CHAPTER 12

Probability and Statistics

BIG Ideas

- Solve problems involving independent events, dependent events, permutations, and combinations.
- Find probability and odds.
- Find statistical measures.
- Use the normal, binomial, and exponential distributions.
- Determine whether a sample is unbiased.

Key Vocabulary

event (p. 684)
probability (p. 697)
sample space (p. 684)

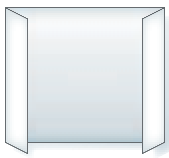
Real-World Link

Approval Polls Polls are often conducted to determine how satisfied the public is with the job being performed by elected officials, such as the President and state governors. Results of these polls may determine on which issues an official focuses his or her efforts.

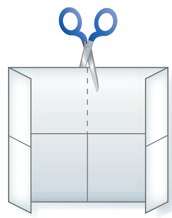
FOLDABLES Study Organizer

Probability and Statistics Make this Foldable to help you organize your notes. Begin with one sheet of 11" by 17" paper.

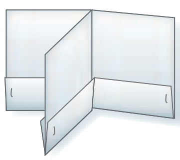
1. **Fold** 2" tabs on each of the short sides.



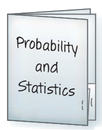
2. **Fold** in half in both directions. Open and cut as shown.



3. **Refold** along the width. Staple each pocket.



4. **Label** pockets as *The Counting Principle, Permutations and Combinations, Probability,* and *Statistics.* Place index cards for notes in each pocket.



Ted S. Warren/Associated Press

GET READY for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Math Online Take the Online Readiness Quiz at algebra2.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Find each probability if a die is rolled once. (Prerequisite Skill)

1. $P(2)$

2. P(numbers greater than 1)

3. $P(5)$

4. P(even number)

5. P(odd number)

6. P(numbers less than 5)

STAMP COLLECTING **Lynette collects stamps from different countries. She has 12 from Mexico, 5 from Canada, 3 from France, 8 from Great Britain, 1 from Russia and 3 from Germany. Find the probability of each of the following if she accidentally loses one stamp.** (Prerequisite Skill)

7. the stamp is from Canada

8. the stamp is not from Germany or Russia

Expand each binomial. (Lesson 11-7)

9. $(a + b)^3$

10. $(c + d)^4$

11. $(m - n)^5$

12. $(x + y)^6$

13. **COINS** A coin is flipped five times. Each time the coin is flipped the outcome is either a head h or a tail t. The terms of the binomial expansion of $(h + t)^5$ can be used to find the probabilities of each combination of heads and tails. Expand the binomial. (Lesson 11-7)

QUICKReview

EXAMPLE 1

Find the probability of rolling a 1 or a 6 if a die is rolled once.

$$P(1 \text{ or } 6) = \frac{\text{number of desired outcomes}}{\text{number of possible outcomes}}$$

There are 2 desired outcomes since 1 or 6 are both desired. There are 6 possible outcomes since there are 6 sides on a die.

$$P(1 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$$

The probability of a 1 or a 6 being rolled is $\frac{1}{3}$, or about 33%.

EXAMPLE 2

Expand $(g - h)^7$.

Remember Pascal's Triangle when expanding a binomial to a large power.

$a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$

Notice the signs in the expansion alternate because the binomial is the difference of two terms. The sum of the exponents of the variables in each term of the expansion is always 7, which is the power the binomial is being raised to. Substitute g for a and h for b.

$g^7 - 7g^6h + 21g^5h^2 - 35g^4h^3 + 35g^3h^4 - 21g^2h^5 + 7gh^6 - h^7$

12-1 The Counting Principle

Main Ideas

- Solve problems involving independent events.
- Solve problems involving dependent events.

New Vocabulary

outcome
sample space
event
independent events
Fundamental Counting Principle
dependent events

GET READY for the Lesson

The number of possible license plates for a state is too great to count by listing all of the possibilities. It is much more efficient to count the number of possibilities by using the Fundamental Counting Principle.

Independent Events An **outcome** is the result of a single trial. For example, the trial of flipping a coin once has two outcomes: head or tail. The set of all possible outcomes is called the **sample space**. An **event** consists of one or more outcomes of a trial. The choices of letters and digits to be put on a license plate are called **independent events** because each letter or digit chosen does *not* affect the choices for the others.

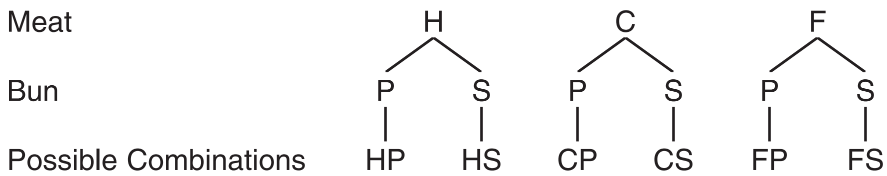
EXAMPLE Independent Events

1 FOOD **A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or a sesame seed bun. How many different combinations of meat and a bun are possible?**

First, note that the choice of the type of meat does not affect the choice of the type of bun, so these events are independent.

Method 1 Tree Diagram

H represents hamburger, C, chicken, F, fish, P, plain, and S, sesame seed.



Method 2 Make a Table

Make a table in which each row represents a type of meat and each column represents a type of bun.

There are six possible outcomes.

		Bun	
		Plain	Sesame
Meal	Hamburger	HP	HS
	Chicken	CP	CS
	Fish	FP	FS

CHECK Your Progress

1. A cafeteria offers drink choices of water, coffee, juice, and milk and salad choices of pasta, fruit, and potato. How many different combinations of drink and salad are possible?

Reading Math

There are both *infinite* and *finite* sample spaces. Finite sample spaces have a countable number of possible outcomes, such as rolling a die. Infinite sample spaces have an uncountable number of possible outcomes, such as the probability of a point on a line.

D.F. Harris

Notice that in Example 1, there are 3 ways to choose the type of meat, 2 ways to choose the type of bun, and 3 • 2 or 6 total ways to choose a combination of the two. This illustrates the **Fundamental Counting Principle**.

KEY CONCEPT *Fundamental Counting Principle*

Words If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Example If event M can occur in 2 ways and event N can occur in 3 ways, then M followed by N can occur in 2 • 3 or 6 ways.

This rule can be extended to any number of events.

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STANDARDIZED TEST EXAMPLE — Fundamental Counting Principle

2 Kim won a contest on a radio station. The prize was a restaurant gift certificate and tickets to a sporting event. She can select one of three different restaurants and tickets to a football, baseball, basketball, or hockey game. How many different ways can she select a restaurant followed by a sporting event?

A 7 **B** 12 **C** 15 **D** 16

Test-Taking Tip

Remember that you can check your answer by making a tree diagram or a table showing the outcomes.

Read the Test Item

Her choice of a restaurant does not affect her choice of a sporting event, so these events are independent.

Solve the Test Item

There are 3 ways she can choose a restaurant and there are 4 ways she can choose the sporting event. By the Fundamental Counting Principle, there are 3 • 4 or 12 total ways she can choose her two prizes. The answer is B.

CHECK Your Progress

2. Dane is renting a tuxedo for prom. Once he has chosen his jacket, he must choose from three types of pants and six colors of vests. How many different ways can he select his attire for the prom?

F 9 **G** 10 **H** 18 **J** 36

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EXAMPLE — More than Two Independent Events

3 **COMMUNICATION** **Many answering machines allow owners to call home and get their messages by entering a 3-digit code. How many codes are possible?**

The choice of any digit does not affect the other two digits, so the choices of the digits are independent events.

There are 10 possible first digits in the code, 10 possible second digits, and 10 possible third digits. So, there are 10 • 10 • 10 or 1000 possible different code numbers.

Reading Math

Independent and *dependent* have the same meaning in mathematics as they do in ordinary language.

Math Online Extra Examples at algebra2.com

CHECK Your Progress

3. If a garage door opener has a 10-digit keypad and the code to open the door is a 4-digit code, how many codes are possible?

Dependent Events Some situations involve dependent events. With **dependent events**, the outcome of one event *does* affect the outcome of another event. The Fundamental Counting Principle applies to dependent events as well as independent events.

EXAMPLE Dependent Events

4 **SCHOOL** **Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?**

When Charlita schedules a given class for a given period, she cannot schedule that class for any other period. Therefore, the choices of which class to schedule each period are dependent events.

There are 6 classes Charlita can take during first period. That leaves 5 classes she can take second period. After she chooses which classes to take the first two periods, there are 4 remaining choices for third period, and so on.

Period	1st	2nd	3rd	4th	5th	6th
Number of Choices	6	5	4	3	2	1

There are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 720 schedules that Charlita could have.

Note that $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$.

Study Tip

Look Back

To review **factorials**, see Lesson 11-7.

CHECK Your Progress

4. Each player in a board game uses one of six different pieces. If four players play the game, how many different ways could the players choose their game pieces?

CONCEPT SUMMARY

Independent Events	**Words**	If the outcome of an event does not affect the outcome of another event, the two events are independent.
	Example	Tossing a coin and rolling a die are independent events.
Dependent Events	**Words**	If the outcome of an event does affect the outcome of another event, the two events are dependent.
	Example	Taking a piece of candy from a jar and then taking a second piece without replacing the first are dependent events because taking the first piece affects what is available to be taken next.

CHECK Your Understanding

Examples 1–4 (pp. 684–686)

State whether the events are *independent* or *dependent*.

1. choosing the color and size of a pair of shoes

2. choosing the winner and runner-up at a dog show

Examples 1, 2 (pp. 684, 685)

3. An ice cream shop offers a choice of two types of cones and 15 flavors of ice cream. How many different 1-scoop ice cream cones can a customer order?

Example 2 (p. 685)

4. **STANDARDIZED TEST PRACTICE** A bookshelf holds 4 different biographies and 5 different mystery novels. How many ways can one book of each type be selected?

A 1 **B** 9 **C** 10 **D** 20

Example 3 (p. 685)

5. Lance's math quiz has eight true-false questions. How many different choices for giving answers to the eight questions are possible?

6. Pizza House offers three different crusts, four sizes, and eight toppings. How many different ways can a customer order a pizza?

Example 4 (p. 686)

7. For a college application, Macawi must select one of five topics on which to write a short essay. She must also select a different topic from the list for a longer essay. How many ways can she choose the topics for the two essays?

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–11	1, 4
12–26	1–4

State whether the events are *independent* or *dependent*.

8. choosing a president, vice-president, secretary, and treasurer for Student Council, assuming that a person can hold only one office

9. selecting a fiction book and a nonfiction book at the library

10. Each of six people guess the total number of points scored in a basketball game. Each person writes down his or her guess without telling what it is.

11. The letters A through Z are written on pieces of paper and placed in a jar. Four of them are selected one after the other without replacing any of them.

12. Tim wants to buy one of three different books he sees in a book store. Each is available in print and on CD. How many book and format choices does he have?

13. A video store has 8 new releases this week. Each is available on videotape and on DVD. How many ways can a customer choose a new release and a format to rent?

14. Carlos has homework in math, chemistry, and English. How many ways can he choose the order in which to do his homework?

15. The menu for a banquet has a choice of 2 types of salad, 5 main courses, and 3 desserts. How many ways can a salad, a main course, and a dessert be selected to form a meal?

16. A baseball glove manufacturer makes gloves in 4 different sizes, 3 different types by position, 2 different materials, and 2 different levels of quality. How many different gloves are possible?

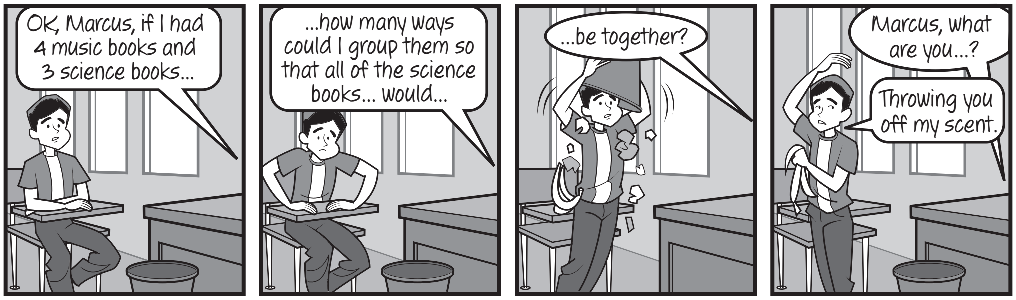
17. Each question on a five-question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

Cross-Curricular Project

Math Online You can use the Fundamental Counting Principle to list possible outcomes in games. Visit algebra2.com to continue work on your project.

18. PASSWORDS Abby is registering at a Web site. She must select a password containing six numerals to be able to use the site. How many passwords are allowed if no digit may be used more than once?

ENTERTAINMENT For Exercises 19 and 20, refer to the comic strip. Assume that the books are all different.



Real-World Link

Before 1995, area codes had the following format.

(XYZ)

X = 2, 3, …, or 9

Y = 0 or 1

Z = 0, 1, 2, …, or 9

Source: www.nanpa.com

19. How many ways can you arrange the science books?

20. Since the science books are to be together, they can be treated like one book and arranged with the music books. Use your answer to Exercise 19 and the Counting Principle to find the answer to the problem in the comic.

AREA CODES For Exercises 21 and 22, refer to the information about telephone area codes at the left.

21. How many area codes were possible before 1995?

22. In 1995, the restriction on the middle digit was removed, allowing any digit in that position. How many total codes were possible after this change was made?

23. How many ways can six different books be arranged on a shelf if one of the books is a dictionary and it must be on an end?

24. In how many orders can eight actors be listed in the opening credits of a movie if the leading actor must be listed first or last?

25. HOME SECURITY How many different 5-digit codes are possible using the keypad shown at the right if the first digit cannot be 0 and no digit may be used more than once?

EXTRA PRACTICE

See pages 916, 937.

Math Online

Self-Check Quiz at algebra2.com

26. RESEARCH Use the Internet or other resource to find the configuration of letters and numbers on license plates in your state. Then find the number of possible plates.

H.O.T. Problems

27. OPEN ENDED Describe a situation in which you can use the Fundamental Counting Principle to show that there are 18 total possibilities.

28. REASONING Explain how choosing to buy a car or a pickup truck and then selecting the color of the vehicle could be dependent events.

29. CHALLENGE The members of the Math Club need to elect a president and a vice president. They determine that there are a total of 272 ways that they can fill the positions with two different members. How many people are in the Math Club?

(t)Mitch Kezar/Getty Images, (b)age fotostock

30. ***Writing in Math*** Use the information on page 684 to explain how you can count the maximum number of license plates a state can issue. Explain how to use the Fundamental Counting Principle to find the number of different license plates in a state such as Oklahoma, which has 3 letters followed by 3 numbers. Also explain how a state can increase the number of possible plates without increasing the length of the plate number.

STANDARDIZED TEST PRACTICE

31. ACT/SAT How many numbers between 100 and 999, inclusive, have 7 in the tens place?

A 90

B 100

C 110

D 120

32. REVIEW A coin is tossed four times. How many possible sequences of heads or tails are possible?

F 4

G 8

H 16

J 32

Spiral Review

33. Prove that $4 + 7 + 10 + \cdots + (3n + 1) = \frac{n(3n + 5)}{2}$ for all positive integers n. (Lesson 11-8)

Find the indicated term of each expansion. (Lesson 11-7)

34. third term of $(x + y)^8$

35. fifth term of $(2a - b)^7$

36. CARTOGRAPHY Edison is located at (9, 3) in the coordinate system on a road map. Kettering is located at (12, 5) on the same map. Each side of a square on the map represents 10 miles. To the nearest mile, what is the distance between Edison and Kettering? (Lesson 10-1)

Solve each equation by factoring. (Lesson 5-3)

37. $x^2 - 16 = 0$

38. $x^2 - 3x - 10 = 0$

39. $3x^2 + 8x - 3 = 0$

Solve each matrix equation. (Lesson 4-1)

40. $[x \quad y] = [y \quad 4]$

41. $\begin{bmatrix} 3y \\ 2x \end{bmatrix} = \begin{bmatrix} x + 8 \\ y - x \end{bmatrix}$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.** (Lesson 11-7)

42. $\frac{5!}{2!}$

43. $\frac{6!}{4!}$

44. $\frac{7!}{3!}$

45. $\frac{6!}{1!}$

46. $\frac{4!}{2!2!}$

47. $\frac{6!}{2!4!}$

48. $\frac{8!}{3!5!}$

49. $\frac{5!}{5!0!}$

12-2 Permutations and Combinations

Main Ideas

- Solve problems involving permutations.
- Solve problems involving combinations.

New Vocabulary

permutation
linear permutation
combination

GET READY for the Lesson

When the manager of a softball team fills out her team's lineup card before the game, the order in which she fills in the names is important because it determines the order in which the players will bat.

Suppose she has 7 possible players in mind for the top 4 spots in the lineup. You know from the Fundamental Counting Principle that there are $7 \cdot 6 \cdot 5 \cdot 4$ or 840 ways that she could assign players to the top 4 spots.

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**. In a permutation, the *order* of the objects is very important. The arrangement of objects or people in a line is called a **linear permutation**.

Notice that $7 \cdot 6 \cdot 5 \cdot 4$ is the product of the first 4 factors of 7!. You can rewrite this product in terms of 7!.

$$7 \cdot 6 \cdot 5 \cdot 4 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$ Multiply by $\frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$ or 1.

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \text{ or } \frac{7!}{3!}$$ $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ and $3! = 3 \cdot 2 \cdot 1$

Notice that 3! is the same as $(7 - 4)!$.

The number of ways to arrange 7 people or objects taken 4 at a time is written $P(7, 4)$. The expression for the softball lineup above is a case of the following formula.

Reading Math

Permutations The expression $P(n, r)$ reads *the number of permutations of n objects taken r at a time.* It is sometimes written as $_nP_r$.

KEY CONCEPT *Permutations*

The number of permutations of n distinct objects taken r at a time is given by

$$P(n, r) = \frac{n!}{(n - r)!}.$$

EXAMPLE Permutation

1 **FIGURE SKATING** **There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?**

Since each winner will receive a different medal, order is important. You must find the number of permutations of 10 things taken 3 at a time.

Study Tip

Alternate Method

Notice that in Example 1, all of the factors of $(n - r)!$ are also factors of $n!$. You can also evaluate the expression in the following way.

$$\frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 \text{ or } 720$$

$$P(n, r) = \frac{n!}{(n-r)!} \quad \text{Permutation formula}$$

$$P(10, 3) = \frac{10!}{(10-3)!} \quad n = 10, r = 3$$

$$= \frac{10!}{7!} \quad \text{Simplify.}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \text{ or } 720 \quad \text{Divide by common factors.}$$

The gold, silver, and bronze medals can be awarded in 720 ways.

CHECK Your Progress

1. A newspaper has nine reporters available to cover four different stories. How many ways can the reporters be assigned to cover the stories?

Suppose you want to rearrange the letters of the word *geometry* to see if you can make a different word. If the two *e*s were not identical, the eight letters in the word could be arranged in $P(8, 8)$ ways. To account for the identical *e*s, divide $P(8, 8)$ by the number of arrangements of *e*. The two *e*s can be arranged in $P(2, 2)$ ways.

$$\frac{P(8, 8)}{P(2, 2)} = \frac{8!}{2!} \quad \text{Divide.}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} \text{ or } 20{,}160 \quad \text{Simplify.}$$

Thus, there are 20,160 ways to arrange the letters in *geometry*.

When some letters or objects are alike, use the rule below to find the number of permutations.

KEY CONCEPT — *Permutations with Repetitions*

The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

This rule can be extended to any number of objects that are repeated.

EXAMPLE Permutation with Repetition

2 **How many different ways can the letters of the word *MISSISSIPPI* be arranged?**

The letter *I* occurs 4 times, *S* occurs 4 times, and *P* occurs twice.

You need to find the number of permutations of 11 letters of which 4 of one letter, 4 of another letter, and 2 of another letter are the same.

$$\frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!4!2!} \text{ or } 34{,}650$$

There are 34,650 ways to arrange the letters.

CHECK Your Progress

2. How many different ways can the letters of the word *DECIDED* be arranged?

Study Tip

Permutations and Combinations

- If order in an arrangement *is* important, the arrangement is a *permutation*
- If order is *not* important, the arrangement is a *combination*.

Combinations An arrangement or selection of objects in which order is *not* important is called a **combination**. The number of combinations of n objects taken r at a time is written $C(n, r)$. *It is sometimes written* ${}_nC_r$

You know that there are $P(n, r)$ ways to select r objects from a group of n if the order is important. There are $r!$ ways to order the r objects that are selected, so there are $r!$ permutations that are all the same combination. Therefore,

$$C(n, r) = \frac{P(n, r)}{r!} \text{ or } \frac{n!}{(n-r)!r!}.$$

KEY CONCEPT — *Combinations*

The number of combinations of n distinct objects taken r at a time is given by

$$C(n, r) = \frac{n!}{(n-r)!r!}.$$

EXAMPLE 3 Combination

A group of seven students working on a project needs to choose two students to present the group's report. How many ways can they choose the two students?

Since the order they choose the students is not important, you must find the number of combinations of 7 students taken 2 at a time.

$C(n, r) = \frac{n!}{(n-r)!r!}$ Combination formula

$C(7, 2) = \frac{7!}{(7-2)!2!}$ $n = 7$ and $r = 2$

$= \frac{7!}{5!2!}$ or 21 Simplify.

There are 21 possible ways to choose the two students.

CHECK Your Progress

3. A family with septuplets assigns different chores to the children each week. How many ways can three children be chosen to help with the laundry?

In more complicated situations, you may need to multiply combinations and/or permutations.

EXAMPLE 4 Multiple Events

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Study Tip

Deck of Cards

In this text, a *standard deck of cards* always means a deck of 52 playing cards. There are 4 suits–clubs (black), diamonds (red), hearts (red), and spades (black)–with 13 cards in each suit.

Five cards are drawn from a standard deck of cards. How many hands consist of three clubs and two diamonds?

By the Fundamental Counting Principle, you can multiply the number of ways to select three clubs and the number of ways to select two diamonds.

Only the cards in the hand matter, not the order in which they were drawn, so use combinations.

$C(13, 3)$ Three of 13 clubs are to be drawn.

$C(13, 2)$ Two of 13 diamonds are to be drawn.

$$C(13, 3) \cdot C(13, 2) = \frac{13!}{(13-3)!13!} \cdot \frac{13!}{(13-2)!13!}$$ Combination formula

$$= \frac{13!}{10!3!} \cdot \frac{13!}{11!2!}$$ Subtract.

$$= 286 \cdot 78 \text{ or } 22{,}308$$ Simplify.

There are 22,308 hands consisting of 3 clubs and 2 diamonds.

CHECK Your Progress

4. How many five-card hands consist of five cards of the same suit?

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CHECK Your Understanding

Examples 1–4 (pp. 690–693)

Evaluate each expression.

1. $P(5, 3)$

2. $P(6, 3)$

3. $C(4, 2)$

4. $C(6, 1)$

Examples 1–3 (pp. 690–692)

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

5. seven shoppers in line at a checkout counter

6. an arrangement of the letters in the word *intercept*

7. an arrangement of 4 blue tiles, 2 red tiles, and 3 black tiles in a row

8. choosing 2 different pizza toppings from a list of 6

Example 3 (p. 692)

9. **SCHEDULING** The Helping Hand Moving Company owns nine trucks. On one Saturday, the company has six customers who need help moving. In how many ways can a group of six trucks be selected from the company's fleet?

Example 4 (pp. 692–693)

10. Six cards are drawn from a standard deck of cards. How many hands will contain three hearts and three spades?

Exercises

HOMEWORK HELP

For Exercises	See Examples
11–14, 21, 22	1
23, 24	2
15–18, 25–28	3
19, 20, 29–31	4

Evaluate each expression.

11. $P(8, 2)$

12. $P(9, 1)$

13. $P(7, 5)$

14. $P(12, 6)$

15. $C(5, 2)$

16. $C(8, 4)$

17. $C(12, 7)$

18. $C(10, 4)$

19. $C(12, 4) \cdot C(8, 3)$

20. $C(9, 3) \cdot C(6, 2)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

21. the winner and first, second, and third runners-up in a contest with 10 finalists

22. placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf

23. an arrangement of the letters in the word *algebra*

24. an arrangement of the letters in the word *parallel*

25. selecting two of eight employees to attend a business seminar

Real-World Link

The Hawaiian language consists of only twelve letters, the vowels a, e, i, o, and u and the consonants h, k, l m, n, p, and w.

Source: andhawaii.com

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

26. selecting nine books to check out of the library from a reading list of twelve

27. choosing two CDs to buy from ten that are on sale

28. selecting three of fifteen flavors of ice cream at the grocery store

29. How many ways can a hand of five cards consisting of four cards from one suit and one card from another suit be drawn from a standard deck of cards?

30. A student council committee must be composed of two juniors and two sophomores. How many different committees can be chosen from seven juniors and five sophomores?

31. How many ways can a hand of five cards consisting of three cards from one suit and two cards from another suit be drawn from a standard deck of cards?

32. MOVIES The manager of a four-screen movie theater is deciding which of 12 available movies to show. The screens are in rooms with different seating capacities. How many ways can she show four different movies on the screens?

33. LANGUAGES How many different arrangements of the letters of the Hawaiian word *aloha* are possible?

34. GOVERNMENT How many ways can five members of the 100-member United States Senate be chosen to serve on a committee?

35. LOTTERIES In a multi-state lottery, the player must guess which five of forty-nine white balls numbered from 1 to 49 will be drawn. The order in which the balls are drawn does not matter. The player must also guess which one of forty-two red balls numbered from 1 to 42 will be drawn. How many ways can the player fill out a lottery ticket?

EXTRA PRACTICE

See pages 917, 937.

Math Online

Self-Check Quiz at algebra2.com

36. CARD GAMES *Hachi-hachi* is a Japanese game that uses a deck of *Hanafuda* cards which is made up of 12 suits, with each suit having four cards. How many 7-card hands can be formed so that 3 are from one suit and 4 are from another?

H.O.T. Problems

37. OPEN ENDED Describe a situation in which the number of outcomes is given by $P(6, 3)$.

38. REASONING Prove that $C(n, n - r) = C(n, r)$.

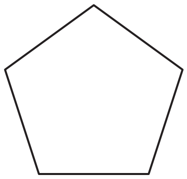
39. REASONING Determine whether the statement $C(n, r) = P(n, r)$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

40. CHALLENGE Show that $C(n - 1, r) + C(n - 1, r - 1) = C(n, r)$.

41. *Writing in Math* Use the information on page 690 to explain how permutations and combinations apply to softball. Explain how to find the number of 9-person lineups that are possible and how many ways there are to choose 9 players if 16 players show up for a game.

STANDARDIZED TEST PRACTICE

42. ACT/SAT How many diagonals can be drawn in the pentagon?



A 5 C 15

B 10 D 20

43. REVIEW How many ways can eight runners in an Olympic race finish in first, second, and third places?

F 8 H 56

G 24 J 336

Spiral Review

44. Darius can do his homework in pencil or pen, using lined or unlined paper, and on one or both sides of each page. How many ways can he do his homework? (Lesson 12-1)

45. A customer in an ice cream shop can order a sundae with a choice of 10 flavors of ice cream, a choice of 4 flavors of sauce, and with or without a cherry on top. How many different sundaes are possible? (Lesson 12-1)

Find a counterexample for each statement. (Lesson 11-8)

46. $1 + 2 + 3 + \cdots + n = 2n - 1$

47. $5^n + 1$ is divisible by 6.

Solve each equation or inequality. (Lesson 9-5)

48. $3e^x + 1 = 2$

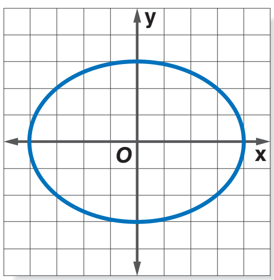
49. $e^{2x} > 5$

50. $\ln (x - 1) = 3$

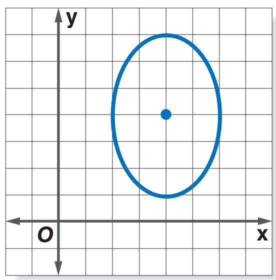
51. CONSTRUCTION A painter works on a job for 10 days and is then joined by an associate. Together they finish the job in 6 more days. The associate could have done the job in 30 days. How long would it have taken the painter to do the job alone? (Lesson 8-6)

Write an equation for each ellipse. (Lesson 10-4)

52.



53.



GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate the expression $\frac{x}{x + y}$ for the given values of x and y.** (Lesson 1-1)

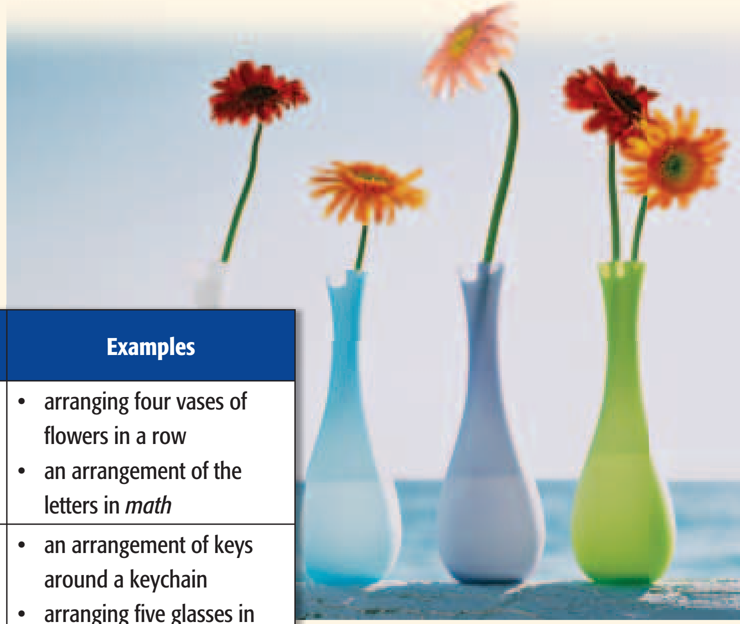
54. $x = 3, y = 2$

55. $x = 4, y = 4$

56. $x = 2, y = 8$

57. $x = 5, y = 10$

READING MATH



Permutations and Combinations

When solving probability problems, it is helpful to be able to determine whether situations involve *permutations* or *combinations.* Often words in a problem give clues as to which type of arrangement is involved.

Type of Arrangement	Description	Clue Words	Examples
Linear Permutation	The order of objects or people in a line is important.	• arranging *x* • an arrangement of first, second, third	• arranging four vases of flowers in a row • an arrangement of the letters in *math*
Circular Permutation	The order of objects or people in a circle is important.	• an arrangement around • arranging in a circle	• an arrangement of keys around a keychain • arranging five glasses in a circle on a tray
Combination	The order of objects or people is not important.	• selecting *x* of *y* • choosing *x* from *y* • forming *x* from *y*	• selecting 3 of 8 flavors • choosing 2 people from a group of 7

Reading to Learn

Determine whether each situation involves a *permutation* or a *combination.* If it is a permutation, identify it as *linear* or *circular.*

1. choosing six students from a class of 25

2. an arrangement of the letters in *drive*

3. selecting two of nine different side dishes

4. choosing three classes from a list of twelve to schedule for first, second, and third periods

5. arranging eighteen students in a circle for a class discussion

6. arranging seven swimmers in the lanes of a swimming pool

7. selecting five volunteers from a group of ten

8. an arrangement of six small photographs around a central photograph

9. forming a team of twelve athletes from a group of 35 who try out

10. **OPEN ENDED** Write a combination problem that involves the numbers 4 and 16.

11. Discuss how the definitions of the words *permanent* and *combine* could help you to remember the difference between permutations and combinations.

12. Describe a real-world situation that involves a permutation and a real-world situation that involves a combination. Explain your reasoning.

12-3 Probability

Main Ideas

- Use combinations and permutations to find probability.
- Create and use graphs of probability distributions.

New Vocabulary

probability
success
failure
random
random variable
probability distribution
uniform distribution
relative-frequency histogram

Reading Math

Notation When *P* is followed by an event in parentheses, *P* stands for *probability.* When there are two numbers in parentheses, *P* stands for *permutations.*

GET READY for the Lesson

The risk of getting struck by lightning in any given year is 1 in 750,000. The chances of surviving a lightning strike are 3 in 4. These risks and chances are a way of describing the probability of an event. The **probability** of an event is a ratio that measures the chances of the event occurring.

Probability and Odds Mathematicians often use tossing of coins and rolling of dice to illustrate probability. When you toss a coin, there are only two possible outcomes—heads or tails. A desired outcome is called a **success**. Any other outcome is called a **failure**.

KEY CONCEPT — *Probability of Success and Failure*

If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows.

$$P(S) = \frac{s}{s+f} \qquad P(F) = \frac{f}{s+f}$$

The probability of an event occurring is always between 0 and 1, inclusive. The closer the probability of an event is to 1, the more likely the event is to occur. The closer the probability of an event is to 0, the less likely the event is to occur. When all outcomes have an equally likely chance of occurring, we say that the outcomes occur at **random**.

EXAMPLE 1 Probability with Combinations

Monifa has a collection of 32 CDs—18 R&B and 14 rap. As she is leaving for a trip, she randomly chooses 6 CDs to take with her. What is the probability that she selects 3 R&B and 3 rap?

Step 1 Determine how many 6-CD selections meet the conditions.

$C(18, 3)$ Select 3 R&B CDs. Their order does not matter.

$C(14, 3)$ Select 3 rap CDs.

Step 2 Use the Fundamental Counting Principle to find s, the number of successes.

$$C(18, 3) \cdot C(14, 3) = \frac{18!}{15!3!} \cdot \frac{14!}{11!3!} \text{ or } 297{,}024$$

(continued on the next page)

Step 3 Find the total number, $s + f$, of possible 6-CD selections.

$C(32, 6) = \frac{32!}{26!6!}$ or 906,192 $\quad s + f = 906{,}192$

Step 4 Determine the probability.

$P(\text{3 R\&B CDs and 3 rap CDs}) = \frac{s}{s+f}$ — Probability formula

$= \frac{297{,}024}{906{,}192}$ — Substitute.

≈ 0.32777 — Use a calculator.

The probability of selecting 3 R&B CDs and 3 rap CDs is about 0.32777 or 33%.

CHECK Your Progress

1. A board game is played with tiles with letters on one side. There are 56 tiles with consonants and 42 tiles with vowels. Each player must choose seven of the tiles at the beginning of the game. What is the probability that a player selects four consonants and three vowels?

Personal Tutor

EXAMPLE Probability with Permutations

2 **Ramon has five books on the floor, one for each of his classes: Algebra 2, chemistry, English, Spanish, and history. Ramon is going to put the books on a shelf. If he picks the books up at random and places them in a row on the same shelf, what is the probability that his English, Spanish, and Algebra 2 books will be the leftmost books on the shelf, but not necessarily in that order?**

Step 1 Determine how many book arrangements meet the conditions.

$P(3, 3)$ — Place the 3 leftmost books.

$P(2, 2)$ — Place the other 2 books.

Step 2 Use the Fundamental Counting Principle to find the number of successes.

$P(3, 3) \cdot P(2, 2) = 3! \cdot 2!$ or 12

Step 3 Find the total number, $s + f$, of possible 5-book arrangements.

$P(5, 5) = 5!$ or 120 $\quad s + f = 120$

Step 4 Determine the probability.

$P(\text{English, Spanish, Algebra 2 followed by other books})$

$= \frac{s}{s+f}$ — Probability formula

$= \frac{12}{120}$ — Substitute.

$= 0.1$ — Use a calculator.

The probability of placing English, Spanish, and Algebra 2 before the other four books is 0.1 or 10%.

CHECK Your Progress

2. What is the probability that English will be the last book on the shelf?

Personal Tutor at algebra2.com

Probability Distributions Many experiments, such as rolling a die, have numerical outcomes. A **random variable** is a variable whose value is the numerical outcome of a random event. For example, when rolling a die we can let the random variable *D* represent the number showing on the die. Then *D* can equal 1, 2, 3, 4, 5, or 6. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. The table below illustrates the probability distribution for rolling a die. *A distribution like this one where all of the probabilities are the same is called a* ***uniform distribution****.*

Reading Math

Random Variables The notation $P(X = n)$ is used with random variables. $P(D = 4) = \frac{1}{6}$ is read *the probability that D equals 4 is one sixth.*

D = Roll	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$P(D = 4) = \frac{1}{6}$

To help visualize a probability distribution, you can use a table of probabilities or a graph, called a **relative-frequency histogram**.

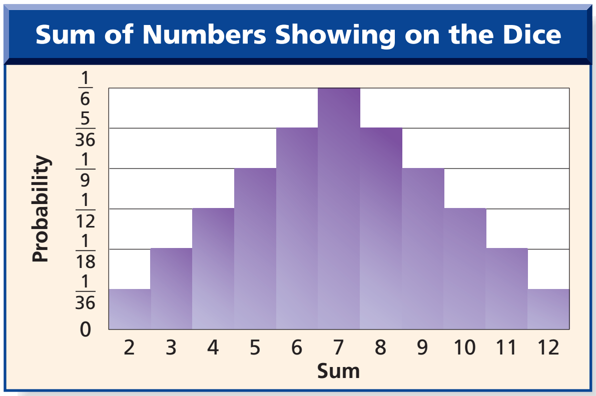
EXAMPLE 3 Probability Distribution

Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the sum of the numbers rolled.

Reading Math

Discrete Random Variables A discrete random variable is a variable that can have a countable number of values. The variable is said to be *random* if the sum of the probabilities is 1.

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$



a. Use the graph to determine which outcome is most likely. What is its probability?

The most likely outcome is a sum of 7, and its probability is $\frac{1}{6}$.

(continued on the next page)

b. Use the table to find $P(S = 9)$. What other sum has the same probability?

According to the table, the probability of a sum of 9 is $\frac{1}{9}$. The other outcome with a probability of $\frac{1}{9}$ is 5.

CHECK Your Progress

3A. Which outcome(s) is least likely? What is its probability?

3B. Use the table to find $P(S = 3)$. What other sum has the same probability?

CHECK Your Understanding

Example 1 (pp. 697–698)

Suppose you select 2 letters at random from the word *compute*. Find each probability.

1. P(2 vowels) **2.** P(2 consonants) **3.** P(1 vowel, 1 consonant)

Example 2 (pp. 698–699)

ORGANIZATION **An administrative assistant has 4 blue file folders, 3 red folders, and 3 yellow folders on her desk. Each folder contains different information, so two folders of the same color should be viewed as being different. She puts the file folders randomly in a box to be taken to a meeting. Find each probability.**

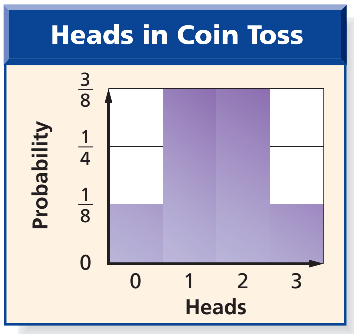
4. P(4 blue, 3 red, 3 yellow, in that order)

5. P(first 2 blue, last 2 blue)

Example 3 (pp. 699–700)

The table and the relative-frequency histogram show the distribution of the number of heads when 3 coins are tossed. Find each probability.

H = Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



6. $P(H = 0)$

7. $P(H = 2)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–15	1
16–21	2
22–27	3

Bob is moving and all of his sports cards are mixed up in a box. Twelve cards are baseball, eight are football, and five are basketball. If he reaches in the box and selects them at random, find each probability.

8. P(3 football) **9.** P(3 baseball)

10. P(1 basketball, 2 football) **11.** P(2 basketball, 1 baseball)

12. P(1 football, 2 baseball) **13.** P(1 basketball, 1 football, 1 baseball)

14. P(2 baseball, 2 basketball) **15.** P(2 football, 1 hockey)

DVDS Janice has 8 DVD cases on a shelf, one for each season of her favorite TV show. Her brother accidentally knocks them off the shelf onto the floor. When her brother puts them back on the shelf, he does not pay attention to the season numbers and puts the cases back on the shelf randomly. Find each probability.

16. P(season 5 in the correct position)

17. P(seasons 1 and 8 in the correct positions)

18. P(seasons 1 through 4 in the correct positions)

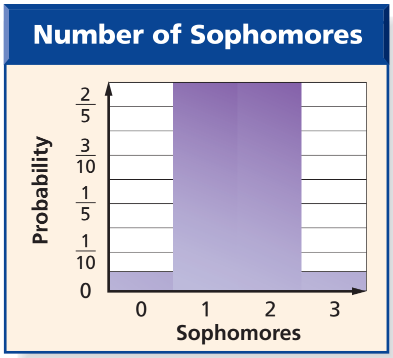
19. P(all even-numbered seasons followed by all odd-numbered seasons)

20. P(all even-numbered seasons in the correct position)

21. P(seasons 5 through 8 in any order followed by seasons 1 through 4 in any order)

Three students are selected at random from a group of 3 sophomores and 3 juniors. The table and relative-frequency histogram show the distribution of the number of sophomores chosen. Find each probability.

Sophomores	0	1	2	3
Probability	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$



22. P(0 sophomores)

23. P(1 sophomore)

24. P(2 sophomores)

25. P(3 sophomores)

26. P(2 juniors)

27. P(1 junior)

Real-World Career

Physician

In addition to the MCAT, most medical schools require applicants to have had one year each of biology, physics, and English, and two years of chemistry in college.

Math Online

For more information, go to algebra2.com.

28. **LOTTERIES** The state of Texas has a lottery in which 5 numbers out of 37 are drawn at random. What is the probability of a given ticket matching all 5 numbers?

ENTRANCE TESTS **For Exercises 29–31, use the table that shows the college majors of the students who took the Medical College Admission Test (MCAT) recently.**

If a student taking the test were randomly selected, find each probability. Express as decimals rounded to the nearest thousandth.

Major	Students
biological sciences	15,819
humanities	963
math or statistics	179
physical sciences	2770
social sciences	2482
specialized health sciences	1431
other	1761

29. P(math or statistics)

30. P(biological sciences)

31. P(physical sciences)

EXTRA PRACTICE

See pages 917, 937.

Math Online

Self-Check Quiz at algebra2.com

32. **CARD GAMES** The game of euchre (YOO ker) is played using only the 9s, 10s, jacks, queens, kings, and aces from a standard deck of cards. Find the probability of being dealt a 5-card hand containing all four suits.

33. **WRITING** Josh types the five entries in the bibliography of his term paper in random order, forgetting that they should be in alphabetical order by author. What is the probability that he actually typed them in alphabetical order?

H.O.T. Problems

34. **OPEN ENDED** Describe an event that has a probability of 0 and an event that has a probability of 1.

Food & Drug Administration/Science Photo Library/Photo Researchers

CHALLENGE **Theoretical probability** is determined using mathematical methods and assumptions about the fairness of coins, dice, and so on. **Experimental probability** is determined by performing experiments and observing the outcomes.

Determine whether each probability is *theoretical* or *experimental*. Then find the probability.

35. Two dice are rolled. What is the probability that the sum will be 12?

36. A baseball player has 126 hits in 410 at-bats this season. What is the probability that he gets a hit in his next at-bat?

37. A hand of 2 cards is dealt from a standard deck of cards. What is the probability that both cards are clubs?

38. ***Writing in Math*** Use the information on page 697 to explain what probability tells you about life's risks. Include a description of the meaning of *success* and *failure* in the case of being struck by lightning and surviving.

STANDARDIZED TEST PRACTICE

39. **ACT/SAT** What is the value of $\frac{6!}{2!}$?

A 3

B 60

C 360

D 720

40. **REVIEW** A jar contains 4 red marbles, 3 green marbles, and 2 blue marbles. If a marble is drawn at random, what is the probability that it is *not* green?

F $\frac{2}{9}$ **G** $\frac{1}{3}$ **H** $\frac{4}{9}$ **J** $\frac{2}{3}$

Spiral Review

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. (Lesson 12-2)

41. arranging 5 different books on a shelf

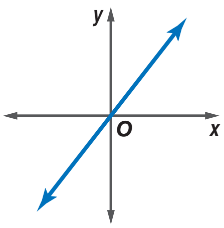
42. arranging the letters of the word *arrange*

43. picking 3 apples from the last 7 remaining at the grocery store

44. How many ways can 4 different gifts be placed into 4 different gift bags if each bag gets exactly 1 gift? (Lesson 12-1)

Identify the type of function represented by each graph. (Lesson 8-5)

45.



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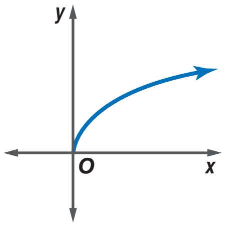


GET READY for the Next Lesson

PREREQUISITE SKILL **Find each product if $a = \frac{3}{5}$, $b = \frac{2}{7}$, $c = \frac{3}{4}$, and $d = \frac{1}{3}$.**

47. ab **48.** bc **49.** cd **50.** bd **51.** ac

12-4 Multiplying Probabilities

Main Ideas

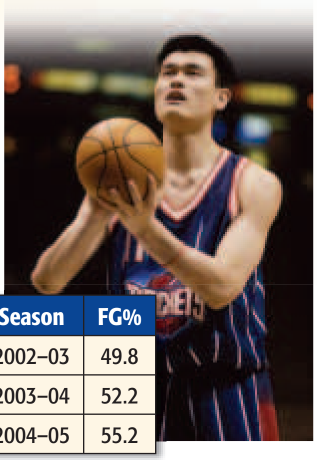
- Find the probability of two independent events.
- Find the probability of two dependent events.

New Vocabulary

area diagram

GET READY for the Lesson

Yao Ming, of the Houston Rockets, has one of the best field-goal percentages in the National Basketball Association. The table shows the field-goal percentages for three years of his career. For any year, you can determine the probability that Yao will make two field goals in a row based on the probability of his making one field goal.



Season	FG%
2002–03	49.8
2003–04	52.2
2004–05	55.2

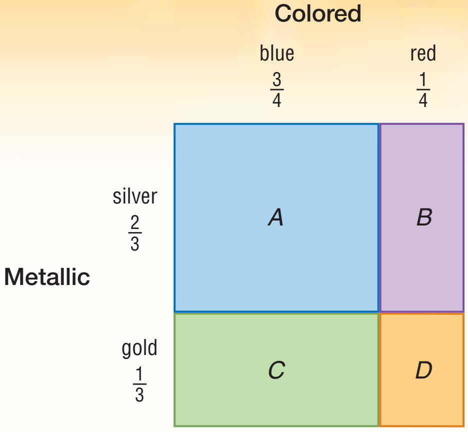
Source: nba.com

Probability of Independent Events In a situation with two events like shooting a field goal and then shooting another, you can find the probability of both events occurring if you know the probability of each event occurring. You can use an **area diagram** to model the probability of the two events occurring.

Algebra Lab

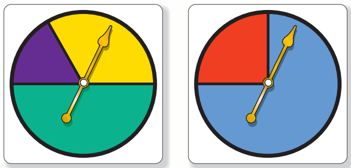
Area Diagrams

Suppose there are 1 red and 3 blue paper clips in one drawer and 1 gold and 2 silver paper clips in another drawer. The area diagram represents the probabilities of choosing one colored paper clip and one metallic paper clip if one of each is chosen at random. For example, rectangle A represents drawing 1 silver clip and 1 blue clip.



MODEL AND ANALYZE

1. Find the areas of rectangles A, B, C, and D. Explain what each represents.
2. Find the probability of choosing a red paper clip and a silver paper clip.
3. What are the length and width of the whole square? What is the area? Why does the area need to have this value?
4. Make an area diagram that represents the probability of each outcome if you spin each spinner once. Label the diagram and describe what the area of each rectangle represents.



In Exercise 4 of the lab, spinning one spinner has no effect on the second spinner. These events are independent.

KEY CONCEPT — Probability of Two Independent Events

If two events, A and B, are independent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

This formula can be applied to any number of independent events.

EXAMPLE Two Independent Events

1 **At a picnic, Julio reaches into an ice-filled cooler containing 8 regular soft drinks and 5 diet soft drinks. He removes a can, then decides he is not really thirsty, and puts it back. What is the probability that Julio and the next person to reach into the cooler both randomly select a regular soft drink?**

> **Study Tip**
>
> **Alternative Method**
>
> You could use the Fundamental Counting Principle to find the number of successes and the number of total outcomes.
>
> both regular = $8 \cdot 8$ or 64
>
> total outcomes = $13 \cdot 13$ or 169
>
> So, $P(\text{both reg.}) = \frac{64}{169}$

Explore These events are independent since Julio replaced the can that he removed. The outcome of the second person's selection is not affected by Julio's selection.

Plan Since there are 13 cans, the probability of each person's getting a regular soft drink is $\frac{8}{13}$.

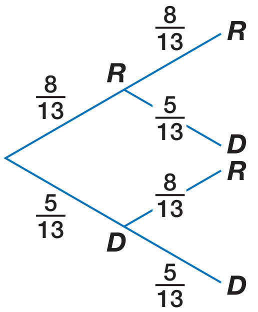
Solve $P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular})$ — Probability of independent events

$= \frac{8}{13} \cdot \frac{8}{13}$ or $\frac{64}{169}$ — Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{64}{169}$ or about 38%.

Check You can verify this result by making a tree diagram that includes probabilities. Let R stand for regular and D stand for diet.

$P(R, R) = \frac{8}{13} \cdot \frac{8}{13}$



CHECK Your Progress

1. At a promotional event, a radio station lets visitors spin a prize wheel. The wheel has 10 sectors of the same size for posters, 6 for T-shirts, and 2 for concert tickets. What is the probability that two consecutive visitors will win posters?

EXAMPLE Three Independent Events

2 **In a board game, three dice are rolled to determine the number of moves for the players. What is the probability that the first die shows a 6, the second die shows a 6, and the third die does not?**

> **Study Tip**
>
> The **complement** of a set is the set of all objects that do *not* belong to the given set. For a six-sided die, showing a 6 is the complement of showing 1, 2, 3, 4, or 5.

Let A be the event that the first die shows a 6. → $P(A) = \frac{1}{6}$

Let B be the event that the second die shows a 6. → $P(B) = \frac{1}{6}$

Let C be the event that the third die does *not* show a 6. → $P(C) = \frac{5}{6}$

$P(A, B, \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$ Probability of independent events

$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}$ or $\frac{5}{216}$ Substitute and multiply.

The probability that the first and second dice show a 6 and the third die does not is $\frac{5}{216}$.

CHECK Your Progress

2. In a state lottery game, each of three cages contains 10 balls. The balls are each labeled with one of the digits 0–9. What is the probability that the first two balls drawn will be even and that the third will be prime?

Study Tip

Conditional Probability

The event of getting a regular soft drink the second time *given* that Julio got a regular soft drink the first time is called a *conditional probability*.

Probability of Dependent Events In Example 1, what is the probability that both people select a regular soft drink if Julio does not put his back in the cooler? In this case, the two events are dependent because the outcome of the first event affects the outcome of the second event.

First selection	Second selection
$P(\text{regular}) = \frac{8}{13}$	$P(\text{regular}) = \frac{7}{12}$

Notice that when Julio removes his can, there is not only one fewer regular soft drink but also one fewer drink in the cooler.

$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular following regular})$

$= \frac{8}{13} \cdot \frac{7}{12}$ or $\frac{14}{39}$ Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{14}{39}$ or about 36%.

KEY CONCEPT *Probability of Two Dependent Events*

If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

This formula can be extended to any number of dependent events.

EXAMPLE Two Dependent Events

3 **The host of a game show is drawing chips from a bag to determine the prizes for which contestants will play. Of the 10 chips in the bag, 6 show *television*, 3 show *vacation*, and 1 shows *car*. If the host draws the chips at random and does not replace them, find the probability that he draws a vacation, then a car.**

Because the first chip is not replaced, the events are dependent. Let T represent a television, V a vacation, and C a car.

$P(V \text{ and } C) = P(V) \cdot P(C \text{ following } V)$ Dependent events

$= \frac{3}{10} \cdot \frac{1}{9}$ or $\frac{1}{30}$ After the first chip is drawn, there are 9 left.

The probability of a vacation and then a car is $\frac{1}{30}$ or about 3%.

CHECK Your Progress

3. Use the information above. What is the probability that the host draws two televisions?

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EXAMPLE Three Dependent Events

4 **Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a diamond, a club, and another diamond in that order.**

Since the cards are not replaced, the events are dependent. Let D represent a diamond and C a club.

$P(D, C, D) = P(D) \cdot P(C \text{ following } D) \cdot P(D \text{ following } D \text{ and } C)$

$= \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50}$ or $\frac{13}{850}$ If the first two cards are a diamond and a club, then 12 of the remaining cards are diamonds.

The probability is $\frac{13}{850}$ or about 1.5%.

CHECK Your Progress

4. Find the probability of drawing three cards of the same suit.

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CHECK Your Understanding

Example 1 (p. 704) **A die is rolled twice. Find each probability.**

1. $P(5, \text{then } 1)$

2. $P(\text{two even numbers})$

Examples 1, 3 (pp. 704, 705) **There are 8 action, 3 comedy, and 5 children's DVDs on a shelf. Suppose two DVDs are selected at random from the shelf. Find each probability.**

3. P(2 action DVDs), if replacement occurs

4. P(2 action DVDs), if no replacement occurs

5. P(a comedy DVD, then a children's DVD), if no replacement occurs

Examples 2, 4 (pp. 704, 706) **Three cards are drawn from a standard deck of cards. Find each probability.**

6. P(3 hearts), if replacement occurs

7. P(3 hearts), if no replacement occurs

Determine whether the events are *independent* or *dependent*. Then find the probability.

8. A black die and a white die are rolled. What is the probability that a 3 shows on the black die and a 5 shows on the white die?

9. Yana has 4 black socks, 6 blue socks, and 8 white socks in his drawer. If he selects three socks at random with no replacement, what is the probability that he will first select a blue sock, then a black sock, and then another blue sock?

Example 3 (p. 705) **Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.**

10. P(two hearts)

11. P(ace, then king)

Exercises

A die is rolled twice. Find each probability.

12. $P(2, \text{then } 3)$

13. P(no 6s)

14. P(two 4s)

15. P(1, then any number)

16. P(two of the same number)

17. P(two different numbers)

HOMEWORK HELP	
For Exercises	See Examples
12–20	1
21–29	3
30–35	1–4

The tiles *E, T, F, U, N, X,* and *P* of a word game are placed face down in the lid of the game. If two tiles are chosen at random, find each probability.

18. $P(E$, then $N)$, if replacement occurs

19. P(2 consonants), if replacement occurs

20. $P(T$, then $D)$, if replacement occurs

21. $P(X$, then $P)$, if no replacement occurs

22. P(2 consonants), if no replacement occurs

23. P(selecting the same letter twice), if no replacement occurs

Anita scores well enough at a carnival game that she gets to randomly draw two prizes out of a prize bag. There are 6 purple T-shirts, 8 yellow T-shirts, and 5 T-shirts with a picture of a celebrity on them in the bag. Find each probability.

24. P(choosing 2 purple)

25. P(choosing 2 celebrity)

26. P(choosing a yellow, then a purple)

27. P(choosing a celebrity, then a yellow)

28. **ELECTIONS** Tami, Sonia, Malik, and Roger are the four candidates for Student Council president. If their names are placed in random order on the ballot, what is the probability that Malik's name will be first on the ballot followed by Sonia's name second?

29. **CHORES** The five children of the Blanchard family get weekly chores assigned to them at random. Their parents put pieces of paper with the names of the five children in a hat and draw them out. The order of the names pulled determines the order in which the children will be responsible for sorting laundry for the next five weeks. What is the probability that Jim will be responsible for the first week and Emily will be responsible for the fifth week?

Real-World Link

Three hamsters domesticated in 1930 are the ancestors of most of the hamsters sold as pets or used for research.

Source: www.ahc.umn.edu

Determine whether the events are *independent* or *dependent*. Then find the probability.

30. There are 3 miniature chocolate bars and 5 peanut butter cups in a candy dish. Judie chooses 2 of them at random. What is the probability that she chose 2 miniature chocolate bars?

31. A cage contains 3 white and 6 brown hamsters. Maggie randomly selects one, puts it back, and then randomly selects another. What is the probability that both selections were white?

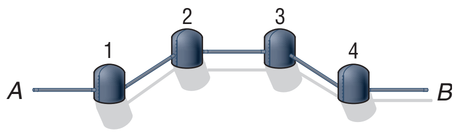
32. A bag contains 7 red, 4 blue, and 6 yellow marbles. If 3 marbles are selected in succession, what is the probability of selecting blue, then yellow, then red, if replacement occurs each time?

33. Jen's purse contains three \$1 bills, four \$5 bills, and two \$10 bills. If she selects three bills in succession, find the probability of selecting a \$10 bill, then a \$5 bill, and then a \$1 bill if the bills are not replaced.

34. What is the probability of getting heads each time if a coin is tossed 5 times?

35. When Ramon plays basketball, he makes an average of two out of every three foul shots he takes. What is the probability that he will make the next three foul shots in a row?

36. **UTILITIES** A city water system includes a sequence of 4 pumps as shown below. Water enters the system at point A, is pumped through the system by pumps at locations 1, 2, 3, and 4, and exits the system at point B.



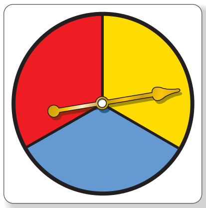
If the probability of failure for any one pump is $\frac{1}{100}$, what is the probability that water will flow all the way through the system from A to B?

Real-World Link

There are 2,598,960 different 5-card hands that can be drawn from a 52-card deck. Of these, only 5108 are hands in which all 5 cards are of the same suit.

37. **FISHING** Suppose a sport fisher has a 35% chance of catching a fish that he can keep each time he goes to a spot. What is the probability that he catches a fish the first 4 times he visits the spot but on the fifth visit he does not?

For Exercises 38–41, suppose you spin the spinner twice.



38. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning red and then blue.

39. Sketch an area diagram of the outcomes. Shade the region on your area diagram corresponding to getting the same color twice.

40. What is the probability that you get the same color on both spins?

41. If you got the same color twice, what is the probability that the color was red?

Find each probability if 13 cards are drawn from a standard deck of cards and no replacement occurs.

42. P(all hearts)

43. P(all red cards)

44. P(all one suit)

45. P(no kings)

EXTRA PRACTICE
See pages 917, 937.

Math Online
Self-Check Quiz at algebra2.com

For Exercises 46–48, use the following information.

A bag contains 10 marbles. In this problem, a *cycle* means that you draw a marble, record its color, and put it back.

46. You go through the cycle 10 times. If you do not record any black marbles, can you conclude that there are no black marbles in the bag?

47. Can you conclude that there are none if you repeat the cycle 50 times?

48. How many times do you have to repeat the cycle to be certain that there are no black marbles in the bag? Explain your reasoning.

H.O.T. Problems

49. **OPEN ENDED** Describe two real-life events that are dependent.

50. **FIND THE ERROR** Mario and Tamara are calculating the probability of getting a 4 and then a 2 if they roll a die twice. Who is correct? Explain your reasoning.

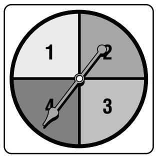
Mario	Tamara
$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$	$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$

51. CHALLENGE If one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance of the whole string lighting?

52. *Writing in Math* Use the information on page 703 to explain how probability applies to basketball. Explain how a value such as one of those in the table could be used to find the chances of Yao Ming making 0, 1, or 2 of 2 successive field goals, assuming the 2 field goals are independent, and a possible reason why 2 field goals might not be independent.

STANDARDIZED TEST PRACTICE

53. ACT/SAT The spinner is spun four times. What is the probability that the spinner lands on 2 each time?



A $\frac{1}{2}$ C $\frac{1}{16}$

B $\frac{1}{4}$ D $\frac{1}{256}$

54. REVIEW A coin is tossed and a die is rolled. What is the probability of a head and a 3?

F $\frac{1}{4}$ H $\frac{1}{12}$

G $\frac{1}{8}$ J $\frac{1}{24}$

Spiral Review

A gumball machine contains 7 red, 8 orange, 9 purple, 7 white, and 5 yellow gumballs. Tyson buys 3 gumballs. Find each probability, assuming that the machine dispenses the gumballs at random. (Lesson 12-3)

55. P(3 red)

56. P(2 white, 1 purple)

57. PHOTOGRAPHY A photographer is taking a picture of a bride and groom together with 6 attendants. How many ways can he arrange the 8 people in a row if the bride and groom stand in the middle? (Lesson 12-2)

Solve each equation. Check your solutions. (Lesson 9-3)

58. $\log_5 5 + \log_5 x = \log_5 30$

59. $\log_{16} c - 2\log_{16} 3 = \log_{16} 4$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials. (Lesson 6-7)

60. $x^3 - x^2 - 10x + 6;\ x + 3$

61. $x^3 - 7x^2 + 12x;\ x - 3$

GET READY for the Next Lesson

PREREQUISITE SKILL Find each sum if $a = \frac{1}{2}$, $b = \frac{1}{6}$, $c = \frac{2}{3}$, and $d = \frac{3}{4}$.

62. $a + b$

63. $b + c$

64. $a + d$

65. $b + d$

66. $c + a$

67. $c + d$

12-5 Adding Probabilities

Main Ideas

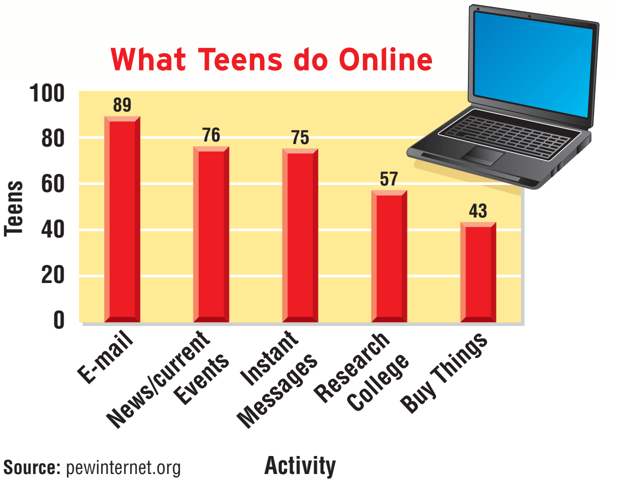
- Find the probability of mutually exclusive events.
- Find the probability of inclusive events.

New Vocabulary

simple event
compound event
mutually exclusive events
inclusive events

GET READY for the Lesson

The graph shows the results of a survey about what teens do online. Determining the probability that a randomly selected teen sends/reads e-mail or buys things online requires adding probabilities.



Source: pewinternet.org

Mutually Exclusive Events When you roll a die, an event such as rolling a 1 is called a **simple event** because it cannot be broken down into smaller events. An event that consists of two or more simple events is called a **compound event**. For example, the event of rolling an odd number or a number greater than 5 is a compound event because it consists of the simple events rolling a 1, rolling a 3, rolling a 5, or rolling a 6.

When there are two events, it is important to understand how they are related before finding the probability of one or the other event occurring. Suppose you draw a card from a standard deck of cards. What is the probability of drawing a 2 or an ace? Since a card cannot be both a 2 *and* an ace, these are called **mutually exclusive events**. That is, the two events cannot occur at the same time. The probability of drawing a 2 or an ace is found by adding their individual probabilities.

$P(2 \text{ or ace}) = P(2) + P(\text{ace})$ Add probabilities.

$= \frac{4}{52} + \frac{4}{52}$ There are 4 twos and 4 aces in a deck.

$= \frac{8}{52} \text{ or } \frac{2}{13}$ Simplify.

The probability of drawing a 2 or an ace is $\frac{2}{13}$.

Study Tip

Formula

This formula can be extended to any number of mutually exclusive events.

KEY CONCEPT *Probability of Mutually Exclusive Events*

Words If two events, A and B, are mutually exclusive, then the probability that A or B occurs is the sum of their probabilities.

Symbols $P(A \text{ or } B) = P(A) + P(B)$

EXAMPLE Two Mutually Exclusive Events

1 **Keisha has a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from the stack, what is the probability that it is a baseball or a soccer card?**

These are mutually exclusive events, since the card cannot be both a baseball card *and* a soccer card. Note that there is a total of 19 cards.

$P(\text{baseball or soccer}) = P(\text{baseball}) + P(\text{soccer})$ — Mutually exclusive events

$= \frac{8}{19} + \frac{6}{19} \text{ or } \frac{14}{19}$ — Substitute and add.

The probability that Keisha selects a baseball or a soccer card is $\frac{14}{19}$.

CHECK Your Progress

1. One teacher must be chosen to supervise a senior class fund-raiser. There are 12 math teachers, 9 language arts teachers, 8 social studies teachers, and 10 science teachers. If the teacher is chosen at random, what is the probability that the teacher is either a language arts teacher or a social studies teacher?

To extend the formula to more than two events, add the probabilities for all of the events.

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EXAMPLE Three Mutually Exclusive Events

2 **There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have at least 2 girls?**

At least 2 girls means that the subcommittee may have 2, 3, or 4 girls. It is not possible to select a group of 2 girls, a group of 3 girls, and a group of 4 girls all in the same 4-member subcommittee, so the events are mutually exclusive. Add the probabilities of each type of committee.

$P(\text{at least 2 girls}) = P(2 \text{ girls}) + P(3 \text{ girls}) + P(4 \text{ girls})$

2 girls, 2 boys; 3 girls, 1 boy; 4 girls, 0 boys

$= \frac{C(7, 2) \cdot C(6, 2)}{C(13, 4)} + \frac{C(7, 3) \cdot C(6, 1)}{C(13, 4)} + \frac{C(7, 4) \cdot C(6, 0)}{C(13, 4)}$

$= \frac{315}{715} + \frac{210}{715} + \frac{35}{715} \text{ or } \frac{112}{143}$ — Simplify.

The probability of at least 2 girls on the subcommittee is $\frac{112}{143}$ or about 0.78.

Study Tip

Choosing a Committee

$C(13, 4)$ refers to choosing 4 subcommittee members from 13 committee members. Since order does not matter, the number of combinations is found.

CHECK Your Progress

2. The Cougar basketball team can send 5 players to a basketball clinic. Six guards and 5 forwards would like to attend the clinic. If the players are selected at random, what is the probability that at least 3 of the players selected to attend the clinic will be forwards?

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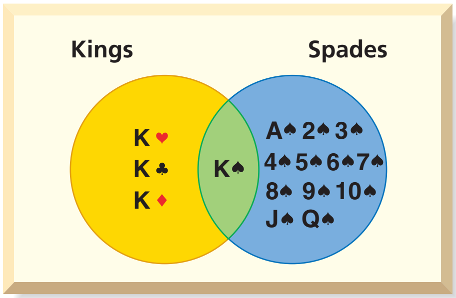
Inclusive Events What is the probability of drawing a king or a spade from a standard deck of cards? Since it is possible to draw a card that is both a king and a spade, these events are not mutually exclusive. These are called **inclusive events**.

P(king)	P(spade)	P(spade, king)
$\frac{4}{52}$	$\frac{13}{52}$	$\frac{1}{52}$
1 king in each suit	spades	king of spades

> **Study Tip**
>
> **Common Misconception**
>
> In mathematics, unlike everyday language, the expression *A* or *B* allows the possibility of both *A* and *B* occurring.

In the first two fractions above, the probability of drawing the king of spades is counted twice, once for a king and once for a spade. To find the correct probability, you must subtract P(king of spades) from the sum of the first two probabilities.



$$P(\text{king or spade}) = P(\text{king}) + P(\text{spade}) - P(\text{king of spades})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \text{ or } \frac{4}{13}$$

The probability of drawing a king or a spade is $\frac{4}{13}$.

> **KEY CONCEPT** *Probability of Inclusive Events*
>
> **Words** If two events, *A* and *B*, are inclusive, then the probability that *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occurring.
>
> **Symbols** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

EXAMPLE Inclusive Events

3 **EDUCATION** **Suppose that of 1400 students, 550 take Spanish, 700 take biology, and 400 take both Spanish and biology. What is the probability that a student selected at random takes Spanish or biology?**

Since some students take both Spanish and biology, the events are inclusive.

$P(\text{Spanish}) = \frac{550}{1400}$ $\quad P(\text{biology}) = \frac{700}{1400}$ $\quad P(\text{Spanish and biology}) = \frac{400}{1400}$

$$P(\text{Spanish or biology}) = P(\text{Spanish}) + P(\text{biology}) - P(\text{Spanish and biology})$$

$$= \frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} \text{ or } \frac{17}{28}$$ Substitute and simplify.

The probability that a student selected at random takes Spanish or biology is $\frac{17}{28}$.

CHECK Your Progress

3. Sixty plastic discs, each with one of the numbers from 1 to 60, are in a bag. LaTanya will win a game if she can pull out any disc with a number divisible by 2 or 3. What is the probability that LaTanya will win?

Concepts in Motion
Interactive Lab
algebra2.com

CHECK Your Understanding

Examples 1–3 (pp. 711–712)

A die is rolled. Find each probability.

1. P(1 or 6) **2.** P(at least 5) **3.** P(less than 3)

4. P(even or prime) **5.** P(multiple of 3 or 4) **6.** P(multiple of 2 or 3)

Examples 2, 3 (pp. 711–712)

A card is drawn from a standard deck of cards. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. P(6 or king) **8.** P(queen or spade)

Example 2 (p. 711)

9. **SCHOOL** There are 8 girls and 8 boys on the Student Senate. Three of the students are seniors. What is the probability that a person selected from the Student Senate is not a senior?

Exercises

HOMEWORK HELP

For Exercises	See Examples
10–19	1, 2
20–23	1–3
24–29	3

Jesse has eight friends who have volunteered to help him with a school fundraiser. Five are boys and 3 are girls. If he randomly selects 3 friends to help him, find each probability.

10. P(2 boys or 2 girls) **11.** P(all boys or all girls)

12. P(at least 2 girls) **13.** P(at least 1 boy)

Six girls and eight boys walk into a video store at the same time. There are six salespeople available to help them. Find the probability that the salespeople will first help the given numbers of girls and boys.

14. P(4 girls, 2 boys or 4 boys, 2 girls) **15.** P(5 girls, 1 boy or 5 boys, 1 girl)

16. P(all girls or all boys) **17.** P(at least 4 boys)

18. P(at least 5 girls or at least 5 boys) **19.** P(at least 3 girls)

For Exercises 20–23, determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

20. There are 4 algebra books, 3 literature books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature book or an algebra book?

21. A die is rolled. What is the probability of rolling a 5 or a number greater than 3?

22. In the Math Club, 7 of the 20 girls are seniors, and 4 of the 14 boys are seniors. What is the probability of randomly selecting a boy or a senior to represent the Math Club at a statewide math contest?

23. A card is drawn from a standard deck of cards. What is the probability of drawing an ace or a face card? (*Hint:* A face card is a jack, queen, or king.)

24. One tile with each letter of the alphabet is placed in a bag, and one is drawn at random. What is the probability of selecting a vowel or a letter from the word *function?*

25. Each of the numbers from 1 to 30 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that the number is a multiple of 2 or a multiple of 3?

Two cards are drawn from a standard deck of cards. Find each probability.

26. P(both queens or both red) **27.** P(both jacks or both face cards)

28. P(both face cards or both black) **29.** P(both either black or an ace)

GAMES For Exercises 30–35, use the following information.
A certain game has two stacks of 30 tiles with pictures on them. In the first stack of tiles, there are 10 dogs, 4 cats, 5 balls, and 11 horses. In the second stack of tiles, there are 3 flowers, 8 fish, 12 balls, 2 cats, and 5 horses. The top tile in each stack is chosen. Find each probability.

30. P(each is a ball)

31. P(neither is a horse)

32. P(exactly one is a ball)

33. P(exactly one is a fish)

34. P(both are a fish)

35. P(one is a dog and one is a flower)

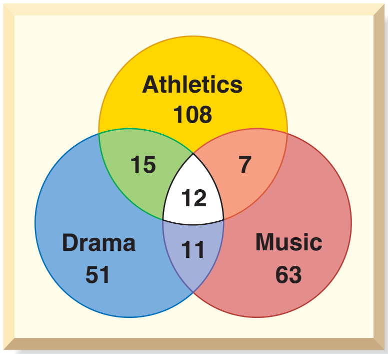
BASEBALL For Exercises 36–38, use the following information.
Albert and Paul are on the school baseball team. Albert has a batting average of .4, and Paul has a batting average of .3. That means that Albert gets a hit 40% of his at bats and Paul gets a hit 30% of his times at bat. What is the probability that—

36. both Albert and Paul are able to get hits their first time at bat?

37. neither Albert nor Paul is able to get a hit their first time at bat?

38. at least one of the two friends is able to get a hit their first time at bat?

SCHOOL For Exercises 39–41, use the Venn diagram that shows the number of participants in extracurricular activities for a junior class of 324 students.
Determine each probability if a student is selected at random from the class.



39. P(drama or music)

40. P(drama or athletics)

41. P(athletics and drama, or music and athletics)

EXTRA PRACTICE
See pages 918, 937.
Math Online
Self-Check Quiz at
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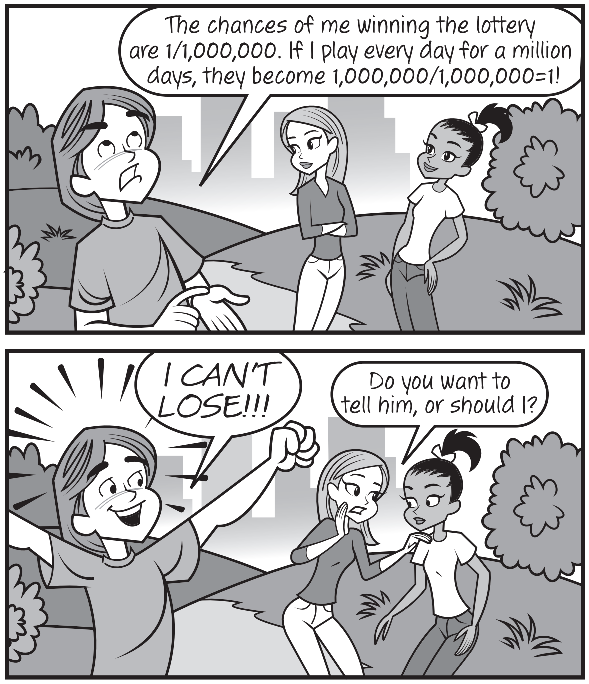
H.O.T. Problems



42. REASONING What is wrong with the conclusion in the comic?

43. OPEN ENDED Describe two mutually exclusive events and two inclusive events.

44. CHALLENGE A textbook gives the following probability equation for events A and B that are mutually exclusive or inclusive.

$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$

Is this correct? Explain.

45. ***Writing in Math*** Use the information on page 710 to explain how probability applies to what teens do online. Include an explanation of whether the events listed in the graphic are mutually exclusive or inclusive.

STANDARDIZED TEST PRACTICE

46. ACT/SAT In a jar of red and white gumballs, the ratio of white gumballs to red gumballs is 5:4. If the jar contains a total of 180 gumballs, how many of them are red?

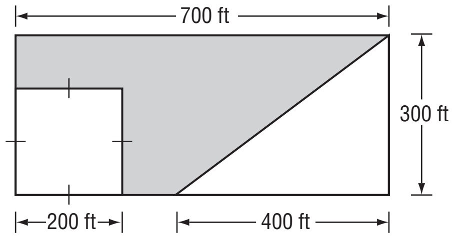
A 45

B 64

C 80

D 100

47. REVIEW What is the area of the shaded part of the rectangle below?



F 90,000 ft^2 **H** 130,000 ft^2

G 110,000 ft^2 **J** 150,000 ft^2

Spiral Review

A die is rolled three times. Find each probability. (Lesson 12-4)

48. P(1, then 2, then 3)

49. P(no 4s)

50. P(three 1s)

51. P(three even numbers)

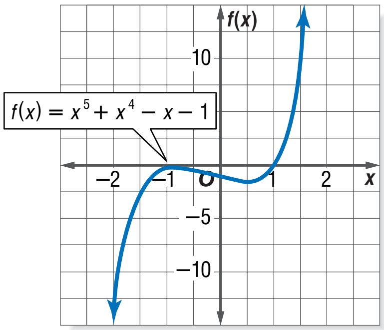
52. BOOKS Dan has twelve books on his shelf that he has not read yet. There are seven novels and five biographies. He wants to take four books with him on vacation. What is the probability that he randomly selects two novels and two biographies? (Lesson 12-3)

Find the sum of each series. (Lessons 11-2 and 11-4)

53. $2 + 4 + 8 + \cdots + 128$

54. $\sum_{n=1}^{3} (5n - 2)$

55. Use the graph of the polynomial function at the right to determine at least one binomial factor of the polynomial. Then find all factors of the polynomial. (Lesson 6-7)



SPEED SKATING **For Exercises 56 and 57, use the following information.**

In 2001, Catriona LeMay Doan set a world record for women's speed skating by skating approximately 13.43 meters per second in the 500-meter race. (Lesson 2-6)

56. Suppose she could maintain that speed. Write an equation that represents how far she could travel in t seconds.

57. What type of function does the equation in Exercise 56 represent?

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the mean, median, mode, and range for each set of data. Round to the nearest hundredth, if necessary.** (Pages 759 and 760)

58. 298, 256, 399, 388, 276

59. 3, 75, 58, 7, 34

60. 4.8, 5.7, 2.1, 2.1, 4.8, 2.1

61. 80, 50, 65, 55, 70, 65, 75, 50

62. 61, 89, 93, 102, 45, 89

63. 13.3, 15.4, 12.5, 10.7

CHAPTER 12

Mid-Chapter Quiz

Lessons 12-1 through 12-5

1. **RESTAURANT** At Burger Hut, you can order your hamburger with or without cheese, onions, or pickles, and rare, medium, or well-done. How many different ways can you order your hamburger? (Lesson 12-1)
2. **AUTOMOBILES** For a particular model of car, a dealer offers 3 sizes of engines, 2 types of stereos, 18 body colors, and 7 upholstery colors. How many different possibilities are available for that model? (Lesson 12-1)
3. **CODES** How many codes consisting of a letter followed by 3 digits can be made if no digit can be used more than once? (Lesson 12-1)
4. **ROUTES** There are 4 different routes a student can bike from his house to school. In how many ways can he make a round trip if he uses a different route coming than going? (Lesson 12-1)

Evaluate each expression. (Lesson 12-2)

5. $P(12, 3)$
6. $C(8, 3)$

Determine whether each situation involves a permutation or a combination. Then find the number of possibilities. (Lesson 12-2)

7. 8 cars in a row parked next to a curb
8. a hand of 6 cards from a standard deck of cards

9. **MULTIPLE CHOICE** A box contains 10 silver, 9 green, 8 blue, 11 pink, and 12 yellow paper clips. If a paperclip is drawn at random, what is the probability that it is *not* yellow? (Lesson 12-3)

   A $\frac{1}{5}$

   B $\frac{6}{25}$

   C $\frac{19}{25}$

   D $\frac{3}{5}$

Two cards are drawn from a standard deck of cards. Find each probability. (Lesson 12-3)

10. P(2 aces)
11. P(1 heart, 1 club)
12. P(1 queen, 1 king)

A bag contains colored marbles as shown in the table below. Two marbles are drawn at random from the bag. Find each probability. (Lesson 12-4)

Color	Number
red	5
green	3
blue	2

13. P(red, then green) if replacement occurs
14. P(red, then green) if no replacement occurs
15. P(2 red) if no replacement occurs
16. P(2 red) if replacement occurs

A twelve-sided die has sides numbered 1 through 12. The die is rolled once. Find each probability. (Lesson 12-5)

17. P(4 or 5)
18. P(even or a multiple of 3)
19. P(odd or a multiple 4)

20. **MULTIPLE CHOICE** In a box of chocolate and yellow cupcakes, the ratio of chocolate cupcakes to yellow cupcakes is 3:2. If the box contains 20 cupcakes, how many of them are chocolate? (Lesson 12-5)

   F 9 H 11

   G 10 J 12

21. **MULTIPLE CHOICE** A company received job applications from 2000 people. Six hundred of the applicants had the desired education, 1200 had the desired work experience, and 400 had both the desired education and work experience. What is the probability that an applicant selected at random will have the desired education or work experience?

   A $\frac{3}{10}$

   B $\frac{1}{2}$

   C $\frac{7}{10}$

   D $\frac{9}{10}$

12-6 Statistical Measures

Main Ideas

- Use measures of central tendency to represent a set of data.
- Find measures of variation for a set of data.

New Vocabulary

univariate data
measure of central tendency
measure of variation
dispersion
variance
standard deviation

GET READY for the Lesson

On Mr. Dent's most recent Algebra 2 test, his students earned the following scores.

72	70	77	76	90	68	81	86	34	94
71	84	89	67	19	85	75	66	80	94

When his students ask how they did on the test, which measure of central tendency should Mr. Dent use to describe the scores?

Measures of Central Tendency Data with one variable, such as the test scores, are called **univariate data**. Sometimes it is convenient to have one number that describes a set of data. This number is called a **measure of central tendency**, because it represents the center or middle of the data. The most commonly used measures of central tendency are the *mean*, *median*, and *mode*.

When deciding which measure of central tendency to use to represent a set of data, look closely at the data itself.

Study Tip

Look Back

To review **outliers**, see Lesson 2-5.

CONCEPT SUMMARY — *Measures of Tendency*

Use	When . . .
mean	the data are spread out, and you want an average of the values
median	the data contain outliers
mode	the data are tightly clustered around one or two values

EXAMPLE 1 Choose a Measure of Central Tendency

SWEEPSTAKES A sweepstakes offers a first prize of $10,000, two second prizes of $100, and one hundred third prizes of $10. Which measure of central tendency best represents the available prizes?

Since 100 of the 103 prizes are $10, the mode ($10) best represents the available prizes. Notice that in this case the median is the same as the mode.

CHECK Your Progress

1. Which measure of central tendency would the organizers of the sweepstakes be most likely to use in their advertising?

Measures of Variation

Measures of variation or **dispersion** measure how spread out or scattered a set of data is. The simplest measure of variation to calculate is the *range*, the difference between the greatest and the least values in a set of data. Variance and standard deviation are measures of variation that indicate how much the data values differ from the mean.

Reading Math

Symbols The symbol σ is the lower case Greek letter *sigma*. $\bar{x}$ is read *x bar*.

To find the **variance** σ^2 of a set of data, follow these steps.

1. Find the mean, $\bar{x}$.
2. Find the difference between each value in the set of data and the mean.
3. Square each difference.
4. Find the mean of the squares.

The **standard deviation** σ is the square root of the variance.

KEY CONCEPT — Standard Deviation

If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation σ is given by the following formula.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

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EXAMPLE 2 Standard Deviation

STATES The table shows the populations in millions of 11 eastern states as of the 2000 Census. Find the variance and standard deviation of the data to the nearest tenth.

State	Population	State	Population	State	Population
NY	19.0	MD	5.3	RI	1.0
PA	12.3	CT	3.4	DE	0.8
NJ	8.4	ME	1.3	VT	0.6
MA	6.3	NH	1.2	–	–

Source: U.S. Census Bureau

Step 1 Find the mean. Add the data and divide by the number of items.

$$\bar{x} = \frac{19.0 + 12.3 + 8.4 + 6.3 + 5.3 + 3.4 + 1.3 + 1.2 + 1.0 + 0.8 + 0.6}{11}$$

$\approx 5.4\overline{18}$ The mean is about 5.4 million people.

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$ Variance formula

$$= \frac{(19.0 - 5.4)^2 + (12.3 - 5.4)^2 + \cdots + (8.0 - 5.4)^2 + (0.6 - 5.4)^2}{11}$$

$$= \frac{344.4}{11}$$ Simplify.

$\approx 31.3\overline{09}$ The variance is about 31.3.

Step 3 Find the standard deviation.

$\sigma^2 \approx 31.3$ Take the square root of each side.

$\sigma \approx 5.594640292$ The standard deviation is about 5.6 million people.

CHECK Your Progress

2. The leading number of home runs in Major League Baseball for the 1994–2004 seasons were 43, 50, 52, 56, 70, 65, 50, 73, 57, 47, and 48. Find the variance and standard deviation of the data to the nearest tenth.

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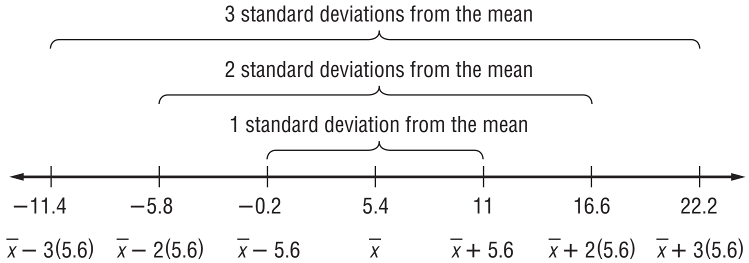
Most of the members of a set of data are within 1 standard deviation of the mean. The data in Example 2 can be broken down as shown below.



Looking at the original data, you can see that most of the states' populations were between 2.4 million and 20.2 million. That is, the majority of members of the data set were within 1 standard deviation of the mean.

You can use a TI-83/84 Plus graphing calculator to find statistics for the data in Example 2.

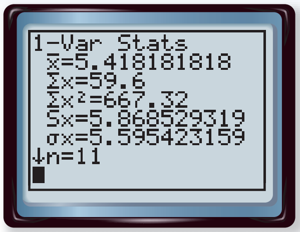
GRAPHING CALCULATOR LAB

One-Variable Statistics

The TI-83/84 Plus can compute a set of one-variable statistics from a list of data. These statistics include the mean, variance, and standard deviation. Enter the data into L1.

KEYSTROKES: [STAT] [ENTER] 19.0 [ENTER] 12.3 [ENTER] . . .

Then use [STAT] [▶] 1 [ENTER] to show the statistics. The mean $\bar{x}$ is about 5.4, the sum of the values $\sum x$ is 59.6, the standard deviation σx is about 5.6, and there are $n = 11$ data items. If you scroll down, you will see the least value (minX = .6), the three quartiles (1, 3.4, and 8.4), and the greatest value (maxX = 19).



THINK AND DISCUSS

1. Find the variance of the data set.

2. Enter the data set in list L1 but without the outlier 19.0. What are the new mean, median, and standard deviation?

3. Did the mean or median change less when the outlier was deleted?

CHECK Your Understanding

Example 1 (pp. 717–718)

EDUCATION For Exercises 1 and 2, use the following information.
The table below shows the amounts of money spent on education per student in a recent year in two regions of the United States.

Pacific States		Southwest Central States	
State	Expenditures per Student ($)	State	Expenditures per Student ($)
Alaska	9564	Texas	6771
California	7405	Arkansas	6276
Washington	7039	Louisiana	6567
Oregon	7642	Oklahoma	6229

Source: *The World Almanac*

1. Find the mean for each region.

2. For which region is the mean more representative of the data? Explain.

Example 2 (pp. 718–719)

Find the variance and standard deviation of each set of data to the nearest tenth.

3. {48, 36, 40, 29, 45, 51, 38, 47, 39, 37}

4. {321, 322, 323, 324, 325, 326, 327, 328, 329, 330}

5. {43, 56, 78, 81, 47, 42, 34, 22, 78, 98, 38, 46, 54, 67, 58, 92, 55}

Exercises

HOMEWORK HELP	
For Exercises	See Examples
6–13, 24–30	2
14–23	1

Find the variance and standard deviation of each set of data to the nearest tenth.

6. {400, 300, 325, 275, 425, 375, 350}

7. {5, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 9}

8. {2.4, 5.6, 1.9, 7.1, 4.3, 2.7, 4.6, 1.8, 2.4}

9. {4.3, 6.4, 2.9, 3.1, 8.7, 2.8, 3.6, 1.9, 7.2}

10. {234, 345, 123, 368, 279, 876, 456, 235, 333, 444}

11. {13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 67, 56, 34, 99, 44, 55}

12.

Stem	Leaf
4	4 5 6 7 7
5	3 5 6 7 8 9
6	7 7 8 9 9 9

4 | 5 = 45

13.

Stem	Leaf
5	7 7 7 8 9
6	3 4 5 5 6 7
7	2 3 4 5 6

6 | 3 = 63

BASKETBALL For Exercises 14 and 15, use the following information.
The table below shows the rebounding totals for the members of the 2005 Charlotte Sting.

162	145	179	37	44	53	70	65	47	35	71	5	5

Source: WNBA

14. Find the mean, median, and mode of the data to the nearest tenth.

15. Which measure of central tendency best represents the data? Explain your answer.

ADVERTISING For Exercises 16–18, use the following information.
An electronics store placed an ad in the newspaper showing five flat-screen TVs for sale. The ad says, "Our flat-screen TVs average $695." The prices of the flat-screen TVs are $1200, $999, $1499, $895, $695, $1100, $1300, and $695.

16. Find the mean, median, and mode of the prices.

17. Which measure is the store using in its ad? Why did they choose it?

18. As a consumer, which measure would you want to see advertised? Explain.

EDUCATION For Exercises 19 and 20, use the following information.
The Millersburg school board is negotiating a pay raise with the teacher's union. Three of the administrators have salaries of $90,000 each. However, a majority of the teachers have salaries of about $45,000 per year.

19. You are a member of the school board and would like to show that the current salaries are reasonable. Would you quote the mean, median, or mode as the "average" salary to justify your claim? Explain.

20. You are the head of the teacher's union and maintain that a pay raise is in order. Which of the mean, median, or mode would you quote to justify your claim? Explain your reasoning.

Real-World Link

While the Mall of America does not have the most gross leasable area, it is the largest fully enclosed retail and entertainment complex in the United States.

Source: Mall of America

SHOPPING MALLS For Exercises 21–23, use the following information.
The table lists the areas of some large shopping malls in the United States.

Mall	Gross Leasable Area (ft^2)
1 Del Amo Fashion Center, Torrance, CA	3,000,000
2 South Coast Plaza/Crystal Court, Costa Mesa, CA	2,918,236
3 Mall of America, Bloomington, MN	2,472,500
4 Lakewood Center Mall, Lakewood, CA	2,390,000
5 Roosevelt Field Mall, Garden City, NY	2,300,000
6 Gurnee Mills, Gurnee, IL	2,200,000
7 The Galleria, Houston, TX	2,100,000
8 Randall Park Mall, North Randall, OH	2,097,416
9 Oakbrook Shopping Center, Oak Brook, IL	2,006,688
10 Sawgrass Mills, Sunrise, FL	2,000,000
10 The Woodlands Mall, The Woodlands, TX	2,000,000
10 Woodfield, Schaumburg, IL	2,000,000

Source: Blackburn Marketing Service

21. Find the mean, median, and mode of the gross leasable areas.

22. You are a realtor who is trying to lease mall space in different areas of the country to a large retailer. Which measure would you talk about if the customer felt that the malls were too large for his store? Explain.

23. Which measure would you talk about if the customer had a large inventory? Explain.

SCHOOL For Exercises 24–26, use the frequency table at the right that shows the scores on a multiple-choice test.

Score	Frequency
90	3
85	2
80	3
75	7
70	6
65	4

24. Find the variance and standard deviation of the scores.

25. What percent of the scores are within one standard deviation of the mean?

26. What percent of the scores are within two standard deviations of the mean?

FOOTBALL **For Exercises 27–30, use the weights in pounds of the starting offensive linemen of the football teams from three high schools.**

Jackson	Washington	King
170, 165, 140, 188, 195	144, 177, 215, 225, 197	166, 175, 196, 206, 219

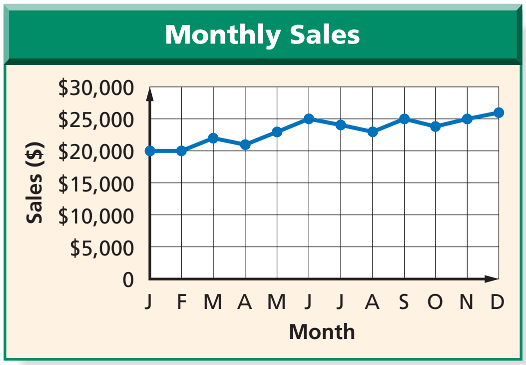
27. Find the standard deviation of the weights for Jackson High.

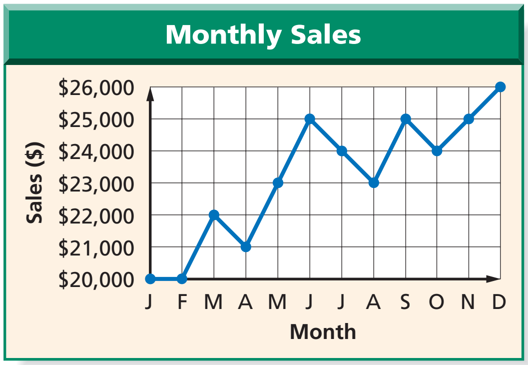
28. Find the standard deviation of the weights for Washington High.

29. Find the standard deviation of the weights for King High.

30. Which team had the most variation in weights? How do you think this variation will impact their play?

For Exercises 31–33, consider the two graphs below.





EXTRA PRACTICE
See pages 918, 937.

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31. Explain why the graphs made from the same data look different.

32. Describe a situation where the first graph might be used.

33. Describe a situation where the second graph might be used.

H.O.T. Problems

34. **OPEN ENDED** Give a sample set of data with a variance and standard deviation of 0.

35. **REASONING** Find a counterexample for the following statement.

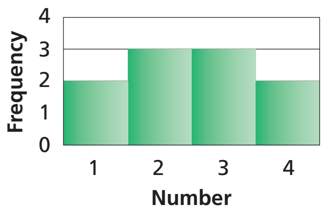
The standard deviation of a set of data is always less than the variance.

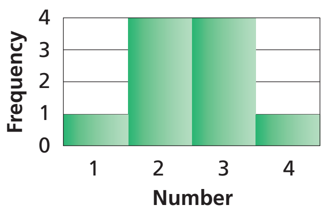
CHALLENGE **For Exercises 36 and 37, consider the two sets of data.**

$A = \{1, 2, 2, 2, 2, 3, 3, 3, 3, 4\}$, $B = \{1, 1, 2, 2, 2, 3, 3, 3, 4, 4\}$

36. Find the mean, median, variance, and standard deviation of each set of data.

37. Explain how you can tell which histogram below goes with each data set without counting the frequencies in the sets.





38. **Which One Doesn't Belong?** Identify the term that does not belong with the other three. Explain your reasoning.

mode | variance | mean | median

39. ***Writing in Math*** Use the information on page 717 to explain what statistics a teacher should tell the class after a test. Include the mean, median, and mode of the given data set and which measure of central tendency you think best represents the test scores and why. How will the measures of central tendency be affected if Mr. Dent adds 5 points to each score?

STANDARDIZED TEST PRACTICE

40. **ACT/SAT** What is the mean of the numbers represented by $x + 1$, $3x - 2$, and $2x - 5$?

A $2x - 2$

B $\frac{6x - 7}{3}$

C $\frac{x + 1}{3}$

D $x + 4$

41. **REVIEW** A school has two backup generators having probabilities of 0.9 and 0.95, respectively, of operating in case of power outage. Find the probability that at least one backup generator operates during a power outage.

F 0.855

G 0.89

H 0.95

J 0.995

Spiral Review

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability. (Lesson 12-5)

42. A card is drawn from a standard deck of cards. What is the probability that it is a 5 or a spade?

43. A jar of change contains 5 quarters, 8 dimes, 10 nickels, and 19 pennies. If a coin is pulled from the jar at random, what is the probability that it is a nickel or a dime?

Two cards are drawn from a standard deck of cards. Find each probability. (Lesson 12-4)

44. P(ace, then king) if replacement occurs

45. P(ace, then king) if no replacement occurs

46. P(heart, then club) if no replacement occurs

47. P(heart, then club) if replacement occurs

48. **BUSINESS** The Energy Booster Company keeps their stock of Health Aid liquid in a tank that is a rectangular prism. Its sides measure $x - 1$ centimeters, $x + 3$ centimeters, and $x - 2$ centimeters. Suppose they would like to bottle their Health Aid in $x - 3$ containers of the same size. How much liquid in cubic centimeters will remain unbottled? (Lesson 6-6)

GET READY for the Next Lesson

PREREQUISITE SKILL **Find each percent.**

49. 68% of 200

50. 68% of 500

51. 95% of 400

52. 95% of 500

53. 99% of 400

54. 99% of 500

12-7 The Normal Distribution

Main Ideas

- Determine whether a set of data appears to be normally distributed or skewed.
- Solve problems involving normally distributed data.

New Vocabulary

discrete probability distribution
continuous probability distribution
normal distribution
skewed distribution

GET READY for the Lesson

The frequency table below lists the heights of the 2005 New England Patriots. However, it does not show how these heights compare to the average height of a professional football player. To make that comparison, you can determine how the heights are distributed.

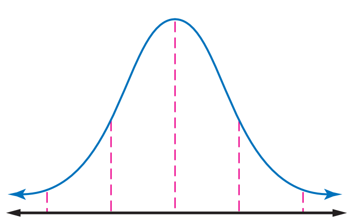
Height (in.)	70	71	72	73	74	75	76	77	78	79	80
Frequency	13	3	5	7	10	9	14	2	4	0	1

Source: www.nfl.com

Normal and Skewed Distributions The probability distributions you have studied thus far are **discrete probability distributions** because they have only a finite number of possible values. A discrete probability distribution can be represented by a histogram. For a **continuous probability distribution**, the outcome can be any value in an interval of real numbers. Continuous probability distributions are represented by curves instead of histograms.

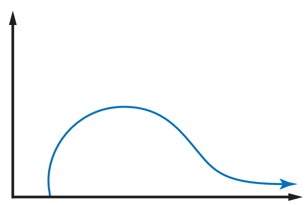
The curve at the right represents a continuous probability distribution. Notice that the curve is symmetric. Such a curve is often called a *bell curve*. Many distributions with symmetric curves or histograms are **normal distributions**.

Normal Distribution

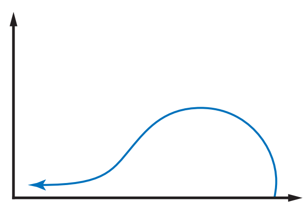


A curve or histogram that is not symmetric represents a **skewed distribution**. For example, the distribution for a curve that is high at the left and has a tail to the right is said to be *positively skewed*. Similarly, the distribution for a curve that is high at the right and has a tail to the left is said to be *negatively skewed*.

Positively Skewed



Negatively Skewed



Study Tip

Skewed Distributions

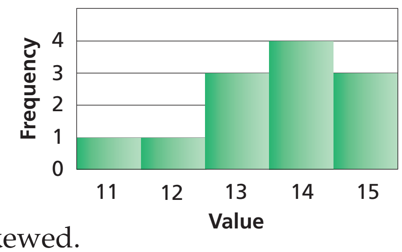
In a positively skewed distribution, the long tail is in the positive direction. These are sometimes said to be *skewed to the right*. In a negatively skewed distribution, the long tail is in the negative direction. These are sometimes said to be *skewed to the left*.

EXAMPLE 1 Classify a Data Distribution

Determine whether the data {14, 15, 11, 13, 13, 14, 15, 14, 12, 13, 14, 15} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Make a frequency table for the data. Then use the table to make a histogram.

Value	11	12	13	14	15
Frequency	1	1	3	4	3



Since the histogram is high at the right and has a tail to the left, the data are negatively skewed.

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CHECK Your Progress

1. Determine whether the data {25, 27, 20, 22, 28, 20, 24, 22, 20, 21, 21, 26} appear to be *positively skewed, negatively skewed,* or *normally distributed.*

Use Normal Distributions Standardized test scores, the lengths of newborn babies, the useful life and size of manufactured items, and production levels can all be represented by normal distributions. In all of these cases, the number of data values must be large for the distribution to be approximately normal.

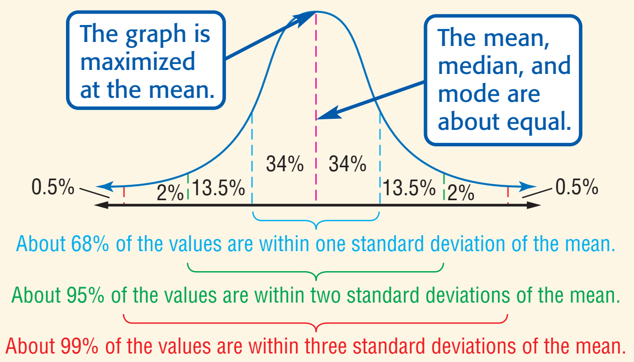
Study Tip

Normal Distributions

If you randomly select an item from data that are normally distributed, the probability that the one you pick will be within one standard deviation of the mean is 0.68. If you do this 1000 times, about 680 of those picked will be within one standard deviation of the mean.

KEY CONCEPT — *Normal Distribution*

Normal distributions have these properties.



EXAMPLE 2 Normal Distribution

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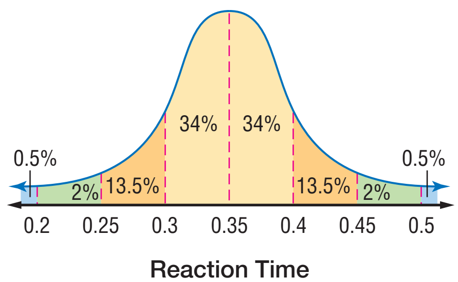
PHYSIOLOGY The reaction times for a hand-eye coordination test administered to 1800 teenagers are normally distributed with a mean of 0.35 second and a standard deviation of 0.05 second.

a. About how many teens had reaction times between 0.25 and 0.45 second?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.



(continued on the next page)

Reading Math

Normally Distributed Random Variable

A *normally distributed random variable* is a variable whose values are arbitrary but whose statistical distribution is normal.

The values 0.25 and 0.45 are 2 standard deviations *below and above* the mean, respectively. Therefore, about 95% of the data are between 0.25 and 0.45. Since $1800 \times 95\% = 1710$, we know that about 1710 of the teenagers had reaction times between 0.25 and 0.45 second.

b. What is the probability that a teenager selected at random had a reaction time greater than 0.4 second?

The value 0.4 is one standard deviation above the mean. You know that about 100% − 68% or 32% of the data are more than one standard deviation away from the mean. By the symmetry of the normal curve, half of 32%, or 16%, of the data are more than one standard deviation above the mean.

The probability that a teenager selected at random had a reaction time greater than 0.4 second is about 16% or 0.16.

CHECK Your Progress

In a recent year, the mean and standard deviation for scores on the ACT were 21.0 and 4.7. Assume that the scores were normally distributed.

2A. If 1,000,000 people took the test, about how many of them scored between 16.3 and 25.7?

2B. What is the probability that a test taker scored higher than 30.4?

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CHECK Your Understanding

Example 1 (p. 725)

1. The table at the right shows recent composite ACT scores. Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Score	Percent of Students
33–36	1
28–32	9
24–27	19
20–23	29
16–19	27
13–15	12

Source: ACT.org

Example 2 (pp. 725–726)

For Exercises 2–4, use the following information.
Mr. Bash gave a quiz in his social studies class. The scores were normally distributed with a mean of 21 and a standard deviation of 2.

2. What percent would you expect to score between 19 and 23?

3. What percent would you expect to score between 23 and 25?

4. What is the probability that a student chosen at random scored between 17 and 25?

QUALITY CONTROL For Exercises 5–8, use the following information.
The useful life of a certain car battery is normally distributed with a mean of 100,000 miles and a standard deviation of 10,000 miles. The company makes 20,000 batteries a month.

5. About how many batteries will last between 90,000 and 110,000 miles?

6. About how many batteries will last more than 120,000 miles?

7. About how many batteries will last less than 90,000 miles?

8. What is the probability that if you buy a car battery at random, it will last between 80,000 and 110,000 miles?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–11	1
12–23	2

Determine whether the data in each table appear to be *positively skewed, negatively skewed,* or *normally distributed.*

9.

U.S. Population	
Age	**Percent**
0–19	28.7
20–39	29.3
40–59	25.5
60–79	13.3
80–99	3.2
100+	0.0

Source: U.S. Census Bureau

10.

Record High U.S. Temperatures	
Temperature (°F)	**Number of States**
100–104	3
105–109	8
110–114	16
115–119	13
120–124	7
125–129	2
130–134	1

Source: *The World Almanac*

11. **SCHOOL** The frequency table at the right shows the grade-point averages (GPAs) of the juniors at Stanhope High School. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

GPA	Frequency
0.0–0.4	4
0.5–0.9	4
1.0–1.4	2
1.5–1.9	32
2.0–2.4	96
2.5–2.9	91
3.0–3.4	110
3.5–4.0	75

HEALTH For Exercises 12 and 13, use the following information.
The cholesterol level for adult males of a specific racial group is normally distributed with a mean of 4.8 and a standard deviation of 0.6.

12. About what percent of the males have cholesterol below 4.2?

13. About how many of the 900 men in a study have cholesterol between 4.2 and 6.0?

Real-World Link

Doctors recommend that people maintain a total blood cholesterol of 200 mg/dL or less.

Source: americanheart.org

VENDING For Exercises 14–16, use the following information.
A vending machine usually dispenses about 8 ounces of coffee. Lately, the amount varies and is normally distributed with a standard deviation of 0.3 ounce.

14. What percent of the time will you get more than 8 ounces of coffee?

15. What percent of the time will you get less than 8 ounces of coffee?

16. What percent of the time will you get between 7.4 and 8.6 ounces of coffee?

MANUFACTURING For Exercises 17–19, use the following information.
The sizes of CDs made by a company are normally distributed with a standard deviation of 1 millimeter. The CDs are supposed to be 120 millimeters in diameter, and they are made for drives 122 millimeters wide.

17. What percent of the CDs would you expect to be greater than 120 millimeters?

18. If the company manufactures 1000 CDs per hour, how many of the CDs made in one hour would you expect to be between 119 and 122 millimeters?

19. About how many CDs per hour will be too large to fit in the drives?

FOOD For Exercises 20–23, use the following information.
The shelf life of a particular snack chip is normally distributed with a mean of 180 days and a standard deviation of 30 days.

20. About what percent of the products last between 150 and 210 days?

21. About what percent of the products last between 180 and 210 days?

22. About what percent of the products last less than 90 days?

23. About what percent of the products last more than 210 days?

EXTRA PRACTICE
See pages 918, 937.
Math Online
Self-Check Quiz at
algebra2.com

RAINFALL For Exercises 24–26, use the table at the right.

24. Find the mean.

25. Find the standard deviation.

26. If the data are normally distributed, what percent of the time will the annual precipitation in these cities be between 16.97 and 7.69 inches?

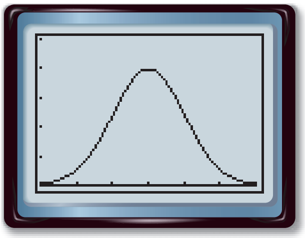
Average Annual Precipitation	
City	**Precipitation (in.)**
Albuquerque	9
Boise	12
Phoenix	8
Reno	7
Salt Lake City	17
San Francisco	20

Source: noaa.gov

H.O.T. Problems

27. **OPEN ENDED** Sketch a positively skewed graph. Describe a situation in which you would expect data to be distributed this way.

28. **CHALLENGE** The graphing calculator screen shows the graph of a normal distribution for a large set of test scores whose mean is 500 and whose standard deviation is 100. If every test score in the data set were increased by 25 points, describe how the mean, standard deviation, and graph of the data would change.



[200, 800] scl: 100 by [0, 0.005] scl: 0.001

29. ***Writing in Math*** Use the information on page 724 to explain how the heights of professional athletes are distributed. Include a histogram of the given data, and an explanation of whether you think the data is normally distributed.

STANDARDIZED TEST PRACTICE

30. **ACT/SAT** If $x + y = 5$ and $xy = 6$, what is the value of $x^2 + y^2$?

A 13

B 17

C 25

D 37

31. **REVIEW** Jessica wants to create several different 7-character passwords. She wants to use arrangements of the first three letters of her name, *followed* by arrangements of 4 digits in 1987, the year of her birth. How many different passwords can she create?

F 672 **G** 288 **H** 576 **J** 144

Spiral Review

Find the variance and standard deviation of each set of data to the nearest tenth. (Lesson 12-6)

32. {7, 16, 9, 4, 12, 3, 9, 4}

33. {12, 14, 28, 19, 11, 7, 10}

A card is drawn from a standard deck of cards. Find each probability. (Lesson 12-5)

34. P(jack or queen)

35. P(ace or heart)

36. P(2 or face card)

GET READY for the Next Lesson

PREREQUISITE SKILL **Use a calculator to evaluate each expression to four decimal places.** (Lesson 9-5)

37. e^{-4}

38. e^3

39. $e^{-\frac{1}{2}}$

12-8 Exponential and Binomial Distribution

Main Ideas

- Use exponential distributions to find exponential probabilities.
- Use binomial distributions to find binomial probabilities.

New Vocabulary

exponential distribution
exponential probability
binomial distribution
binomial probability

GET READY for the Lesson

The average length of time that a student at East High School spends talking on the phone per day is 1 hour. What is the probability that a randomly chosen student talks on the phone for more than 2 hours?

Exponential Distributions You can use exponential distributions to predict the probabilities of events based on time. They are most commonly used to measure *reliability*, which is the amount of time that a product lasts. Exponential distributions apply to situations where the time spent on an event, or the amount of time that an event lasts, is important.

Exponential distributions are represented by the following functions.

KEY CONCEPT — *Exponential Distribution Functions*

The formula $f(x) = e^{-mx}$ gives the probability $f(x)$ that something lasts longer or costs more than the given value x, where m is the multiplicative inverse of the mean amount of time.

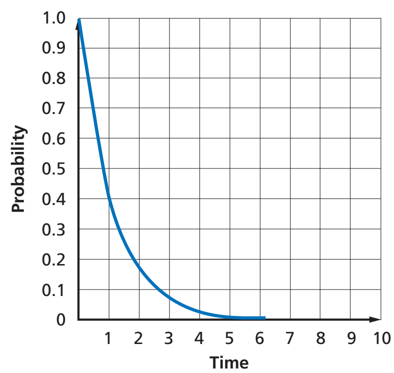
The formula $f(x) = 1 - e^{-mx}$ gives the probability $f(x)$ that something does not last as long or costs less than the given value x, where m is the multiplicative inverse of the mean amount of time.

Study Tip

Look Back

To review inverses, see Lesson 1–2.

Exponential distributions are represented by a curve similar to the one shown. The x-axis usually represents length of time, or money. The y-axis represents probability, so the range will be from 0 to 1.



EXAMPLE 1 Exponential Distribution

Refer to the application above. What is the probability that a randomly chosen student talks on the phone for more than 2 hours?

First, find the m, the inverse of the mean. Because the mean is 1, the multiplicative inverse is 1.

$f(x) = e^{-mx}$	Exponential Distribution Function
$f(2) = e^{-1(2)}$	Replace x with 2 and m with 1.
$= e^{-2}$	Simplify.
≈ 0.135 or 13.5%	Use a calculator.

There is a 13.5% chance that a randomly selected East High School student talks on the phone for more than 2 hours a day. This appears to be a reasonable solution because few students spend either a short amount of time or a long amount of time on the phone.

CHECK Your Progress

1. If computers last an average of 3 years, what is the probability that a randomly selected computer will last more than 4 years?

EXAMPLE Exponential Distribution

2 **If athletic shoes last an average of 1.5 years, what is the probability that a randomly selected pair of athletic shoes will last less than 6 months?**

The question asks for the probability that a pair of shoes lasts *less* than 6 months, so we will use the second exponential distribution function. The mean is 1.5 or $\frac{3}{2}$, so the multiplicative inverse m is $\frac{2}{3}$.

$f(x) = 1 - e^{-mx}$ Exponential Distribution Function

$f\left(\frac{1}{2}\right) = 1 - e^{-\frac{2}{3}\left(\frac{1}{2}\right)}$ Replace x with $\frac{1}{2}$ (6 mo $= \frac{1}{2}$ yr) and m with $\frac{2}{3}$.

$= 1 - e^{-\frac{1}{3}}$ Simplify.

≈ 0.2835 or 28.35% Use a calculator.

There is a 28.35% chance that a randomly selected pair of athletic shoes will last less than 6 months.

CHECK Your Progress

2. If the average lifespan of a dog is 12 years, what is the probability that a randomly selected dog will live less than 2 years?

Binomial Distributions In a binomial distribution, all of the trials are independent and have only two possible outcomes, success or failure. The probability of success is the same in every trial. The outcome of one trial does not affect the probabilities of any future trials. The random variable is the number of successes in a given number of trials.

KEY CONCEPT ***Binomial Distribution Functions***

The probability of x successes in n independent trials is

$$P(x) = C(n, x)\, p^x q^{n-x},$$

where p is the probability of success of an individual trial and q is the probability of failure on that same individual trial ($p + q = 1$).

The expectation for a binomial distribution is

$$E(X) = np,$$

where n is the total number of trials and p is the probability of success.

EXAMPLE 3 Binomial Probability

A chocolate company makes boxes of assorted chocolates, 40% of which are dark chocolate on average. The production line mixes the chocolates randomly and packages 10 per box.

a. What is the probability that at least 3 chocolates in a given box are dark chocolates?

A success is a dark chocolate, so $p = 0.4$ and $q = 1 - 0.4$ or 0.6. You could add the probabilities of having exactly 3, 4, 5, 6, 7, 8, 9, or 10 dark chocolates, but it is easier to calculate the probability of the box having exactly 0, 1, or 2 chocolates and then subtracting that sum from 1.

$P(\geq 3 \text{ dark chocolates})$

$= 1 - P(< 3)$

$= 1 - P(0) - P(1) - P(2)$ Mutually exclusive events subtracted from 1

$= 1 - C(10, 0)(0.4)^0(0.6)^{10} - C(10, 1)(0.4)^1(0.6)^9 - C(10, 2)(0.4)^2(0.6)^8$

$= 0.8327$ or 83.27% Simplify.

The probability of at least three chocolates being dark chocolates is 0.8327 or 83.27%.

b. What is the expected number of dark chocolates in a box?

$E(X) = np$ Expectation for a binomial distribution

$= 10(0.4)$ $n = 10$ and $p = 0.4$

$= 4$ Multiply.

The expected number of dark chocolates in a box is 4.

CHECK Your Progress

3. If 20% of the chocolates are white chocolates, what is the probability that at least one chocolate in a given box is a white chocolate?

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CHECK Your Understanding

Examples 1, 2 (pp. 729–730)

For Exercises 1 and 2, use the following information.
The average amount of time high school students spend on homework is 2 hours per day.

1. What is the probability that a randomly selected student spends more than 3 hours per day on homework?

2. What is the probability that a randomly selected student spends less than 1 hour per day on homework?

Examples 3 (p. 731)

For Exercises 3 and 4, use the following information.
Mary's cat is having kittens. The probability of a kitten being male is 0.5.

3. If Mary's cat has 4 kittens, what is the probability that at least 3 will be male?

4. What is the expected number of males in a litter of 6?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
5-6, 9–14, 18–20	1–2
7–8, 15–17	3

For Exercises 5 and 6, use the following information.
The average life span of a certain type of car tire is 4 years.

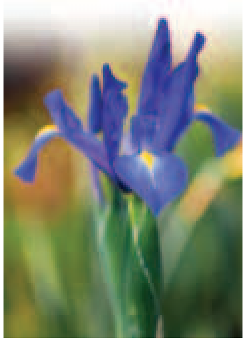
5. What is the probability that a randomly selected set of 4 tires will last more than 9 years?

6. What is the probability that a randomly selected set of tires will last fewer than 2.5 years?

GARDENING **For Exercises 7 and 8, use the following information.**
Dan is planting 24 irises in his front yard. The flowers he bought were a combination of two varieties, blue and white. The flowers are not blooming yet, but Dan knows that the probability of having a blue flower is 75%.

7. What is the probability that at least 20 of the flowers will be blue?

8. What is the expected number of white irises in Dan's garden?



Real-World Link
There are hundreds of species and cultivations of iris in all colors of the rainbow. Iris vary from tiny woodland ground covers, to 4-feet-tall flowers that flourish in the sun, to species that thrive in swampy soil. There is an iris that will do well in virtually every garden.
Source: hgic.clemson.edu

For Exercises 9–14, use the following information.
An exponential distribution has a mean of 0.5. Find each probability.

9. $x > 1.5$ **10.** $x > 3$ **11.** $x > \frac{1}{4}$

12. $x < 1$ **13.** $x < \frac{1}{3}$ **14.** $x < 2.5$

For Exercises 15–17, use the following information.
A binomial distribution has a 60% rate of success. There are 18 trials.

15. What is the probability that there will be at least 12 successes?

16. What is the probability that there will be 12 failures?

17. What is the expected number of successes?

RELIABILITY **For Exercises 18–20, use the following information.**
A light bulb has an average life of 8 months.

18. What is the probability that a randomly chosen bulb will last more than 13 months?

19. What is the probability that a randomly chosen bulb will last less than 6 months?

20. There is an 80% chance that a randomly chosen light bulb will last more than how long?

JURY DUTY **For Exercises 21–23, use the following information.**
A jury of twelve people is being selected for trial. The probability that a juror will be male is 0.5. The probability that a juror will vote to convict is 0.75.

21. What is the probability that more than 3 jurors will be men?

22. What is the probability that fewer than 6 jurors will vote to convict?

23. What is the expected number of votes for conviction?

EXTRA PRACTICE
See pages 919, 937.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

24. OPEN ENDED Sketch the graph of an exponential distribution function. Describe a situation in which you would expect data to be distributed in this way.

25. **REASONING** An exponential distribution function has a mean of 2. A fellow student says that the probability that $x > 2$ is 0.5. Determine whether this is *sometimes*, *always*, or *never* true. Explain your reasoning.

26. **CHALLENGE** The average amount of money spent per day by students in Mrs. Ross's class for lunch is \$2. In this class, 90% of students spend less than what amount per day?

27. ***Writing in Math*** Your school has received a grant, and the administration is considering adding a new science wing to the building. You have been asked to poll a sample of your classmates to find out if they support using the funding for the science wing project. Describe how you could use binomial distribution to predict the number of people in the school who would support the science wing project.

STANDARDIZED TEST PRACTICE

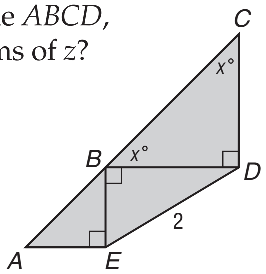
28. **ACT/SAT** In rectangle $ABCD$, what is $x + y$ in terms of z?

A $90 + z$

B $190 - z$

C $180 + z$

D $270 - z$



29. **REVIEW** Your gym teacher is randomly distributing 15 red dodge balls and 10 yellow dodge balls. What is the probability that the first ball that she hands out will be yellow and the second will be red?

F $\frac{1}{24}$

G $\frac{1}{4}$

H $\frac{2}{5}$

J $\frac{23}{25}$

Spiral Review

A set of 260 data values is normally distributed with a mean of 50 and a standard deviation of 5.5. (Lesson 12-7)

30. What percent of the data lies between 39 and 61?

31. What is the probability that a data value selected at random is greater than 39?

A die is rolled, Find each probability. (Lesson 12-5)

32. P(even)

33. P (1 or 6)

34. P(prime number)

Simplify each expression. (Lesson 6-2)

35. $(x - 7)(x + 9)$

36. $(4b^2 + 7)^2$

37. $(3q - 6) - (q + 13) + (-2q + 11)$

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the indicated term of each expression.** (Lesson 11-7)

38. third term of $(a + b)^7$

39. fourth term of $(c + d)^8$

40. fifth term of $(x + y)^9$

EXTEND 12-8

Algebra Lab
Simulations

A **simulation** uses a probability experiment to mimic a real-life situation. You can use a simulation to solve the following problem about **expected value**.

A brand of cereal is offering one of six different prizes in every box. If the prizes are equally and randomly distributed within the cereal boxes, how many boxes, on average, would you have to buy in order to get a complete set?

ACTIVITY 1

Work in pairs or small groups to complete Steps 1 through 4.

Step 1 Use the six numbers on a die to represent the six different prizes.

Step 2 Roll the die and record which prize was in the first box of cereal. Use a tally sheet like the one shown.

Step 3 Continue to roll the die and record the prize number until you have a complete set of prizes. Stop as soon as you have a complete set. This is the end of one trial in your simulation. Record the number of boxes required for this trial.

Step 4 Repeat steps 1, 2, and 3 until your group has carried out 25 trials. Use a new tally sheet for each trial.

Simulation Tally Sheet	
Prize Number	**Boxes Purchased**
1	
2	
3	
4	
5	
6	
Total Needed	

Analyze the Results

1. Create two different statistical graphs of the data collected for 25 trials.
2. Determine the mean, median, maximum, minimum, and standard deviation of the total number of boxes needed in the 25 trials.
3. Combine the small-group results and determine the mean, median, maximum, minimum, and standard deviation of the number of boxes required for all the trials conducted by the class.
4. If you carry out 25 additional trials, will your results be the same as in the first 25 trials? Explain.
5. Should the small-group results or the class results give a better idea of the average number of boxes required to get a complete set of prizes? Explain.
6. If there were 8 prizes instead of 6, would you need to buy more boxes of cereal or fewer boxes of cereal on average?
7. **DESIGN A SIMULATION** What if one of the 6 prizes was more common than the other 5? Suppose one prize appears in 25% of all the boxes and the other 5 prizes are equally and randomly distributed among the remaining 75% of the boxes? Design and carry out a new simulation to predict the average number of boxes you would need to buy to get a complete set. Include some measures of central tendency and dispersion with your data.

12-9 Binomial Experiments

Main Ideas

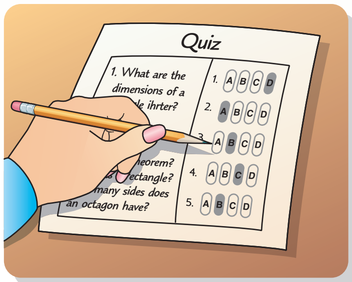
- Use binomial experiments to find probabilities.
- Find probabilities for binomial experiments.

New Vocabulary

binomial experiment

GET READY for the Lesson

What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice quiz if you guess at every question?



Binomial Expansions You can use the Binomial Theorem to find probabilities in certain situations where there are two possible outcomes. The 5 possible ways of getting 4 questions right r and 1 question wrong w are shown at the right. This chart shows the combination of 5 things (answer choices) taken 4 at a time (right answers) or $C(5, 4)$.

w	r	r	r	r
r	w	r	r	r
r	r	w	r	r
r	r	r	w	r
r	r	r	r	w

The terms of the binomial expansion of $(r + w)^5$ can be used to find the probabilities of each combination of right and wrong.

$$(r + w)^5 = r^5 + 5r^4w + 10r^3w^2 + 10r^2w^3 + 5rw^4 + w^5$$

Study Tip

Look Back
To review the **Binomial Theorem**, see Lesson 11-7.

Coefficient	Term	Meaning
$C(5, 5) = 1$	r^5	1 way to get all 5 questions right
$C(5, 4) = 5$	$5r^4w$	5 ways to get 4 questions right and 1 question wrong
$C(5, 3) = 10$	$10r^3w^2$	10 ways to get 3 questions right and 2 questions wrong
$C(5, 2) = 10$	$10r^2w^3$	10 ways to get 2 questions right and 3 questions wrong
$C(5, 1) = 5$	$5rw^4$	5 ways to get 1 question right and 4 questions wrong
$C(5, 0) = 1$	w^5	1 way to get all 5 questions wrong

The probability of getting a question right that you guessed on is $\frac{1}{4}$. So, the probability of getting the question wrong is $\frac{3}{4}$. To find the probability of getting 4 questions right and 1 question wrong, substitute $\frac{1}{4}$ for r and $\frac{3}{4}$ for w in the term $5r^4w$.

$$P(\text{4 right, 1 wrong}) = 5r^4w$$

$$= 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) \qquad r = \frac{1}{4}, w = \frac{3}{4}$$

$$= \frac{15}{1024} \qquad \text{Multiply.}$$

The probability of getting exactly 4 questions correct is $\frac{15}{1024}$ or about 1.5%.

EXAMPLE Binomial Theorem

1 **If a family has 4 children, what is the probability that they have 3 boys and 1 girl?**

There are two possible outcomes for the gender of each of their children: boy or girl. The probability of a boy b is $\frac{1}{2}$, and the probability of a girl g is $\frac{1}{2}$.

$(b + g)^4 = b^4 + 4b^3g + 6b^2g^2 + 4bg^3 + g^4$

The term $4b^3g$ represents 3 boys and 1 girl.

$P(3 \text{ boys}, 1 \text{ girl}) = 4b^3g$

$= 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$ $b = \frac{1}{2}, g = \frac{1}{2}$

$= \frac{1}{4}$ The probability is $\frac{1}{4}$ or 25%.

CHECK Your Progress

1. If a coin is flipped six times, what is the probability that the coin lands heads up four times and tails up two times?

Real-World Link

As of 2005, the National Hockey League record for most goals in a game by one player is seven. A player has scored five or more goals in a game 53 times in league history.

Source: NHL

Binomial Experiments Problems like Example 1 that can be solved using binomial expansion are called **binomial experiments**.

KEY CONCEPT — *Binomial Experiments*

A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The probabilities for each trial are the same.

A binomial experiment is sometimes called a *Bernoulli experiment*.

EXAMPLE Binomial Experiment

Personal Tutor

2 **SPORTS Suppose that when hockey star Martin St. Louis takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game.**

a. What is the probability that he will score exactly 2 goals?

The probability that he scores on a given shot is $\frac{1}{7}$, and the probability that he does not is $\frac{6}{7}$. There are $C(6, 2)$ ways to choose the 2 shots that score.

$P(2 \text{ goals}) = C(6, 2)\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4$ If he scores on 2 shots, he fails to score on 4 shots.

$= \frac{6 \cdot 5}{2}\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4$ $C(6, 2) = \frac{6!}{4!2!}$

$= \frac{19,440}{117,649}$ Simplify.

The probability of exactly 2 goals is $\frac{19,440}{117,649}$ or about 17%.

b. What is the probability that he will score at least 2 goals?

Instead of adding the probabilities of getting exactly 2, 3, 4, 5, and 6 goals, it is easier to subtract the probabilities of getting exactly 0 or 1 goal from 1.

$$P(\text{at least 2 goals}) = 1 - P(0 \text{ goals}) - P(1 \text{ goal})$$

$$= 1 - C(6, 0)\left(\frac{1}{7}\right)^0\left(\frac{6}{7}\right)^6 - C(6, 1)\left(\frac{1}{7}\right)^1\left(\frac{6}{7}\right)^5$$

$$= 1 - \frac{46{,}656}{117{,}649} - \frac{46{,}656}{117{,}649} \quad \text{Simplify.}$$

$$= \frac{24{,}337}{117{,}649} \quad \text{Subtract.}$$

The probability that Martin will score at least 2 goals is $\frac{24{,}337}{117{,}649}$ or about 21%.

CHECK Your Progress

A basketball player has a free-throw percentage of 75% before the last game of the season. The player takes 5 free throws in the final game.

2A. What is the probability that he will make exactly two free throws?

2B. What is the probability that he will make at least two free throws?

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CHECK Your Understanding

Examples 1, 2
(pp. 730–731)

Find each probability if a coin is tossed 3 times.

1. P(exactly 2 heads) **2.** P(0 heads) **3.** P(at least 1 head)

Four cards are drawn from a standard deck of cards. Each card is replaced before the next one is drawn. Find each probability.

4. P(4 jacks) **5.** P(exactly 3 jacks) **6.** P(at most 1 jack)

SPORTS **Lauren Wible of Bucknell University was the 2005 NCAA Division I women's softball batting leader with a batting average of .524. This means that the probability of her getting a hit in a given at-bat was 0.524.**

7. Find the probability of her getting 4 hits in 4 at-bats.

8. Find the probability of her getting exactly 2 hits in 4 at-bats.

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–30	1, 2

Find each probability if a coin is tossed 5 times.

9. P(5 tails) **10.** P(0 tails)

11. P(exactly 2 tails) **12.** P(exactly 1 tail)

13. P(at least 4 tails) **14.** P(at most 2 tails)

Find each probability if a die is rolled 4 times.

15. P(exactly one 3) **16.** P(exactly three 3s)

17. P(at most two 3s) **18.** P(at least three 3s)

Real-World Link

The word *Internet* was virtually unknown until the mid-1980s. By 1997, 19 million Americans were using the Internet. That number tripled in 1998 and passed 100 million in 1999.

Source: UCLA

As a maintenance manager, Jackie Thomas is responsible for managing the maintenance of an office building. When entering a room after hours, the probability that she selects the correct key on the first try is $\frac{1}{5}$. If she enters 6 rooms in an evening, find each probability.

19. P(never the correct key)
20. P(always the correct key)
21. P(correct exactly 4 times)
22. P(correct exactly 2 times)
23. P(no more than 2 times correct)
24. P(at least 4 times correct)

Prisana guesses at all 10 true/false questions on her history test. Find each probability.

25. P(exactly 6 correct)
26. P(exactly 4 correct)
27. P(at most half correct)
28. P(at least half correct)

29. CARS According to a recent survey, about 1 in 3 new cars is leased rather than bought. What is the probability that 3 of 7 randomly selected new cars are leased?

30. INTERNET In a recent year, it was estimated that 55% of U.S. adult Internet users had access to high-speed Internet connections at home or on the job. What is the probability that exactly 2 out of 5 randomly selected U.S. adults had access to high-speed Internet connections?

If a thumbtack is dropped, the probability of it landing point-up is 0.3. If 10 tacks are dropped, find each probability.

31. P(at least 8 points up)
32. P(at most 3 points up)

33. COINS A fair coin is tossed 6 times. Find the probability of each outcome.

Graphing Calculator

BINOMIAL DISTRIBUTION For Exercises 34 and 35, use the following information. You can use a TI-83/84 Plus graphing calculator to investigate the graph of a binomial distribution.

Step 1 Enter the number of trials in L1. Start with 10 trials.

KEYSTROKES: STAT 1 ▲ 2nd [LIST] ▶ 5 X,T,θ,n , X,T,θ,n , 0 , 10) ENTER

Step 2 Calculate the probability of success for each trial in L2.

KEYSTROKES: ▶ ▲ 2nd [DISTR] 0 10 , .5 , 2nd [L1]) ENTER

Step 3 Graph the histogram.

KEYSTROKES: 2nd [STAT PLOT]

Use the arrow and ENTER keys to choose ON, the histogram, L1 as the Xlist, and L2 as the frequency. Use the window [0, 10] scl: 1 by [0, 0.5] scl: 0.1.

34. Replace the 10 in the keystrokes for steps 1 and 2 to graph the binomial distribution for several values of n less than or equal to 47. You may have to adjust your viewing window to see all of the histogram. Make sure Xscl is 1.

35. What type of distribution does the binomial distribution start to resemble as n increases?

EXTRA PRACTICE
See pages 919, 937.
Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

36. OPEN ENDED Describe a situation for which the P(2 or more) can be found by using a binomial expansion.

37. REASONING Explain why each experiment is not a binomial experiment.

a. rolling a die and recording whether a 1, 2, 3, 4, 5, or 6 comes up

b. tossing a coin repeatedly until it comes up heads

c. removing marbles from a bag and recording whether each one is black or white, if no replacement occurs

38. CHALLENGE Find the probability of exactly m successes in n trials of a binomial experiment where the probability of success in a given trial is p.

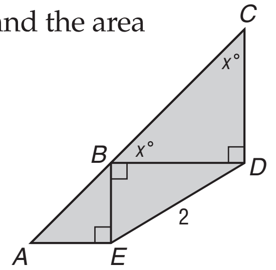
39. *Writing in Math* Use the information on page 735 to explain how you can determine whether guessing is worth it. Explain how to find the probability of getting any number of questions right on a 5-question multiple-choice quiz when guessing and the probability of each score.

STANDARDIZED TEST PRACTICE

40. ACT/SAT If $DE = 2$, what is the sum of the area of $\triangle ABE$ and the area of $\triangle BCD$?

A 2 **C** 4

B 3 **D** 5



41. REVIEW An examination consists of 10 questions. A student must answer only one of the first two questions and only six of the remaining ones. How many choices of questions does the student have?

F 112 **H** 44

G 56 **J** 30

Spiral Review

A set of 400 test scores is normally distributed with a mean of 75 and a standard deviation of 8. (Lesson 12-7)

42. What percent of the test scores lie between 67 and 83?

43. How many of the test scores are greater than 91?

44. What is the probability that a randomly-selected score is less than 67?

45. A salesperson had sales of \$11,000, \$15,000, \$11,000, \$16,000, \$12,000, and \$12,000 in the last six months. Which measure of central tendency would he be likely to use to represent these data when he talks with his supervisor? Explain. (Lesson 12-6)

Graph each inequality. (Lesson 2-7)

46. $x \geq -3$ **47.** $x + y \leq 4$ **48.** $y > |5x|$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate** $2\sqrt{\frac{p(1-p)}{n}}$ **for the given values of p and n. Round to the nearest thousandth if necessary.** (Lesson 5-2)

49. $p = 0.5, n = 100$ **50.** $p = 0.5, n = 400$ **51.** $p = 0.25, n = 500$

52. $p = 0.75, n = 1000$ **53.** $p = 0.3, n = 500$ **54.** $p = 0.6, n = 1000$

EXPLORE 12-10

Algebra Lab
Testing Hypotheses

A **hypothesis** is a statement to be tested. Testing a hypothesis to determine whether it is supported by the data involves five steps.

Step 1 State the hypothesis. The statement should include a *null hypothesis,* which is the hypothesis to be tested, and an *alternative hypothesis.*

Step 2 Design the experiment.

Step 3 Conduct the experiment and collect the data.

Step 4 Evaluate the data. Decide whether to reject the null hypothesis.

Step 5 Summarize the results.

ACTIVITY **Test the following hypothesis.**

People react to sound and touch at the same rate.

You can measure reaction time by having someone drop a ruler and then having someone else catch it between their fingers. The distance the ruler falls will depend on their reaction time. Half of the class will investigate the time it takes to react when someone is told the ruler has dropped. The other half will measure the time it takes to react when the catcher is alerted by touch.

Step 1 The null hypothesis H_0 and alternative hypothesis H_1 are as follows. These statements often use $=, \neq, <, >, \geq$, and $\leq$.

- H_0: reaction time to sound = reaction time to touch
- H_1: reaction time to sound $\neq$ reaction time to touch

Step 2 You will need to decide the height from which the ruler is dropped, the position of the person catching the ruler, the number of practice runs, and whether to use one try or the average of several tries.

Step 3 Conduct the experiment in each group and record the results.

Step 4 Organize the results so that they can be compared.

Step 5 Based on the results of your experiment, do you think the hypothesis is true? Explain.

Analyze the Results

State the null and alternative hypotheses for each conjecture.

1. A teacher feels that playing classical music during a math test will cause the test scores to change (either up or down). In the past, the average test score was 73.
2. An engineer thinks that the mean number of defects can be decreased by using robots on an assembly line. Currently, there are 18 defects for every 1000 items.
3. **MAKE A CONJECTURE** Design an experiment to test the following hypothesis. *Pulse rates increase 20% after moderate exercise.*

12-10 Sampling and Error

Main Ideas

- Determine whether a sample is unbiased.
- Find margins of sampling error.

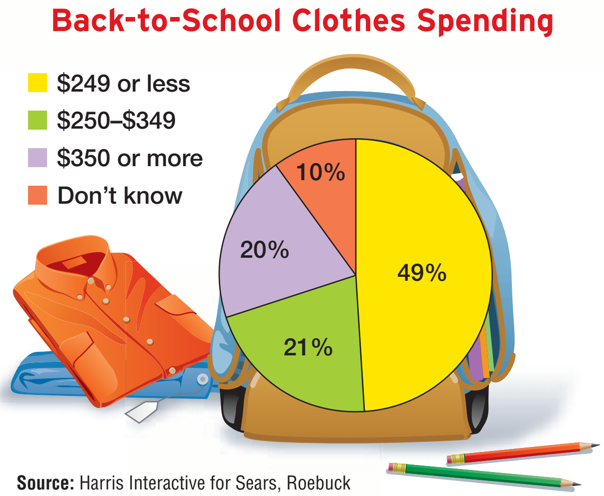
New Vocabulary

unbiased sample
margin of sampling error

GET READY for the Lesson

A survey was conducted asking mothers how much they spend per student on back-to-school clothing. The results of the survey are shown.

When polling organizations want to find how the public feels about an issue, they survey a small portion of the population.



Source: Harris Interactive for Sears, Roebuck

Bias To be sure that survey results are representative of the population, polling organizations need to make sure that they poll a random or **unbiased sample** of the population.

Study Tip

Random Sample
A sample of size *n* is random when every possible sample of size *n* has an equal chance of being selected.

EXAMPLE 1 Biased and Unbiased Samples

State whether each method would produce a random sample. Explain.

a. asking every tenth person coming out of a gym how many times a week they exercise to determine how often city residents exercise

This would not result in a random sample because the people surveyed probably exercise more often than the average person.

b. surveying people going into an Italian restaurant to find out people's favorite type of food

This would probably not result in a random sample because the people surveyed would probably be more likely than others to prefer Italian food.

CHECK Your Progress

1. asking every player at a golf course what sport they prefer to watch on TV

Margin of Error The **margin of sampling error (ME)** gives a limit on the difference between how a sample responds and how the total population would respond.

KEY CONCEPT — Margin of Sampling Error

If the percent of people in a sample responding in a certain way is p and the size of the sample is n, then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where

$$ME = 2\sqrt{\frac{p(1-p)}{n}}.$$

That is, the probability is 0.95 that $p \pm ME$ will contain the true population results.

EXAMPLE Find a Margin of Error

2 **In a survey of 1000 randomly selected adults, 37% answered "yes" to a particular question. What is the margin of error?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ — Formula for margin of sampling error

$= 2\sqrt{\frac{0.37(1-0.37)}{1000}}$ — $p = 37\%$ or 0.37, $n = 1000$

≈ 0.030535 — Use a calculator.

The margin of error is about 3%. This means that there is a 95% chance that the percent of people in the whole population who would answer "yes" is between 37 − 3 or 34% and 37 + 3 or 40%.

CHECK Your Progress

2. In a survey of 625 randomly selected teens, 78% said that they purchase music. What is the margin of error in this survey?

EXAMPLE Analyze a Margin of Error

Personal Tutor

Real-World Link

The percent of smokers in the United States population declined from 38.7% in 1985 to 23.3% in 2000. New therapies, like the nicotine patch, are helping more people to quit.

Source: U.S. Department of Health and Human Services

3 **HEALTH** **In a recent Gallup Poll, 25% of the people surveyed said they had smoked cigarettes in the past week. The margin of error was 3%. How many people were surveyed?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ — Formula for margin of sampling error

$0.03 = 2\sqrt{\frac{0.25(1-0.25)}{n}}$ — $ME = 0.03$, $p = 0.25$

$0.015 = \sqrt{\frac{0.25(0.75)}{n}}$ — Divide each side by 2.

$0.000225 = \frac{0.25(0.75)}{n}$ — Square each side.

$n = \frac{0.25(0.75)}{0.000225}$ — Multiply by n and divide by 0.000225.

$n \approx 833.33$ — About 833 people were surveyed.

CHECK Your Progress

3. In a recent survey, 15% of the people surveyed said they had missed a class or a meeting because they overslept. The margin of error was 4%. How many people were surveyed?

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CHECK Your Understanding

Example 1 (p. 741) **Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.**

1. the government sending a tax survey to everyone whose social security number ends in a particular digit
2. surveying college students in the honors program to determine the average time students at the college study each day

Example 2 (p. 742) **For Exercises 3–5, find the margin of sampling error to the nearest percent.**

3. $p = 72\%, n = 100$
4. $p = 31\%, n = 500$
5. In a survey of 350 randomly selected homeowners, 54% stated that they are planning a major home improvement project in the next six months.

Example 3 (p. 742) **MEDIA For Exercises 6 and 7, use the following information.**
A survey found that 57% of consumers said they will not have any debt from holiday spending. Suppose the survey had a margin of error of 3%.

6. What does the 3% indicate about the results?
7. How many people were surveyed?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–11	1
12–21	2, 3

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

8. pointing with your pencil at a class list with your eyes closed as a way to find a sample of students in your class
9. putting the names of all seniors in a hat, then drawing names from the hat to select a sample of seniors
10. asking every twentieth person on a list of registered voters to determine which political candidate is favored
11. finding the heights of all the boys on the varsity basketball team to determine the average height of all the boys in your school

For Exercises 12–21, find the margin of sampling error to the nearest percent.

12. $p = 81\%, n = 100$
13. $p = 16\%, n = 400$
14. $p = 54\%, n = 500$
15. $p = 48\%, n = 1000$
16. $p = 33\%, n = 1000$
17. $p = 67\%, n = 1500$
18. A poll asked people to name the most serious problem facing the country. Forty-six percent of the 800 randomly selected people said crime.
19. In a recent survey, 431 full-time employees were asked if the Internet has made them more or less productive at work. 27% said it made them more productive.
20. Three hundred sixty-seven of 425 high school students said pizza was their favorite food in the school cafeteria.
21. Nine hundred thirty-four of 2150 subscribers to a particular newspaper said their favorite sport was football.
22. **SHOPPING** According to a recent poll, 33% of shoppers planned to spend \$1000 or more during a holiday season. The margin of error was 3%. How many people were surveyed?
23. **ELECTION PREDICTION** One hundred people were asked whether they would vote for Candidate A or Candidate B in an upcoming election. How many said "Candidate A" if the margin of error was 9.6%?

EXTRA PRACTICE
See pages 919, 937.

Math Online
Self-Check Quiz at algebra2.com

24. **ECONOMICS** In a recent poll, 83% of the 1020 people surveyed said they supported raising the minimum wage. What was the margin of error?

25. **PHYSICIANS** In a recent poll, 61% of the 1010 people surveyed said they considered being a physician to be a very prestigious occupation. What was the margin of error?

H.O.T. Problems

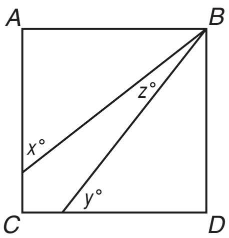
26. **OPEN ENDED** Give examples of a biased sample and an unbiased sample. Explain your reasoning.

27. **REASONING** Explain what happens to the margin of sampling error when the size of the sample n increases. Why does this happen?

28. ***Writing in Math*** Use the information on page 742 to explain how surveys are used in marketing. Find the margin of error for those who spend $249 or less if 807 mothers were surveyed. Explain what this margin of error means.

STANDARDIZED TEST PRACTICE

29. **ACT/SAT** In rectangle $ABCD$, what is $x + y$ in terms of z?



A $90 + z$

B $190 - z$

C $180 + z$

D $270 - z$

30. **REVIEW** If $xy^{-2} + y^{-1} = y^{-2}$, then the value of x *cannot* equal which of the following?

F -1

G 0

H 1

J 2

Spiral Review

A student guesses at all 5 questions on a true-false quiz. Find each probability. (Lesson 12-8)

31. P(all 5 correct)

32. P(exactly 4 correct)

33. P(at least 3 correct)

A set of 250 data values is normally distributed with a mean of 50 and a standard deviation of 5.5. (Lesson 12-7)

34. What percent of the data lies between 39 and 61?

35. What is the probability that a data value selected at random is greater than 39?

Cross-Curricular Project

Algebra and Social Studies

Math from the Past It is time to complete your project. Use the information and data you have gathered about the history of mathematics to prepare a presentation or web page. Be sure to include transparencies and a sample mathematics problem or idea in the presentation.

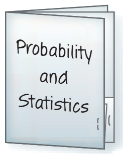
Math Online Cross-Curricular Project at algebra2.com

CHAPTER 12 Study Guide and Review

Download Vocabulary Review from algebra2.com

FOLDABLES™ Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

The Counting Principle, Permutations, and Combinations (Lessons 12-1 and 12-2)

- Fundamental Counting Principle: If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.
- Permutation: order of objects is important.
- Combination: order of objects is not important.

Probability (Lessons 12-3 and 12-4)

- Two independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
- Two dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$
- Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
- Inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Statistical Measures (Lesson 12-5)

- To represent a set of data, use the mean if the data are spread out, the median when the data has outliers, or the mode when the data are tightly clustered around one or two values.
- Standard deviation for n values: $\bar{x}$ is the mean,

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

The Normal Distribution (Lesson 12-6)

- The graph is maximized at the mean and the data are symmetric about the mean.

Binomial Experiments, Sampling, and Error (Lessons 12-7 and 12-8)

- A binomial experiment exists if and only if there are exactly two possible outcomes, a fixed number of independent trials, and the possibilities for each trial are the same.

Key Vocabulary

binomial distribution (p. 730)
binomial experiment (p. 730)
binomial probability (p 731)
combination (p. 692)
compound event (p. 710)
dependent events (p. 686)
event (p. 684)
exponential distribution (p 729)
exponential probability (p. 729)
inclusive events (p. 712)
independent events (p. 684)
measure of variation (p. 718)
mutually exclusive events (p. 710)
normal distribution (p. 724)
outcome (p. 684)
permutation (p. 690)
probability (p. 697)
probability distribution (p. 699)
random (p. 697)
random variable (p. 699)
relative-frequency histogram (p. 699)
sample space (p. 684)
simple event (p. 710)
standard deviation (p. 718)
unbiased sample (p. 741)
uniform distribution (p. 699)
univariate data (p. 717)
variance (p. 718)

Vocabulary Check

Choose the term that best matches each statement or phrase. Choose from the list above.

1. the ratio of the number of ways an event can succeed to the number of possible outcomes
2. an arrangement of objects in which order does not matter
3. two or more events in which the outcome of one event affects the outcome of another event
4. a function that is used to predict the probabilities of an event based on time
5. two events in which the outcome can never be the same
6. an arrangement of objects in which order matters
7. the set of all possible outcomes
8. an event that consists of two or more simple events

Math Online Vocabulary Review at algebra2.com

CHAPTER 12

Study Guide and Review

Lesson-by-Lesson Review

12–1 The Counting Principle (pp. 684–689)

9. PASSWORDS The letters a, c, e, g, i, and k are used to form 6-letter passwords. How many passwords can be formed if the letters can be used more than once in any given password?

Example 1 How many different license plates are possible with two letters followed by three digits?

There are 26 possibilities for each letter. There are 10 possibilities, the digits 0–9, for each number. Thus, the number of possible license plates is as follows.

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^3$ or 676,000

12–2 Permutations and Combinations (pp. 690–695)

10. A committee of 3 is selected from Jillian, Miles, Mark, and Nikia. How many committees contain 2 boys and 1 girl?

11. Five cards are drawn from a standard deck of cards. How many different hands consist of four queens and one king?

12. A box of pencils contains 4 red, 2 white, and 3 blue pencils. How many different ways can 2 red, 1 white, and 1 blue pencil be selected?

Example 2 A basket contains 3 apples, 6 oranges, 7 pears, and 9 peaches. How many ways can 1 apple, 2 oranges, 6 pears, and 2 peaches be selected?

This involves the product of four combinations, one for each type of fruit.

$$C(3, 1) \cdot C(6, 2) \cdot C(7, 6) \cdot C(9, 2)$$

$$= \frac{3!}{(3-1)!1!} \cdot \frac{6!}{(6-2)!2!} \cdot \frac{7!}{(7-6)!6!} \cdot \frac{9!}{(9-2)!2!}$$

$$= 3 \cdot 15 \cdot 7 \cdot 36 \text{ or } 11{,}340 \text{ ways}$$

12–3 Probability (pp. 697–702)

13. A bag contains 4 blue marbles and 3 green marbles. One marble is drawn from the bag at random. What is the probability that the marble drawn is blue?

14. COINS The table shows the distribution of the number of heads occurring when four coins are tossed. Find $P(H = 3)$.

H = Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Example 3 A bag of golf tees contains 23 red, 19 blue, 16 yellow, 21 green, 11 orange, 19 white, and 17 black tees. What is the probability that if you choose a tee from the bag at random, you will choose a green tee?

There are 21 ways to choose a green tee and 23 + 19 + 16 + 11 + 19 + 17 or 105 ways not to choose a green tee. So, s is 21 and f is 105.

$$P(\text{green tee}) = \frac{s}{s+f}$$

$$= \frac{21}{21 + 105} \text{ or } \frac{1}{6}$$

12–4 Multiplying Probabilities (pp. 703–709)

Determine whether the events are *independent* or *dependent*. Then find the probability.

15. Two dice are rolled. What is the probability that each die shows a 4?

16. Two cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart and a club in that order.

17. Luz has 2 red, 2 white, and 3 blue marbles in a cup. If she draws two marbles at random and does not replace the first one, find the probability of a white marble and then a blue marble.

Example 4 **There are 3 dimes, 2 quarters, and 5 nickels in Langston's pocket. If he reaches in and selects three coins at random without replacing any of them, what is the probability that he will choose a dime d, then a quarter q, and then a nickel n?**

Because the outcomes of the first and second choices affect the later choices, these are dependent events.

$$P(d, \text{ then } q, \text{ then } n) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{5}{8} \text{ or } \frac{1}{24}$$

The probability is $\frac{1}{24}$ or about 4.2%.

12–5 Adding Probabilities (pp. 710–715)

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

18. A die is rolled. What is the probability of rolling a 6 or a number less than 4?

19. A die is rolled. What is the probability of rolling a 6 or a number greater than 4?

20. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a red card?

21. There are 5 English, 2 math, and 3 chemistry books on a shelf. If a book is randomly selected, what is the probability of selecting a math book or a chemistry book?

Example 5 **Trish has four \$1 bills and six \$5 bills. She takes three bills from her wallet at random. What is the probability that Trish will select at least two \$1 bills?**

$P(\text{at least two } \$1)$

$$= P(\text{two } \$1, \$5) + P(\text{three } \$1, \text{ no } \$5)$$

$$= \frac{C(4, 2) \cdot C(6, 1)}{C(10, 3)} + \frac{C(4, 3) \cdot C(6, 0)}{C(10, 3)}$$

$$= \frac{\frac{4! \cdot 6!}{(4-2)!2!(6-1)!1!}}{C(10, 3)} + \frac{\frac{4! \cdot 6!}{(4-3)!3!(6-0)!0!}}{C(10, 3)}$$

$$= \frac{36}{120} + \frac{4}{120} \text{ or } \frac{1}{3}$$

The probability is $\frac{1}{3}$ or about 33%.

Study Guide and Review

12-6 Statistical Measures (pp. 717–723)

FOOD **For Exercises 22 and 23, use the frequency table that shows the number of dried apricots per box.**

Apricot Count	Frequency
19	1
20	3
21	5
22	4

22. Find the mean, median, mode, and standard deviation of the apricots to the nearest tenth.

23. For how many boxes is the number of apricots within one standard deviation of the mean?

Example 6 **Find the variance and standard deviation for {100, 156, 158, 159, 162, 165, 170, 190}.**

Step 1 Find the mean.

$$\frac{100 + 156 + 158 + 159 + 162 + 165 + 170 + 190}{8}$$

$$= \frac{1260}{8} \text{ or } 157.5$$

Step 2 Find the standard deviation.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{(100 - 157.5)^2 + \ldots + (190 - 157.5)^2}{8}$$

$\sigma^2 = \frac{4600}{8}$ Simplify.

$\sigma^2 = 575$ Divide.

$\sigma \approx 23.98$ Take the square root of each side.

12-7 The Normal Distribution (pp. 724–728)

UTILITIES **For Exercises 24–27, use the following information.**

The utility bills in a city of 5000 households are normally distributed with a mean of \$180 and a standard deviation of \$16.

24. About how many utility bills were between \$164 and \$196?

25. About how many bills exceeded \$212?

26. About how many bills were under \$164?

27. What is the probability that a random bill is between \$164 and \$180?

BASEBALL **For Exercises 28 and 29, use the following information.**

The average age of a major league baseball player is normally distributed with a mean of 28 and a standard deviation of 4 years.

28. About what percent of major league baseball players are younger than 24?

29. If a team has 35 players, about how many are between the ages of 24 and 32?

Example 7 **Mr. Byrum gave an exam to his 30 Algebra 2 students at the end of the first semester. The scores were normally distributed with a mean score of 78 and a standard deviation of 6.**

a. What percent of the class would you expect to have scored between 72 and 84?
Since 72 and 84 are 1 standard deviation to the left and right of the mean, respectively, 34% + 34% or 68% of the students scored within this range.

b. What percent of the class would you expect to have scored between 90 and 96?
90 to 96 on the test includes 2% of the students.

c. Approximately how many students scored between 84 and 90?
84 to 90 on the test includes 13.5% of the students; $0.135 \times 30 = 4$ students.

Mixed Problem Solving
For mixed problem-solving practice, see page 937.

12-8 Exponential and Binomial Distribution (pp. 729-733)

30. The average person has a pair of automobile windshield wiper blades for 6 months. What is the probability that a randomly selected automobile has a pair of windshield wiper blades older than one year?

LAWS For Exercises 31 and 32, use the following information.

A polling company wants to estimate how many people are in favor of a new environmental law. The polling company polls 20 people. The probability that a person is in favor of the law is 0.5.

31. What is the probability that exactly 12 people are in favor of the new law?

32. What is the expected number of people in favor of the law?

Example 8 According to a recent survey, the average teenager spends one hour a day on an outdoor activity. What is the probability that a randomly selected teenager spends more than 1.5 hours per day outside?

Use the first exponential distribution function. The mean is 1, and the inverse of the mean is 1.

$f(x) = e^{-mx}$ Exponential Distribution Function

$= e^{-1(1.5)}$ Replace m with 1 and x with 1.5.

$= e^{-1.5}$ Simplify.

≈ 0.2231 or 22.31% Use a calculator.

There is a 22.31% chance that a randomly selected teenager spends more than 1.5 hours a day outside.

12-9 Binomial Experiments (pp. 735–739)

Find each probability if a number cube is rolled twelve times.

33. P(twelve 3s) **34.** P(exactly one 3)

35. WORLD CULTURES The Cayuga Indians played a game of chance called *Dish*, in which they used 6 flattened peach stones blackened on one side. They placed the peach stones in a wooden bowl and tossed them. The winner was the first person to get a prearranged number of points. Assume that each face (black or neutral) of each stone has an equal chance of showing up. Find the probability of each possible outcome.

Example 9 To practice for a jigsaw puzzle competition, Laura and Julian completed four jigsaw puzzles. The probability that Laura places the last piece is $\frac{3}{5}$, and the probability that Julian places the last piece is $\frac{2}{5}$. What is the probability that Laura will place the last piece of at least two puzzles?

$P = L^4 + 4L^3J + 6L^2J^2$

$= \left(\frac{3}{5}\right)^4 + 4\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right) + 6\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$

$= \frac{81}{625} + \frac{216}{625} + \frac{216}{625}$ or 0.8208

The probability is about 82%.

Study Guide and Review

12–10 Sampling and Error (pp. 741–744)

36. ELECTION According to a poll of 300 people, 39% said that they favor Mrs. Smith in an upcoming election. What is the margin of sampling error?

37. FREEDOMS In a poll asking people to name their most valued freedom, 51% of the randomly selected people said it was the freedom of speech. Find the margin of sampling error if 625 people were randomly selected.

38. SPORTS According to a recent survey of mothers with children who play sports, 63% of them would prefer that their children not play football. Suppose the margin of error is 4.5%. How many mothers were surveyed?

Example 10 **In a survey taken at a local high school, 75% of the student body stated that they thought school lunches should be free. This survey had a margin of error of 2%. How many people were surveyed?**

$$ME = 2\sqrt{\frac{p(1-p)}{n}}$$ Margin of sampling error

$$0.02 = 2\sqrt{\frac{0.75(1-0.75)}{n}}$$ $ME = 0.02$, $p = 0.75$

$$0.01 = \sqrt{\frac{0.75(1-0.75)}{n}}$$ Divide each side by 2.

$$0.0001 = \frac{0.75(0.25)}{n}$$ Square each side.

$$n = \frac{0.75(0.25)}{0.0001}$$ Multiply and divide.

$$= 1875$$ Simplify.

There were about 1875 people in the survey.

CHAPTER 12 Practice Test

Evaluate each expression.

1. $P(7, 3)$
2. $C(7, 3)$
3. $P(13, 5)$
4. $C(13, 5)$
5. How many ways can 9 bowling balls be arranged on the upper rack of a bowling ball shelf?
6. How many different outfits can be made if you choose 1 each from 11 skirts, 9 blouses, 3 belts, and 7 pairs of shoes?
7. How many ways can the letters of the word *probability* be arranged?
8. How many different soccer teams consisting of 11 players can be formed from 18 players?
9. Eleven points are equally spaced on a circle. How many ways can five of these points be chosen as the vertices of a pentagon?
10. A number is drawn at random from a hat that contains all the numbers from 1 to 100. What is the probability that the number is less than 16?
11. Two cards are drawn in succession from a standard deck of cards without replacement. What is the probability that both cards are greater than 2 and less than 9?
12. A shipment of 10 television sets contains 3 defective sets. How many ways can a hospital purchase 4 of these sets and receive at least 2 of the defective sets?
13. In a row of 10 parking spaces in a parking lot, how many ways can 4 cars park?
14. While shooting arrows, William Tell can hit an apple 9 out of 10 times. What is the probability that he will hit it exactly 4 out of 7 times?
15. Ten people are going on a camping trip in three cars that hold 5, 2, and 4 passengers, respectively. How many ways is it possible to transport the people to their campsite?
16. The number of colored golf balls in a box is shown in the table below.

Color	Number of Golf Balls
white	5
red	3

Three golf balls are drawn from the box in succession, each being replaced in the box before the next draw is made. What is the probability that all 3 golf balls are the same color?

For Exercises 17–19, use the following information.

In a ten-question multiple-choice test with four choices for each question, a student who was not prepared guesses on each item. Find each probability.

17. 6 questions correct
18. at least 8 questions correct
19. fewer than 8 questions correct
20. **MULTIPLE CHOICE** The average amount of money that a student spends for lunch is \$4. What is the probability that a randomly selected student spends less than \$3 on lunch?

    A 0.36 **C** 0.49

    B 0.47 **D** 0.52

21. **MULTIPLE CHOICE** A mail-order computer company offers a choice of 4 amounts of memory, 2 sizes of hard drives, and 2 sizes of monitors. How many different systems are available to a customer?

    F 8

    G 16

    H 32

    J 64

Standardized Test Practice

Cumulative, Chapters 1–12

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Ms. Rudberg has a list of the yearly salaries of the staff members in her department. Which measure of data describes the middle income value of the salaries?

A mean

B median

C mode

D range

2. A survey of 90 physical trainers found that 15 said they went for a run at least 5 times per week. Of that group, 5 said they also swim during the week and at least 25% run and swim every week. Which conclusion is valid based on the information given?

F The report is accurate because 15 out of 90 is 25%.

G The report is accurate because 5 out of 15 is 33%, which is at least 25%.

H The report is inaccurate because 5 out of 90 is only 3.3%.

J The report is inaccurate because she does not know if the swimming is really exercising.

3. GRIDDABLE Anna is training to run a 10-kilometer race. The table below lists the times she received in different races. The times are listed in minutes. What was her mean time in minutes for a 10-kilometer race?

7.25	8.10
7.40	6.75
7.20	7.35
7.10	7.25
8.00	7.45

4. Mariah has 6 books on her bookshelf. Two are literature books, one is a science book, two are math books, and one is a dictionary. What is the probability that she randomly chooses a science book and the dictionary?

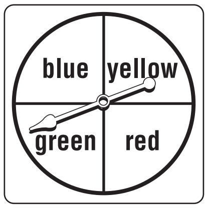
A $\frac{1}{3}$

B $\frac{1}{4}$

C $\frac{1}{12}$

D $\frac{1}{36}$

5. Peter is playing a game where he spins the spinner pictured below and then rolls a die. What is the probability that the spinner lands on yellow and he rolls an even number on the die?



F $\frac{3}{4}$

G $\frac{5}{12}$

H $\frac{1}{8}$

J $\frac{1}{24}$

6. Lynette has a 4-inch by 6-inch picture of her brother. She gets an enlargement made so that the new print has dimensions that are 4 times the dimensions of her original picture. How does the area of the enlargement compare to the area of the original picture?

TEST-TAKING TIP

Question 1 To prepare for a standardized test, make flash cards of key mathematical terms, such as *mean* and *median*. Use the glossary in this text to determine the important terms and their correct definitions.

Math Online Standardized Test Practice at algebra2.com

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 941–956.

7. Lauren works 8-hour shifts at a book store. She makes \$7 an hour and receives a 20% commission on her sales. How much does she need to sell in one shift to earn exactly \$80 before taxes are deducted?

F \$30

G \$86

H \$120

J \$400

8. A rectangular solid has a volume of 35 cubic inches. If the length, width and height are all changed to 3 times their original size, what will be the volume of the new rectangular solid?

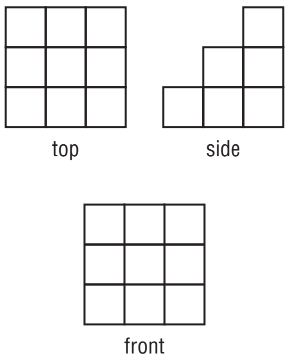
A 38 in^3

B 105 in^3

C 315 in^3

D 945 in^3

9. The top, side and front views of an object built with cubes are shown below.



How many cubes are needed to construct this object?

F 9

G 11

H 18

J 21

10. Petra has made a game for her daughter's birthday party. The playing board is a circle divided evenly into 8 sectors. If the circle has a radius of 18 inches, what is the approximate area of one of the sectors?

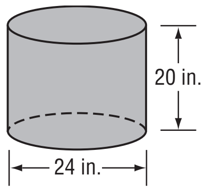
A 4 in^2

B 32 in^2

C 127 in^2

D 254 in^2

11. **GRIDDABLE** Kara has a cylindrical container that she needs to fill with dirt so she can plant some flowers.



What is the volume of the cylinder in cubic inches rounded to the nearest cubic inch?

Pre-AP

Record your answers on a sheet of paper. Show your work.

12. When working at Taco King, Naomi wears a uniform that consists of a shirt, a pair of pants, and a tie. She has 6 uniform shirts, 3 uniform pants, and 4 uniform ties.

a. How many different combinations of shirt, pants, and tie can she make?

b. How many different combinations of shirts and pants can she make?

c. Two of her shirts are red, 3 are blue, and 1 is white. If she wears a different shirt for six days in a row and chooses the shirts at random, what is the probability that she wears a red shirt the first two days?

NEED EXTRA HELP?												
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	12–6	12–4	12–6	12–3	12–4	1–1	2–4	1–1	2–7	10–3	6–8	12–1