equations.

$$x^2 + y^2 + 2x - 12y + 12 = 0$$

$$y + x = 0$$

Use substitution to solve the system.

First, rewrite  $y + x = 0$  as  $y = -x$ .

$$x^2 + y^2 + 2x - 12y + 12 = 0$$

$$x^2 + (-x)^2 + 2x - 12(-x) + 12 = 0$$

$$2x^2 + 14x + 12 = 0$$

$$x^2 + 7x + 6 = 0$$

$$(x + 6)(x + 1) = 0$$

$$x + 6 = 0 \quad \text{or} \quad x + 1 = 0 \quad \text{Zero Product Property}$$

$$x = -6 \quad \quad \quad x = -1 \quad \text{Solve for } x.$$

Now solve for  $y$ .

$$y = -x \quad \quad y = -x \quad \text{Equation for } y \text{ in terms of } x$$

$$= -(-6) \quad = -(-1) \quad \text{Substitute.}$$

$$= 6 \quad \quad = 1$$

The solutions of the system are  $(-6, 6)$  and  $(-1, 1)$ .

Find the midpoint of the line segment with endpoints at the given coordinates.

1.  $(7, 1), (-5, 9)$
2.  $\left(\frac{3}{8}, -1\right), \left(-\frac{8}{5}, 2\right)$
3.  $(-13, 0), (-1, -8)$

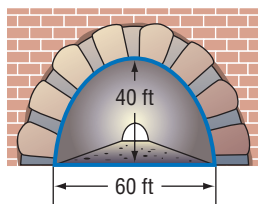
Find the distance between each pair of points with the given coordinates.

4.  $(-6, 7), (3, 2)$
5.  $\left(\frac{1}{2}, \frac{5}{2}\right), \left(-\frac{3}{4}, -\frac{11}{4}\right)$
6.  $(8, -1), (8, -9)$

State whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

7.  $x^2 + 4y^2 = 25$
8.  $x^2 = 36 - y^2$
9.  $4x^2 - 26y^2 + 10 = 0$
10.  $-(y^2 - 24) = x^2 + 10x$
11.  $\frac{1}{3}x^2 - 4 = y$
12.  $y = 4x^2 + 1$
13.  $(x + 4)^2 = 7(y + 5)$
14.  $25x^2 + 49y^2 = 1225$
15.  $5x^2 - y^2 = 49$
16.  $\frac{y^2}{9} - \frac{x^2}{25} = 1$

17. **TUNNELS** The opening of a tunnel is in the shape of a semielliptical arch. The arch is 60 feet wide and 40 feet high. Find the height of the arch 12 feet from the edge of the tunnel.



18. Solve the system of inequalities by graphing.

$$x^2 - y^2 \geq 1$$

$$x^2 + y^2 \leq 16$$

Find the exact solution(s) of each system of equations.

19.  $x^2 + y^2 = 100$

$$y = 2 - x$$

20.  $x^2 + 2y^2 = 6$

$$x + y = 1$$

21.  $x^2 - y^2 - 12x + 12y = 36$

$$x^2 + y^2 - 12x - 12y + 36 = 0$$

**FORESTRY** For Exercises 22 and 23, use the following information.

A forest ranger at an outpost in the Fishlake National Forest in Utah and another ranger at the primary station both heard an explosion. The outpost and the primary station are 6 kilometers apart.

22. If one ranger heard the explosion 6 seconds before the other, write an equation that describes all the possible locations of the explosion. Place the two ranger stations on the  $x$ -axis with the midpoint between the stations at the origin. The transverse axis is horizontal. (*Hint*: The speed of sound is about 0.35 kilometer per second.)
23. Draw a sketch of the possible locations of the explosion. Include the ranger stations in the drawing.
24. **MULTIPLE CHOICE** Which is NOT the equation of a parabola?

A  $y = 2x^2 + 4x - 9$

B  $3x + 2y^2 + y + 1 = 0$

C  $x^2 + 2y^2 + 8y = 8$

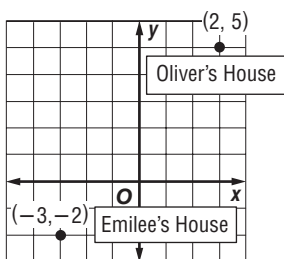
D  $x = \frac{1}{2}(y - 1)^2 + 5$

# Standardized Test Practice

Cumulative, Chapters 1–10

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A coordinate grid is placed over a map. Emilee's house is located at  $(-3, -2)$  and Oliver's house is located at  $(2, 5)$ . A side of each square represents one block.



What is the approximate distance between Emilee's house and Oliver's house?

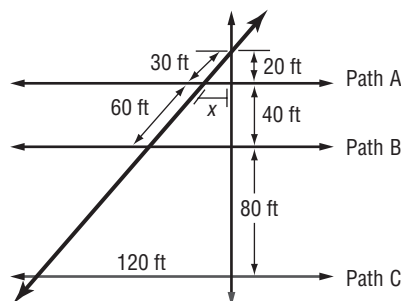
- A 3.2 blocks      C 12.0 blocks  
B 8.6 blocks      D 17.2 blocks
2. A diameter of a circle has endpoints  $A(4, 6)$  and  $B(-3, -1)$ . Find the approximate length of the radius.
- F 2.5 units  
G 4.9 units  
H 5.1 units  
J 9.9 units

## TEST-TAKING TIP

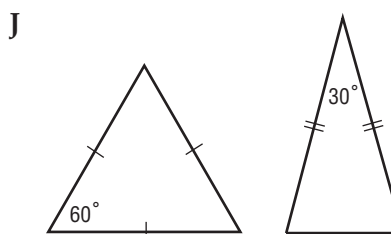
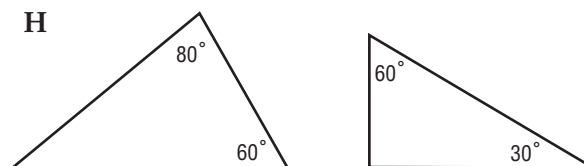
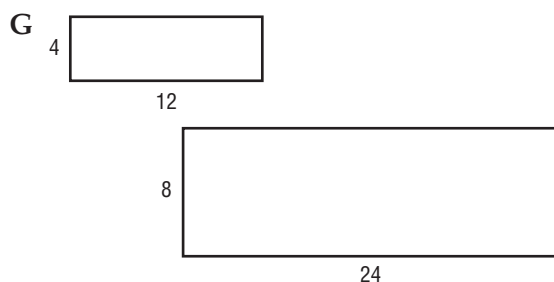
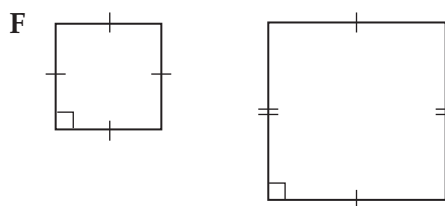
**Question 2** Always write down your calculations on scrap paper or in the test booklet, even if you think you can do the calculations in your head. Writing down your calculations will help you avoid making simple mistakes.

3. Two parallel lines have equations  $y = -2x + 3$  and  $y = mx - 4$ . What is the value of  $m$  in the second linear equation?
- A  $-2$   
B  $-\frac{1}{2}$   
C  $\frac{1}{2}$   
D  $2$

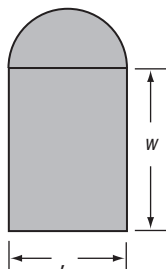
4. **GRIDDABLE** Carla received a map of some walking paths through her college campus. Paths A, B, and C are parallel. What is the length  $x$  to the nearest tenth of a foot?



5. Use the information in each diagram to find the pair of similar polygons.



6. Find the equation that can be used to determine the total area of the composite figure below.



- A  $A = lw + \frac{1}{2}lw$   
 B  $A = lw + \pi\left(\frac{1}{2}l\right)^2$   
 C  $A = lw + \frac{1}{2}\pi l^2$   
 D  $A = lw + \pi\left(\frac{1}{2}l\right)^2\left(\frac{1}{2}\right)$
7. The table shows one of the dimensions of a square tent and the number of people that can fit in the tent.

Length of Tent (yards)	Number of People
2	7
5	28
6	39
8	67
12	147

Let  $\ell$  represent the length of the tent and  $n$  represent the number of people that can fit in the tent. Identify the equation that best represents the relationship between the length of the tent and the number of people that can fit in the tent.

- F  $\ell = n^2 + 3$       H  $\ell = 3n + 1$   
 G  $n = \ell^2 + 3$       J  $n = 3\ell + 1$

8. Margo took her brother to lunch. The bill with tax was \$38.69. If the sales tax was 6%, what was her bill before the sales tax?
- A \$2.32  
 B \$36.37  
 C \$36.50  
 D \$41.01
9. How many faces, edges, and vertices does a pentagonal pyramid have?
- F 5 faces, 8 edges, and 5 vertices  
 G 6 faces, 10 edges, and 6 vertices  
 H 7 faces, 15 edges, and 10 vertices  
 J 6 faces, 12 edges, and 8 vertices

### Pre-AP

**Record your answers on a sheet of paper.  
Show your work.**

10. The endpoints of a diameter of a circle are at  $(-1, 0)$  and  $(5, -8)$ .
- a. What are the coordinates of the center of the circle? Explain your method.  
 b. Find the radius of the circle. Explain your method.  
 c. Write an equation of the circle.

NEED EXTRA HELP?									
If You Missed Question...	1	2	4	5	6	7	8	9	10
Go to Lesson or Page...	10-1	10-1	1-4	2-4	1-1	2-4	750	754	10-3