# Student Handbook

## Built-In Workbooks


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# How to Use the Student Handbook

The Student Handbook is the additional skill and reference material found at the end of the text. This handbook can help you answer these questions.

## What if I Forget What I Learned Last Year?

Use the **Prerequisite Skills** section to refresh your memory about things you have learned in other math classes. Here's a list of the topics covered in your book.

1. The FOIL Method
2. Factoring Polynomials
3. Congruent and Similar Figures
4. Pythagorean Theorem
5. Mean, Median, and Mode
6. Bar and Line Graphs
7. Frequency Tables and Histograms
8. Stem-and-Leaf Plots
9. Box-and-Whisker Plots

## What If I Need More Practice?

You, or your teacher, may decide that working through some additional problems would be helpful. The **Extra Practice** section provides these problems for each lesson so you have ample opportunity to practice new skills.

## What If I Have Trouble with Word Problems?

The **Mixed Problem Solving** portion of the book provides additional word problems that use the skills presented in each lesson. These problems give you real-world situations where math can be applied.

## What If I Need to Practice for a Standardized Test?

You can review the types of problems commonly used for standardized tests in the **Preparing for Standardized Tests** section. This section includes examples and practice with multiple-choice, griddable or grid-in, and extended-response test items.

## What If I Forget a Vocabulary Word?

The **English-Spanish Glossary** provides a list of important or difficult words used throughout the textbook. It provides a definition in English and Spanish as well as the page number(s) where the word can be found.

## What If I Need to Check a Homework Answer?

The answers to odd-numbered problems are included in **Selected Answers**. Check your answers to make sure you understand how to solve all of the assigned problems.

## What If I Need to Find Something Quickly?

The **Index** alphabetically lists the subjects covered throughout the entire textbook and the pages on which each subject can be found.

## What if I Forget a Formula?

Inside the back cover of your math book is a list of **Formulas and Symbols** that are used in the book.

# Prerequisite Skills

## 1 The FOIL Method

The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

### EXAMPLE 1

**Find $(x + 3)(x - 5)$.**

F L

$$(x + 3)(x - 5) = \underbrace{x \cdot x}_{\text{First}} + \underbrace{(-5) \cdot x}_{\text{Outer}} + \underbrace{3 \cdot x}_{\text{Inner}} + \underbrace{(-3) \cdot 5}_{\text{Last}}$$

I

O

$$= x^2 - 5x + 3x - 15$$
$$= x^2 - 2x - 15$$

### EXAMPLE 2

**Find $(3y + 2)(5y + 4)$.**

$$(3y + 2)(5y + 4) = y \cdot y + 4 \cdot 3y + 2 \cdot 5y + 2 \cdot 4$$
$$= y^2 + 12y + 10y + 8$$
$$= y^2 + 22y + 8$$

**Exercises** **Find each product.**

1. $(a + 2)(a + 4)$
2. $(v - 7)(v - 1)$
3. $(h + 4)(h - 4)$
4. $(d - 1)(d + 1)$
5. $(b + 4)(b - 3)$
6. $(s - 9)(s + 11)$
7. $(r + 3)(r - 8)$
8. $(k - 2)(k + 5)$
9. $(p + 8)(p + 8)$
10. $(x - 15)(x - 15)$
11. $(2c + 1)(c - 5)$
12. $(7n - 2)(n + 3)$
13. $(3m + 4)(2m - 5)$
14. $(5g + 1)(6g + 9)$
15. $(2q - 17)(q + 2)$
16. $(4t - 7)(3t - 12)$

**NUMBER** **For Exercises 17 and 18, use the following information.**

I'm thinking of two integers. One is 7 less than a number, and the other is 2 greater than the same number.

17. Write expressions for the two numbers.
18. Write a polynomial expression for the product of the numbers.

**OFFICE SPACE** **For Exercises 19–21, use the following information.**

Monica's current office is square. Her office in the company's new building will be 3 feet wider and 5 feet longer.

19. Write expressions for the dimensions of Monica's new office.
20. Write a polynomial expression for the area of Monica's new office.
21. Suppose Monica's current office is 7 feet by 7 feet. How much larger will her new office be?

## 2 Factoring Polynomials

Some polynomials can be factored using the Distributive Property.

**EXAMPLE**

**1 Factor $4a^2 + 8a$.**

Find the GCF of $4a^2$ and $8a$.

$4a^2 = 2 \cdot 2 \cdot a \cdot a$ $\quad$ $8a = 2 \cdot 2 \cdot 2 \cdot a$ $\quad$ GCF: $2 \cdot 2 \cdot a$ or $4a$

$4a^2 + 8a = 4a(a) + 4a(2)$ Rewrite each term using the GCF.

$= 4a(a + 2)$ Distributive Property

To factor quadratic trinomials of the form $x^2 + bx + c$, find two integers $m$ and $n$ with a product of $c$ and with a sum of $b$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

**EXAMPLE**

**2 Factor each polynomial.**

**a.** $x^2 + 5x + 6$ ← Both $b$ and $c$ are positive.

In this trinomial, $b$ is 5 and $c$ is 6. Find two numbers with a product of 6 and a sum of 5.

| Factors of 6 | Sum of Factors |
|---|---|
| 1, 6 | 7 |
| 2, 3 | 5 |

The correct factors are 2 and 3.

$x^2 + 5x + 6 = (x + m)(x + n)$ Write the pattern.

$= (x + 2)(x + 3)$ $\quad m = 2$ and $n = 3$

**CHECK** Multiply the binomials to check the factorization.

$(x + 2)(x + 3) = x^2 + 3x + 2x + 2(3)$ FOIL

$= x^2 + 5x + 6$ ✓

**b.** $x^2 - 8x + 12$ ← $b$ is negative and $c$ is positive.

In this trinomial, $b = -8$ and $c = 12$. This means that $m + n$ is negative and $mn$ is positive. So $m$ and $n$ must both be negative.

| Factors of 12 | Sum of Factors |
|---|---|
| −1, −12 | −13 |
| −2, −6 | −8 |

The correct factors are −2 and −6.

$x^2 - 8x + 12 = (x + m)(x + n)$ Write the pattern.

$= [x + (-2)][x + (-6)]$ $\quad m = -2$ and $n = -6$

$= (x - 2)(x - 6)$ Simplify.

**c.** $x^2 + 14x - 15$ ← $b$ is positive and $c$ is negative.

In this trinomial, $b = 14$ and $c = -15$. This means that $m + n$ is positive and $mn$ is negative. So either $m$ or $n$ must be negative, but not both.

| Factors of 12 | Sum of Factors |
|---|---|
| 1, −15 | −14 |
| −1, 15 | 14 |

The correct factors are −1 and 15.

$x^2 + 14x - 15 = (x + m)(x + n)$ Write the pattern.

$= [x + (-1)](x + 15)$ $\quad m = -1$ and $n = 15$

$= (x - 1)(x + 15)$ Simplify.

To factor quadratic trinomials of the form $ax^2 + bx + c$, find two integers $m$ and $n$ whose product is equal to $ac$ and whose sum is equal to b. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

**EXAMPLE**

**3** **Factor $6x^2 + 7x - 3$.**

In this trinomial, $a = 6$, $b = 7$ and $c = -3$. Find two numbers with a product of $6 \cdot (-3)$ or $-18$ and a sum of 7.

| Factors of $-18$ | Sum of Factors | |
|---|---|---|
| $1, -18$ | $-17$ | |
| $-1, 18$ | $17$ | |
| $2, -9$ | $-7$ | |
| $-2, 9$ | $7$ | The correct factors are $-2$ and 9. |

$6x^2 + 7x - 3 = 6x^2 + mx + nx - 3$ Write the pattern.

$= 6x^2 + (-2)x + 9x - 3$ $m = -2$ and $n = 9$

$= (6x^2 - 2x) + (9x - 3)$ Group terms with common factors.

$= 2x(3x - 1) + 3(3x - 1)$ Factor the GCF from each group.

$= (2x + 3)(3x - 1)$ Distributive Property

Here are some special products.

**Perfect Square Trinomials**

$(a + b)^2 = (a + b)(a + b)$
$= a^2 + 2ab + b^2$

$(a - b)^2 = (a - b)(a - b)$
$= a^2 - 2ab + b^2$

**Difference of Squares**

$a^2 - b^2 = (a + b)(a - b)$

**EXAMPLE**

**4** **Factor each polynomial.**

**a.** $4x^2 + 20x + 25$

The first and last terms are perfect squares.
The middle term is equal to $2(2x)(5)$.
This is a perfect square trinomial of the form $(a + b)^2$.

$4x^2 + 20x + 25 = (2x)^2 + 2(2x)(5) + 5^2$ Write as $a^2 + 2ab + b^2$.

$= (2x + 5)^2$ Factor using the pattern.

**b.** $x^2 - 4$

This is a difference of squares.

$x^2 - 4 = x^2 - (2)^2$ Write in the form $a^2 - b^2$.

$= (x + 2)(x - 2)$ Factor the difference of squares.

## Exercises

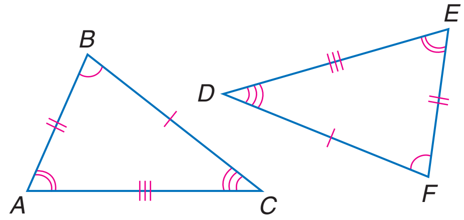
**Factor the following polynomials.**

1. $12x^2 + 4x$
2. $6x^2 y + 2x$
3. $8ab^2 - 12ab$
4. $x^2 + 5x + 4$
5. $y^2 + 12y + 27$
6. $x^2 + 6x + 8$
7. $3y^2 + 13y + 4$
8. $7x^2 + 51x + 14$
9. $3x^2 + 28x + 32$
10. $x^2 - 5x + 6$
11. $y^2 - 5y + 4$
12. $6x^2 - 13x + 5$
13. $6a^2 - 50ab + 16b^2$
14. $11x^2 - 78x + 7$
15. $18x^2 - 31xy + 6y^2$
16. $x^2 + 4xy + 4y^2$
17. $9x^2 - 24x + 16$
18. $4a^2 + 12ab + 9b^2$
19. $x^2 - 144$
20. $4c^2 - 9$
21. $16y^2 - 1$
22. $25x^2 - 4y^2$
23. $36y^2 - 16$
24. $9a^2 - 49b^2$

## 3 Congruent and Similar Figures

**Congruent** figures have the same size and the same shape.

Two polygons are congruent if their corresponding sides are congruent and their corresponding angles are congruent.




$\triangle ABC \cong \triangle EFD$

The order of the vertices indicates the corresponding parts.

| Congruent Angles | Congruent Sides |
|---|---|
| $\angle A \cong \angle E$ | $\overline{AB} \cong \overline{EF}$ |
| $\angle B \cong \angle F$ | $\overline{BC} \cong \overline{FD}$ |
| $\angle C \cong \angle D$ | $\overline{AC} \cong \overline{ED}$ |

Read the symbol $\cong$ as *is congruent to*.

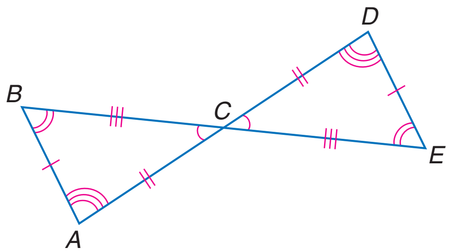
### EXAMPLE

**1 The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.**

List the congruent angles and sides.

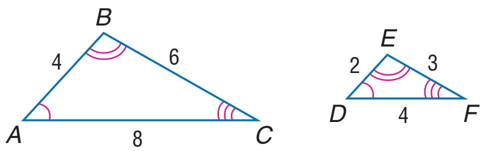
| | |
|---|---|
| $\angle A \cong \angle D$ | $\overline{AB} \cong \overline{DE}$ |
| $\angle B \cong \angle E$ | $\overline{AC} \cong \overline{DC}$ |
| $\angle ACB \cong \angle DCE$ | $\overline{BC} \cong \overline{EC}$ |

Match the vertices of the congruent angles. Therefore, $\triangle ABC \cong \triangle DEC$.




**Similar** figures have the same shape, but not necessarily the same size.

In similar figures, corresponding angles are congruent, and the measures of corresponding sides are proportional. (They have equivalent ratios.)




**Congruent Angles**

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

**Proportional Sides**

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

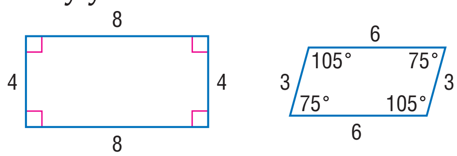
$\triangle ABC \sim \triangle DEF$

Read the symbol $\sim$ as *is similar to.*

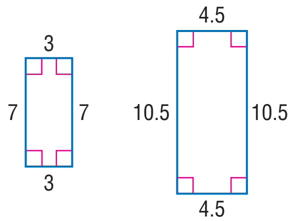
### EXAMPLE

**2 Determine whether the polygons are similar. Justify your answer.**

**a.** Since $\frac{4}{3} = \frac{8}{6} = \frac{4}{3} = \frac{8}{6}$, the measures of the sides of the polygons are proportional. However, the corresponding angles are not congruent. The polygons are not similar.

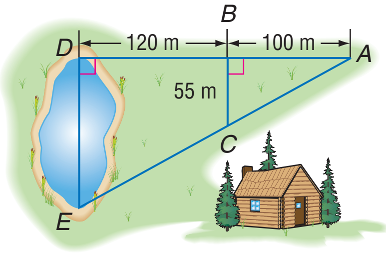



**b.** Since $\frac{7}{10.5} = \frac{3}{4.5} = \frac{7}{10.5} = \frac{3}{4.5}$, the measures of the sides of the polygons are proportional. The corresponding angles are congruent. Therefore, the polygons are similar.



## EXAMPLE

**3** **CIVIL ENGINEERING** The city of Mansfield plans to build a bridge across Pine Lake. Use the information in the diagram to find the distance across Pine Lake.




$\triangle ABC \sim \triangle ADE$

$\frac{AB}{AD} = \frac{BC}{DE}$ Definition of similar polygons

$\frac{100}{220} = \frac{55}{DE}$ $AB = 100$, $AD = 100 + 120 = 220$, $BC = 55$

$100DE = 220(55)$ Cross products

$100DE = 12{,}100$ Simplify.

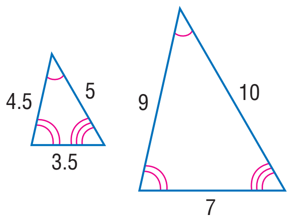
$DE = 121$ Divide each side by 100.

The distance across the lake is 121 meters.

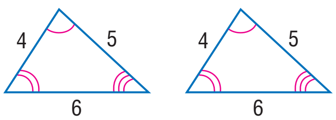
## Exercises

**Determine whether each pair of figures is *similar, congruent,* or *neither.***

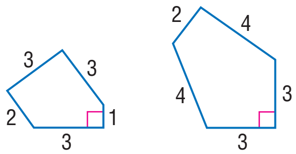
**1.**



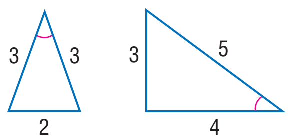

**2.**



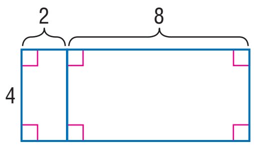

**3.**



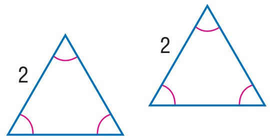

**4.**




**5.**

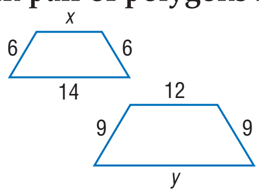



**6.**

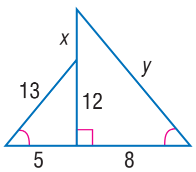



**Each pair of polygons is similar. Find the values of *x* and *y*.**

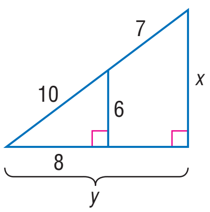
**7.**



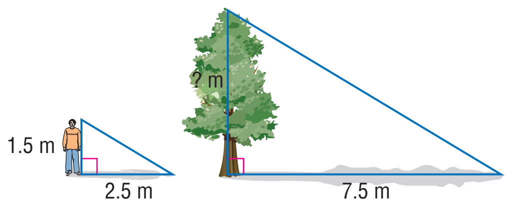

**8.**




**9.**




**10.** **SHADOWS** On a sunny day, Jason measures the length of his shadow and the length of a tree's shadow. Use the figures at the right to find the height of the tree.



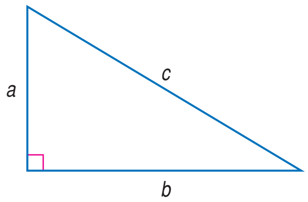

**11.** **PHOTOGRAPHY** A photo that is 4 inches wide by 6 inches long must be reduced to fit in a space 3 inches wide. How long will the reduced photo be?

**12.** **SURVEYING** Surveyors use instruments to measure objects that are too large or too far away to measure by hand. They can use the shadows that objects cast to find the height of the objects without measuring them. A surveyor finds that a telephone pole that is 25 feet tall is casting a shadow 20 feet long. A nearby building is casting a shadow 52 feet long. What is the height of the building?

## 4 Pythagorean Theorem

The **Pythagorean Theorem** states that in a right triangle, the square of the length of the hypotenuse $c$ is equal to the sum of the squares of the lengths of the legs $a$ and $b$.

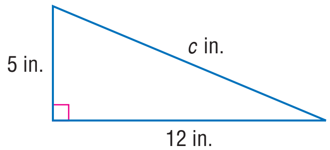
That is, in any right triangle, $c^2 = a^2 + b^2$.




**EXAMPLE**

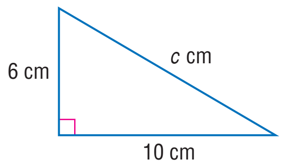
**1 Find the length of the hypotenuse of each right triangle.**

**a.**




| | |
|---|---|
| $c^2 = a^2 + b^2$ | Pythagorean Theorem |
| $c^2 = 5^2 + 12^2$ | Replace $a$ with 5 and $b$ with 12. |
| $c^2 = 25 + 144$ | Simplify. |
| $c^2 = 169$ | Add. |
| $c = \sqrt{169}$ | Take the square root of each side. |
| $c = 13$ | The length of the hypotenuse is 13 inches. |

**b.**

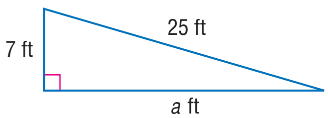



| | |
|---|---|
| $c^2 = a^2 + b^2$ | Pythagorean Theorem |
| $c^2 = 6^2 + 10^2$ | Replace $a$ with 6 and $b$ with 10. |
| $c^2 = 36 + 100$ | Simplify. |
| $c^2 = 136$ | Add. |
| $c = \sqrt{136}$ | Take the square root of each side. |
| $c \approx 11.7$ | Use a calculator. To the nearest tenth, the length of the hypotenuse is 11.7 centimeters. |

**EXAMPLE**

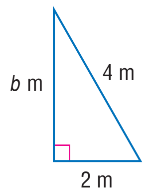
**2** Find the length of the missing leg in each right triangle.

**a.**




| | |
|---|---|
| $c^2 = a^2 + b^2$ | Pythagorean Theorem |
| $25^2 = a^2 + 7^2$ | Replace $c$ with 25 and $b$ with 7. |
| $625 = a^2 + 49$ | Simplify. |
| $625 - 49 = a^2 + 49 - 49$ | Subtract 49 from each side. |
| $576 = a^2$ | Simplify. |
| $\sqrt{576} = a$ | Take the square root of each side. |
| $24 = a$ | The length of the leg is 24 feet. |

**b.**




$c^2 = a^2 + b^2$ Pythagorean Theorem

$4^2 = 2^2 + b^2$ Replace $c$ with 4 and $a$ with 2.

$16 = 4 + b^2$ Simplify.

$12 = b^2$ Subtract 4 from each side.

$\sqrt{12} = b$ Take the square root of each side.

$3.5 \approx b$ Use a calculator to find the square root of 12. Round to the nearest tenth.

To the nearest tenth, the length of the leg is 3.5 meters.

## EXAMPLE

**3 The lengths of the three sides of a triangle are 5, 7, and 9 inches. Determine whether this triangle is a right triangle.**

Since the longest side is 9 inches, use 9 as $c$, the measure of the hypotenuse.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$9^2 \stackrel{?}{=} 5^2 + 7^2$ Replace $c$ with 9, $a$ with 5, and $b$ with 7.

$81 \stackrel{?}{=} 25 + 49$ Evaluate $9^2$, $5^2$, and $7^2$.

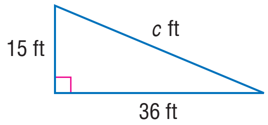
$81 \neq 74$ Simplify.

Since $c^2 \neq a^2 + b^2$, the triangle is *not* a right triangle.

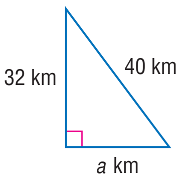
## Exercises

**Find each missing measure. Round to the nearest tenth, if necessary.**

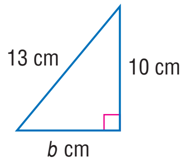
**1.**



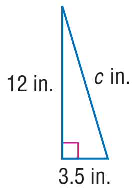

**2.**



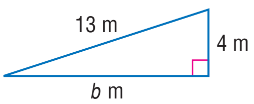

**3.**



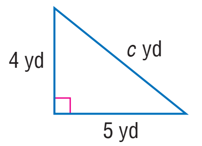

**4.**




**5.**




**6.**




**7.** $a = 3, b = 4, c = ?$

**8.** $a = ?, b = 12, c = 13$

**9.** $a = 14, b = ?, c = 50$

**10.** $a = 2, b = 9, c = ?$

**11.** $a = 6, b = ?, c = 13$

**12.** $a = ?, b = 7, c = 11$

**The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.**

**13.** 5 in., 7 in., 8 in.

**14.** 9 m, 12 m, 15 m

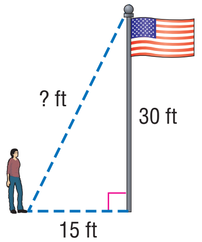
**15.** 6 cm, 7 cm, 12 cm

**16.** 11 ft, 12 ft, 16 ft

**17.** 10 yd, 24 yd, 26 yd

**18.** 11 km, 60 km, 61 km

**19. FLAGPOLES** Mai-Lin wants to find the distance from her feet to the top of the flagpole. If the flagpole is 30 feet tall and Mai-Lin is standing a distance of 15 feet from the flagpole, what is the distance from her feet to the top of the flagpole?




**20. CONSTRUCTION** The walls of the Downtown Recreation Center are being covered with paneling. The doorway into one room is 0.9 meter wide and 2.5 meters high. What is the width of the widest rectangular panel that can be taken through this doorway?

## 5 Mean, Median, and Mode

Mean, median, and mode are measures of central tendency that are often used to represent a set of data.

- To find the **mean**, find the sum of the data and divide by the number of items in the data set. (The mean is often called the average.)
- To find the **median**, arrange the data in numerical order. The median is the middle number. If there is an even number of data, the median is the mean of the two middle numbers.
- The **mode** is the number (or numbers) that appears most often in a set of data. If no item appears most often, the set has no mode.

**EXAMPLE**

**1 Michelle is saving to buy a car. She saved $200 in June, $300 in July, $400 in August, and $150 in September. What was her mean (or average) monthly savings?**

mean = sum of monthly savings/number of months

$$= \frac{\$200 + \$300 + \$400 + \$150}{4}$$

$$= \frac{\$1050}{4} \text{ or } \$262.50$$

Michelle's mean monthly savings was $262.50.

**EXAMPLE**

**2 Find the median of the data.**

To find the median, order the numbers from least to greatest. The median is in the middle. The two middle numbers are 3.7 and 4.1.

$$\frac{3.7 + 4.1}{2} = 3.9$$

There is an even number of data. Find the mean of the middle two.

| Peter's Best Running Times | |
|---|---|
| **Week** | **Minutes to Run a Mile** |
| 1 | 4.5 |
| 2 | 3.7 |
| 3 | 4.1 |
| 4 | 4.1 |
| 5 | 3.6 |
| 6 | 3.4 |

**EXAMPLE**

**3 GOLF Four players tied for first in the 2001 PGA Tour Championship. The scores for each player for each round are shown in the table below. What is the mode score?**

| Player | Round 1 | Round 2 | Round 3 | Round 4 |
|---|---|---|---|---|
| Mike Weir | 68 | 66 | 68 | 68 |
| David Toms | 73 | 66 | 64 | 67 |
| Sergio Garcia | 69 | 67 | 66 | 68 |
| Ernie Els | 69 | 68 | 65 | 68 |

**Source:** ESPN

The mode is the score that occurred most often. Since the score of 68 occurred 6 times, it is the mode of these data.

The range of a set of data is the difference between the greatest and the least values of the set. It describes how a set of data varies.

**EXAMPLE**

**4** **Find the range of the data.** **{6, 11, 18, 4, 9, 15, 6, 3}**

The greatest value is 18 and the least value is 3. So, the range is 18 − 3 or 15.

## Exercises

**Find the mean, median, mode, and range for each set of data. Round to the nearest tenth if necessary.**

1. {2, 8, 12, 13, 15}
2. {66, 78, 78, 64, 34, 88}
3. {87, 95, 84, 89, 100, 82}
4. {99, 100, 85, 96, 94, 99}
5. {9.9, 9.9, 10, 9.9, 8.8, 9.5, 9.5}
6. {501, 503, 502, 502, 502, 504, 503, 503}
7. {7, 19, 15, 13, 11, 17, 9}
8. {6, 12, 21, 43, 1, 3, 13, 8}
9. {0.8, 0.04, 0.9, 1.1, 0.25}
10. $\left\{2\frac{1}{2}, 1\frac{7}{8}, 2\frac{5}{8}, 2\frac{3}{4}, 2\frac{1}{8}\right\}$

11. **CHARITY** The table shows the amounts collected by classes at Jackson High School. Find the mean, median, mode, and range of the data.

**Amounts Collected for Charity**

| Class | Amount | Class | Amount |
|---|---|---|---|
| A | \$150 | E | \$10 |
| B | \$300 | F | \$25 |
| C | \$55 | G | \$200 |
| D | \$40 | H | \$100 |

12. **SCHOOL** The table shows Pilar's grades in chemistry class for the semester. Find her mean, median, and mode scores, and the range of her scores.

**Chemistry Grades**

| Assignment | Grade (out of 100) |
|---|---|
| Homework | 100 |
| Electron Project | 98 |
| Test I | 87 |
| Atomic Mass Project | 95 |
| Test II | 88 |
| Phase Change Project | 90 |
| Test III | 95 |

13. **WEATHER** The table shows the precipitation for the month of July in Cape Hatteras, North Carolina, in various years. Find the mean, median, mode, and range of the data.

**July Precipitation in Cape Hatteras, North Carolina**

| Year | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Inches | 4.22 | 8.58 | 5.28 | 2.03 | 3.93 | 1.08 | 9.54 | 4.94 | 10.85 | 2.66 | 6.04 | 3.26 |

**Source:** National Climatic Data Center

14. **SCHOOL** Kaitlyn's scores on her first five algebra tests are 88, 90, 91, 89, and 92. What test score must Kaitlyn earn on the sixth test so that her mean score will be at least 90?

15. **GOLF** Colin's average for three rounds of golf is 94. What is the highest score he can receive for the fourth round to have an average (mean) of 92?

16. **SCHOOL** Mika has a mean score of 21 on his first four Spanish quizzes. If each quiz is worth 25 points, what is the highest possible mean score he can have after the fifth quiz?

17. **SCHOOL** To earn a grade of B in math, Latisha must have an average (mean) score of at least 84 on five math tests. Her scores on the first three tests are 85, 89, and 82. What is the lowest total score that Latisha must have on the last two tests to earn a B test average?

## 6 Bar and Line Graphs

A **bar graph** compares different categories of data by showing each as a bar whose length is related to the frequency. A **double bar graph** compares two sets of data. Another way to represent data is by using a **line graph**. A line graph usually shows how data changes over a period of time.

### EXAMPLE

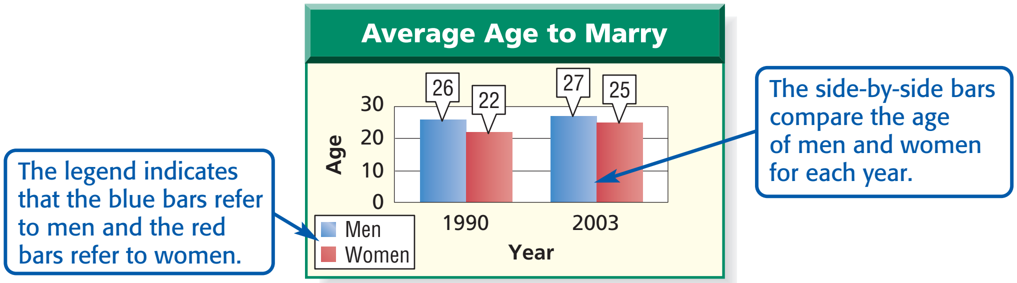
**1 MARRIAGE The table shows the average age at which Americans marry for the first time. Make a double bar graph to display the data.**

| Average Age to Marry | | |
|---|---|---|
| **Year** | **1990** | **2003** |
| Men | 26 | 27 |
| Women | 22 | 25 |

**Source:** U.S. Census Bureau

**Step 1** Draw a horizontal and a vertical axis and label them as shown.

**Step 2** Draw side-by-side bars to represent each category.




### EXAMPLE

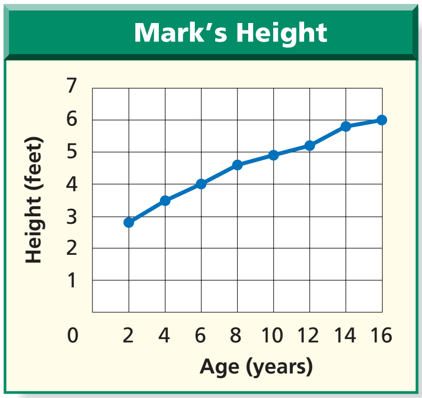
**2 HEALTH The table shows Mark's height at 2-year intervals. Make a line graph to display the data.**

| Age | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|
| Height (feet) | 2.8 | 3.5 | 4.0 | 4.6 | 4.9 | 5.2 | 5.8 | 6 |

**Step 1** Draw a horizontal and a vertical axis. Label them as shown.

**Step 2** Plot the points.

**Step 3** Draw a line connecting each pair of consecutive points.




## Exercises

1. **HEALTH** The table below shows the life expectancy for Americans born in each year listed. Make a double-bar graph to display the data.

| Life Expectancy | | |
|---|---|---|
| **Year of Birth** | **Male** | **Female** |
| 1980 | 70.0 | 77.5 |
| 1985 | 71.2 | 78.2 |
| 1990 | 71.8 | 78.8 |
| 1995 | 72.5 | 78.9 |
| 1998 | 73.9 | 79.4 |

2. **MONEY** The amount of money in Becky's savings account from August through March is shown in the table below. Make a line graph to display the data.

| Month | Amount | Month | Amount |
|---|---|---|---|
| August | $300 | December | $780 |
| September | $400 | January | $800 |
| October | $700 | February | $950 |
| November | $780 | March | $900 |

# 7 Frequency Tables and Histograms

A **frequency table** shows how often an item appears in a set of data. A tally mark is used to record each response. The total number of marks for a given response is the *frequency* of that response. Frequencies can be shown in a bar graph called a histogram. A **histogram** differs from other bar graphs in that no space is between the bars and the bars usually represent numbers grouped by intervals.

## EXAMPLE

**1 TELEVISION** **Use the frequency table of Brad's data.**

**a. How many more chose sports programs than news?**

**b. Which two programs together have the same frequency as adventures?**

**Favorite Television Shows**

| Program | Tally | Frequency |
|---|---|---|
| Sports | 𝍸 II | 7 |
| Mysteries | IIII | 4 |
| Soap operas | 𝍸 | 5 |
| News | 𝍸 | 5 |
| Quiz shows | 𝍸 I | 6 |
| Music videos | II | 2 |
| Adventure | 𝍸 IIII | 9 |
| Comedies | 𝍸 II | 7 |

a. Seven people chose sports. Five people chose news. $7 - 5 = 2$, so 2 more people chose sports than news.

b. As many people chose adventures as the following pairs of programs.

sports and music videos
mysteries and news
mysteries and soap operas
comedies and music videos

## EXAMPLE

**2 FITNESS** **A gym teacher tested the number of sit-ups students in two classes could do in 1 minute. The results are shown.**

**a. Make a histogram of the data. Title the histogram.**

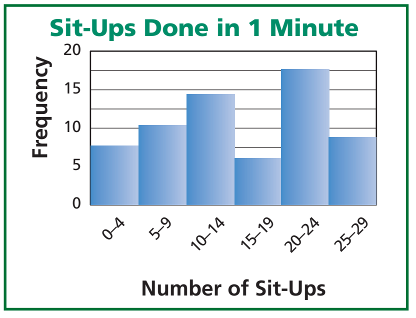
**b. How many students were able to do 25–29 sit-ups in 1 minute?**

**c. How many students were unable to do 10 sit-ups in 1 minute?**

**d. Between which two consecutive intervals does the greatest increase in frequency occur? What is the increase?**

| Number of Sit-Ups | Frequency |
|---|---|
| 0–4 | 8 |
| 5–9 | 12 |
| 10–14 | 15 |
| 15–19 | 6 |
| 20–24 | 18 |
| 25–29 | 10 |

a. Use the same intervals as those in the frequency table on the horizontal axis. Label the vertical axis with a scale that includes the frequency numbers from the table.




b. Ten students were able to do 25–29 sit-ups in 1 minute.

c. Add the students who did 0–4 sit-ups and 5–9 sit-ups. So $8 + 12$, or 20, students were unable to do 10 sit-ups in 1 minute.

d. The greatest increase is between intervals 15–19 and 20–24. These frequencies are 6 and 18. So the increase is $18 - 6 = 12$.

## Exercises

**ART** **For Exercises 1–4, use the following information.**

**The prices in dollars of paintings sold at an art auction are shown.**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1800** | **750** | **600** | **600** | **1800** | **1350** | **300** | **1200** | **750** | **600** | **750** | **2700** |
| **600** | **750** | **300** | **750** | **600** | **450** | **2700** | **1200** | **600** | **450** | **450** | **300** |

1. Make a frequency table of the data.
2. What price was paid most often for the artwork?
3. What is the average price paid for artwork at this auction?
4. How many works of art sold for at least \$600 and no more than \$1200?

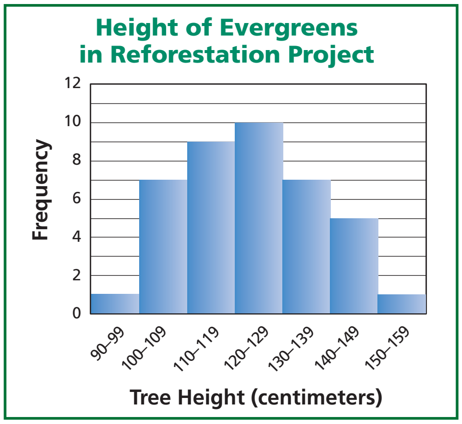
**PETS** **For Exercises 5–9, use the following information.**

**Number of Pets per Family**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 0 | 2 | 1 | 0 |
| 1 | 0 | 1 | 4 | 1 | 2 | 0 | 0 |
| 0 | 1 | 1 | 2 | 2 | 5 | 1 | 0 |

5. Use a frequency table to make a histogram of the data.
6. How many families own two to three pets?
7. How many families own more than three pets?
8. To the nearest percent, what percent of families own no pets?
9. Name the median, mode, and range of the data.

**TREES** **For Exercises 10–12, use the histogram shown.**




10. Which interval contains the most evergreen seedlings?
11. Which intervals contain an equal number of trees?
12. Which intervals contain 95% of the data?
13. Between which two consecutive intervals does the greatest increase in frequency occur? What is the increase?
14. **MARKET RESEARCH** A civil engineer is studying traffic patterns. She counts the number of cars that make it through one rush hour green light cycle. Organize her data into a frequency table, and then make a histogram.

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 16 | 10 | 8 | 8 | 14 | 9 | 7 | 6 | 9 | 10 | 11 | 14 | 10 | 7 | 8 | 9 | 11 | 14 | 10 |

Prerequisite Skills

## 8 Stem-and-Leaf Plots

In a **stem-and-leaf plot**, data are organized in two columns. The greatest place value of the data is used for the stems. The next greatest place value forms the leaves. Stem-and-leaf plots are useful for organizing long lists of numbers.

### EXAMPLE

**SCHOOL Isabella has collected data on the GPAs (grade point average) of the 16 students in the art club. Display the data in a stem-and-leaf plot.**
**{4.0, 3.9, 3.1, 3.9, 3.8, 3.7, 1.8, 2.6, 4.0, 3.9, 3.5, 3.3, 2.9, 2.5, 1.1, 3.5}**

**Step 1** Find the least and the greatest number. Then identify the greatest place-value digit in each number. In this case, ones.
least data: 1.1 greatest data: 4.0

The least number has 1 in the ones place.

The greatest number has 4 in the ones place.

**Step 2** Draw a vertical line and write the stems from 1 to 4 to the left of the line.

**Step 3** Write the leaves to the right of the line, with the corresponding stem. For example, write 0 to the right of 4 for 4.0.

| Stem | Leaf |
|---|---|
| 1 | 8 1 |
| 2 | 5 6 9 |
| 3 | 9 1 9 8 7 9 5 3 5 |
| 4 | 0 0 |

**Step 4** Rearrange the leaves so they are ordered from least to greatest.

**Step 5** Include a key or an explanation.

| Stem | Leaf |
|---|---|
| 1 | 1 8 |
| 2 | 5 6 9 |
| 3 | 1 3 5 5 7 8 9 9 9 |
| 4 | 0 0 $3\|1 = 3.1$ |

### Exercises

**GAMES For Exercises 1–4, use the following information.**
The stem-and-leaf plot at the right shows Charmaine's scores for her favorite computer game.

| Stem | Leaf |
|---|---|
| 9 | 0 0 0 1 3 4 5 5 7 8 8 8 9 9 |
| 10 | 0 3 4 4 5 6 9 |
| 11 | 0 3 9 9 |
| 12 | 1 2 6 |
| 13 | 0 $12\|6 = 126$ |

1. What are Charmaine's highest and lowest scores?
2. Which score(s) occurred most frequently?
3. How many scores were above 115?
4. Has Charmaine ever scored 123?
5. **SCHOOL** The class scores on a 50-item test are shown in the table at the right. Make a stem-and-leaf plot of the data.

| Test Scores | | | | | |
|---|---|---|---|---|---|
| 45 | 15 | 30 | 40 | 28 | 35 |
| 39 | 29 | 38 | 18 | 43 | 49 |
| 46 | 44 | 48 | 35 | 36 | 30 |

6. **GEOGRAPHY** The table shows the land area of each county in Wyoming. Round each area to the nearest hundred square miles and organize the data in a stem-and-leaf plot.

| County | Area $(mi)^2$ | County | Area $(mi)^2$ | County | Area $(mi)^2$ |
|---|---|---|---|---|---|
| Albany | 4273 | Hot Springs | 2004 | Sheridan | 2523 |
| Big Horn | 3137 | Johnson | 4166 | Sublette | 4883 |
| Campbell | 4797 | Laramie | 2686 | Sweetwater | 10,425 |
| Carbon | 7896 | Lincoln | 4069 | Teton | 4008 |
| Converse | 4255 | Natrona | 5340 | Unita | 2082 |
| Crook | 2859 | Niobrara | 2626 | Washakie | 2240 |
| Fremont | 9182 | Park | 6942 | Weston | 2398 |
| Goshen | 2225 | Platte | 2085 | | |

**Source:** *The World Almanac*

## 9 Box-and-Whisker Plots

In a set of data, **quartiles** are values that divide the data into four equal parts.

median

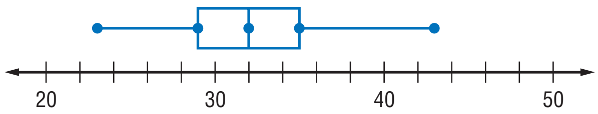
lower half — upper half

23 24 29 30 31 | 33 34 35 39 43

The median of the lower half of a set of data is the **lower quartile**, or LQ.

The median of the upper half of a set of data is the **upper quartile**, or UQ.

To make a **box-and-whisker plot**, draw a box around the quartile values, and lines or *whiskers* to represent the values in the lower fourth of the data and the upper fourth of the data.

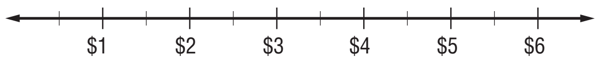



### EXAMPLE

**1** **MONEY** **The amount spent in the cafeteria by 20 students is shown. Display the data in a box-and-whisker plot.**

| Amount Spent | | | |
|---|---|---|---|
| \$2.00 | \$2.00 | \$1.00 | \$4.00 |
| \$1.00 | \$2.50 | \$2.50 | \$2.00 |
| \$2.50 | \$1.00 | \$4.00 | \$2.50 |
| \$3.50 | \$2.00 | \$3.00 | \$2.50 |
| \$4.00 | \$4.00 | \$5.50 | \$1.50 |

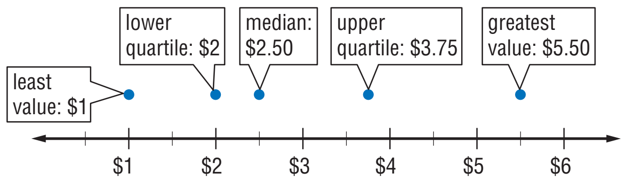
**Step 1** Find the least and greatest number. Then draw a number line that covers the range of the data. In this case, the least value is 1 and the greatest value is 5.5.



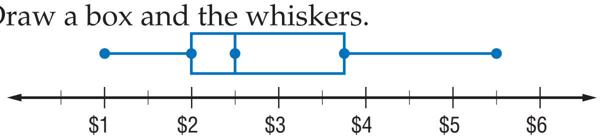

**Step 2** Find the median, the extreme values, and the upper and lower quartiles. Mark these points above the number line.

1, 1, 1, 1.5, 2, 2, 2, 2, 2.5, 2.5, 2.5, 2.5, 2.5, 3, 3.5, 4, 4, 4, 4, 5.5

$LQ = \frac{2 + 2}{2}$ or 2 $M = \frac{2.5 + 2.5}{2}$ or 2.5 $UQ = \frac{3.5 + 4}{2}$ or 3.75




**Step 3** Draw a box and the whiskers.




The **interquartile range (IQR)** is the range of the middle half of the data and contains 50% of the data in the set.

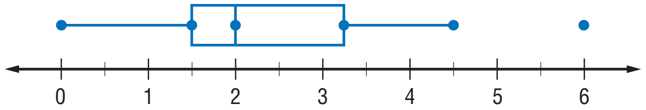
Interquartile range = UQ − LQ

The interquartile range of the data in Example 1 is 3.75 − 2 or 1.75.

An **outlier** is any element of a set that is at least 1.5 interquartile ranges less than the lower quartile or greater than the upper quartile. The whisker representing the data is drawn from the box to the least or greatest value that is not an outlier.

**EXAMPLE**

**2** **SCHOOL** **The number of hours José studied each day for the last month is shown in the box-and-whisker plot below.**



**a. What percent of the data lies between 1.5 and 3.25?**
The value 1.5 is the lower quartile and 3.25 is the upper quartile. The values between the lower and upper quartiles represent 50% of the data.

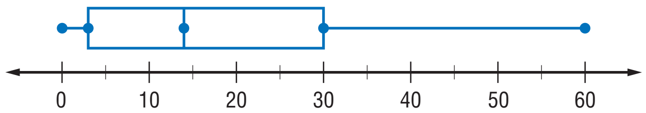
**b. What was the greatest amount of time José studied in a day?**
The greatest value in the plot is 6, so the greatest amount of time José studied in a day was 6 hours.

**c. What is the interquartile range of this box-and-whisker plot?**
The interquartile range is UQ − LQ. For this plot, the interquartile range is 3.25 − 1.5 or 1.75 hours.

**d. Identify any outliers in the data.**
An outlier is at least 1.5(1.75) less than the lower quartile or more than the upper quartile. Since $3.25 + (1.5)(1.75) = 5.875$, and $6 > 5.875$, the value 6 is an outlier, and was not included in the whisker.

## Exercises

**DRIVING** **For Exercises 1–3, use the following information.**
Tyler surveyed 20 randomly chosen students at his school about how many miles they drive in an average day. The results are shown in the box-and-whisker plot.



1. What percent of the students drive more than 30 miles in a day?
2. What is the interquartile range of the box-and-whisker plot?
3. Does a student at Tyler's school have a better chance to meet someone who drives the same mileage they do if they drive 50 miles in a day or 15 miles in a day? Why?
4. **SOFT DRINKS** Carlos surveyed his friends to find the number of cans of soft drink they drink in an average week. Make a box-and-whisker plot of the data.
{0, 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 5, 7, 10, 10, 10, 11, 11}
5. **BASEBALL** The table shows the number of sacrifice hits made by teams in the National Baseball League in one season. Make a box-and-whisker plot of the data.
6. **ANIMALS** The average life span of some animals commonly found in a zoo are as follows: {1, 7, 7, 10, 12, 12, 15, 15, 18, 20, 20, 20, 25, 40, 100}. Make a box-and-whisker plot of the data.

| Team | Home Runs | Team | Home Runs |
|---|---|---|---|
| Arizona | 71 | Milwaukee | 65 |
| Atlanta | 64 | Montreal | 64 |
| Chicago | 117 | New York | 52 |
| Cincinnati | 66 | Philadelphia | 67 |
| Colorado | 81 | Pittsburgh | 60 |
| Florida | 60 | San Diego | 29 |
| Houston | 71 | San Francisco | 67 |
| Los Angeles | 57 | St. Louis | 83 |

Source: ESPN

# Extra Practice

## Lesson 1-1

(pages 6–10)

**Evaluate each expression if $q = \frac{1}{2}$, $r = 1.2$, $s = -6$, and $t = 5$.**

1. $qr - st$
2. $qr \div st$
3. $qrst$
4. $qr + st$
5. $\frac{3q}{4s}$
6. $\frac{5qr}{t}$
7. $\frac{2r(4s - 1)}{t}$
8. $\frac{4q^3s + 1}{t - 1}$

**Evaluate each expression if $a = -0.5$, $b = 4$, $c = 5$, and $d = -3$.**

9. $3b + 4d$
10. $ab^2 + c$
11. $bc + d \div a$
12. $7ab - 3d$
13. $ad + b^2 - c$
14. $\frac{4a + 3c}{3b}$
15. $\frac{3ab^2 - d^3}{a}$
16. $\frac{5a + ad}{bc}$

## Lesson 1-2

(pages 11–17)

**Name the sets of numbers to which each number belongs. (Use N, W, Z, Q, I, and R.)**

1. 8.2
2. $-9$
3. $\sqrt{36}$
4. $-\frac{1}{3}$
5. $\sqrt{2}$
6. $-0.\overline{24}$

**Name the property illustrated by each equation.**

7. $(4 + 9a)2b = 2b(4 + 9a)$
8. $3\left(\frac{1}{3}\right) = 1$
9. $a(3 - 2) = a \cdot 3 - a \cdot 2$
10. $(-3b) + 3b = 0$
11. $jk + 0 = jk$
12. $(2a)b = 2(ab)$

**Simplify each expression.**

13. $7s + 9t + 2s - 7t$
14. $6(2a + 3b) + 5(3a - 4b)$
15. $4(3x - 5y) - 8(2x + y)$
16. $0.2(5m - 8) + 0.3(6 - 2m)$
17. $\frac{1}{2}(7p + 3q) + \frac{3}{4}(6p - 4q)$
18. $\frac{4}{5}(3v - 2w) - \frac{1}{5}(7v - 2w)$

## Lesson 1-3

(pages 18–26)

**Write an algebraic expression to represent each verbal expression.**

1. twelve decreased by the square of a number
2. twice the sum of a number and negative nine
3. the product of the square of a number and 6
4. the square of the sum of a number and 11

**Name the property illustrated by each statement.**

5. If $a + 1 = 6$, then $3(a + 1) = 3(6)$.
6. If $x + (4 + 5) = 21$, then $x + 9 = 21$.
7. If $7x = 42$, then $7x - 5 = 42 - 5$.
8. If $3 + 5 = 8$ and $8 = 2 \cdot 4$, then $3 + 5 = 2 \cdot 4$.

**Solve each equation. Check your solution.**

9. $5t + 8 = 88$
10. $27 - x = -4$
11. $\frac{3}{4}y = \frac{2}{3}y + 5$
12. $8s - 3 = 5(2s + 1)$
13. $3(k - 2) = k + 4$
14. $0.5z + 10 = z + 4$
15. $8q - \frac{q}{3} = 46$
16. $-\frac{2}{7}r + \frac{3}{7} = 5$
17. $d - 1 = \frac{1}{2}(d - 2)$

**Solve each equation or formula for the specified variable.**

18. $C = \pi r$; for $r$
19. $I = Prt$, for $t$
20. $m = \frac{n - 2}{n}$, for $n$

## Lesson 1-4
(pages 27–31)

**Evaluate each expression if $x = -5$, $y = 3$, and $z = -2.5$.**

1. $|2x|$
2. $|-3y|$
3. $|2x + y|$
4. $|y + 5z|$
5. $-|x + z|$
6. $8 - |5y - 3|$
7. $2|x| - 4|2 + y|$
8. $|x + y| - 6|z|$

**Solve each equation. Check your solutions.**

9. $|d + 1| = 7$
10. $|a - 6| = 10$
11. $2|x - 5| = 22$
12. $|t + 9| - 8 = 5$
13. $|p + 1| + 10 = 5$
14. $6|g - 3| = 42$
15. $2|y + 4| = 14$
16. $|3b - 10| = 2b$
17. $|3x + 7| + 4 = 0$
18. $|2c + 3| - 15 = 0$
19. $7 - |m - 1| = 3$
20. $3 + |z + 5| = 10$
21. $2|2d - 7| + 1 = 35$
22. $|3t + 6| + 9 = 30$
23. $|d - 3| = 2d + 9$
24. $|4y - 5| + 4 = 7y + 8$
25. $|2b + 4| - 3 = 6b + 1$
26. $|5t| + 2 = 3t + 18$

## Lesson 1-5
(pages 33–39)

**Solve each inequality. Then graph the solution set on a number line.**

1. $2z + 5 \leq 7$
2. $3r - 8 > 7$
3. $0.75b < 3$
4. $-3x > 6$
5. $2(3f + 5) \geq 28$
6. $-33 > 5g + 7$
7. $-3(y - 2) \geq -9$
8. $7a + 5 > 4a - 7$
9. $5(b - 3) \leq b - 7$
10. $3(2x - 5) < 5(x - 4)$
11. $8(2c - 1) > 11c + 22$
12. $2(d + 4) - 5 \geq 5(d + 3)$
13. $8 - 3t < 4(3 - t)$
14. $-x \geq \frac{x + 4}{7}$
15. $\frac{a + 8}{4} \leq \frac{7 + a}{3}$
16. $-y < \frac{y + 5}{2}$
17. $5(x - 1) - 4x \geq 3(3 - x)$
18. $6s - (4s + 7) > 5 - s$

**Define a variable and write an inequality for each problem. Then solve.**

19. The product of 7 and a number is greater than 42.
20. The difference of twice a number and 3 is at most 11.
21. The product of $-10$ and a number is greater than or equal to 20.
22. Thirty increased by a number is less than twice the number plus three.

## Lesson 1-6
(pages 41–48)

**Write an absolute value inequality for each of the following. Then graph the solution set on a number line.**

1. all numbers less than $-9$ and greater than 9
2. all numbers between $-5.5$ and 5.5
3. all numbers greater than or equal to $-2$ and less than or equal to 2

**Solve each inequality. Graph the solution set on a number line.**

4. $3m - 2 < 7$ or $2m + 1 > 13$
5. $2 < n + 4 < 7$
6. $-3 \leq s - 2 \leq 5$
7. $5t + 3 \leq -7$ or $5t - 2 \geq 8$
8. $7 \leq 4x + 3 \leq 19$
9. $4x + 7 < 5$ or $2x - 4 > 12$
10. $|7x| \geq 21$
11. $|8p| \leq 16$
12. $|7d| \geq -42$
13. $|a + 3| < 1$
14. $|t - 4| > 1$
15. $|2y - 5| < 3$
16. $|3d + 6| \geq 3$
17. $|4x - 1| < 5$
18. $|6v + 12| > 18$
19. $|2r + 4| < 6$
20. $|5w - 3| \geq 9$
21. $|z + 2| \geq 0$
22. $12 + |2q| < 0$
23. $|3h| + 15 < 0$
24. $|5n - 16| \geq 4$

## Lesson 2-1

(pages 58–64)

**State the domain and range of each relation. Then determine whether each relation is a function. Write *yes* or *no*.**

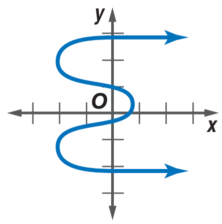
1.

| Year | Population |
|---|---|
| 1970 | 11,605 |
| 1980 | 13,468 |
| 1990 | 15,630 |
| 2000 | 18,140 |

2.

| $x$ | $y$ |
|---|---|
| 1 | 5 |
| 2 | 5 |
| 3 | 5 |
| 4 | 5 |

3.




**Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether *discrete* or *continuous*.**

4. $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$
5. $\{(0, 3), (0, 2), (0, 1), (0, 0)\}$
6. $y = -x$
7. $y = 2x - 1$
8. $y = 2x^2$
9. $y = -x^2$

**Find each value if $f(x) = x + 7$ and $g(x) = (x + 1)^2$.**

10. $f(2)$
11. $f(-4)$
12. $f(a + 2)$
13. $g(4)$
14. $g(-2)$
15. $f(0.5)$
16. $g(b - 1)$
17. $g(3c)$

## Lesson 2-2

(pages 66–70)

**State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.**

1. $\frac{x}{2} - y = 7$
2. $\sqrt{x} = y + 5$
3. $g(x) = \frac{2}{x - 3}$
4. $f(x) = 7$

**Write each equation in standard form. Identify *A*, *B*, and *C*.**

5. $x + 7 = y$
6. $x = -3y$
7. $5x = 7y + 3$
8. $y = \frac{2}{3}x + 8$
9. $-0.4x = 10$
10. $0.75y = -6$

**Find the $x$-intercept and the $y$-intercept of the graph of each equation. Then graph the equation.**

11. $2x + y = 6$
12. $3x - 2y = -12$
13. $y = -x$
14. $x = 3y$
15. $\frac{3}{4}y - x = 1$
16. $y = -3$

## Lesson 2-3

(pages 71–77)

**Find the slope of the line that passes through each pair of points.**

1. $(0, 3), (5, 0)$
2. $(2, 3), (5, 7)$
3. $(2, 8), (2, -8)$
4. $(1.5, -1), (3, 1.5)$
5. $\left(-\frac{1}{2}, \frac{3}{5}\right), \left(\frac{3}{10}, -\frac{1}{4}\right)$
6. $(-3, c), (4, c)$

**Graph the line passing through the given point with the given slope.**

7. $(0, 3); 1$
8. $(2, 3); 0$
9. $(-1, 1); -\frac{1}{3}$

**Graph the line that satisfies each set of conditions.**

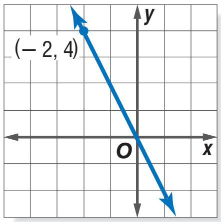
10. passes through $(0, 1)$, parallel to a line with a slope of $-2$
11. passes through $(4, -5)$, perpendicular to the graph of $-2x + 5y = 1$

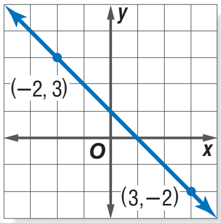
Extra Practice

## Lesson 2-4

(pages 79–84)

**Write an equation in slope-intercept form for each graph.**

1. 


2. 


**Write an equation in slope-intercept form for the line that satisfies each set of conditions.**

3. slope $-1$, passes through $(7, 2)$
4. slope $\frac{3}{4}$, passes through the origin
5. passes through $(1, -3)$ and $(-1, 2)$
6. $x$-intercept $-5$, $y$-intercept 2
7. passes through $(1, 1)$, parallel to the graph of $2x + 3y = 5$
8. passes through $(0, 0)$, perpendicular to the graph of $2y + 3x = 4$

## Lesson 2-5

(pages 86–91)

**Complete parts a–c for each set of data in Exercises 1–3.**

a. **Draw a scatter plot and describe the correlation.**
b. **Use two ordered pairs to write a prediction equation.**
c. **Use your prediction equation to predict the missing value.**

1.

| Telephone Costs | |
|---|---|
| Minutes | Cost ($) |
| 1 | 0.20 |
| 3 | 0.52 |
| 4 | 0.68 |
| 6 | 1.00 |
| 9 | 1.48 |
| 15 | ? |

2.

| Washington | |
|---|---|
| Year | Population |
| 1960 | 2,853,214 |
| 1970 | 3,413,244 |
| 1980 | 4,132,353 |
| 1990 | 4,866,669 |
| 2000 | 5,894,121 |
| 2010 | ? |

**Source:** *The World Almanac*

3.

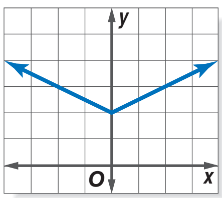
| Federal Minimum Wage | |
|---|---|
| Year | Wage |
| 1981 | $3.35 |
| 1990 | $3.80 |
| 1991 | $4.25 |
| 1996 | $4.75 |
| 1997 | $5.15 |
| 2015 | ? |

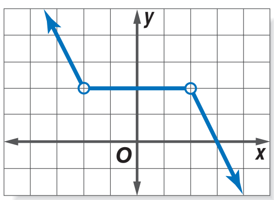
**Source:** *The World Almanac*

## Lesson 2-6

(pages 95–101)

**Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.**

1. 


2. 


**Graph each function. Identify the domain and range.**

3. $f(x) = [\![x + 5]\!]$
4. $g(x) = [\![x]\!] - 2$
5. $f(x) = -2[\![x]\!]$
6. $h(x) = |x| - 3$
7. $h(x) = |x - 1|$
8. $g(x) = |2x| + 2$
9. $h(x) = \begin{cases} x \text{ if } x < -2 \\ 4 \text{ if } x \geq -2 \end{cases}$
10. $f(x) = \begin{cases} -3 \text{ if } x \leq 1 \\ -x \text{ if } x > 1 \end{cases}$

## Lesson 2-7

(pages 102–105)

**Graph each inequality.**

1. $y \geq x - 2$
2. $y < -3x - 1$
3. $4y \leq -3x + 8$
4. $3x > y$
5. $x + 2 \geq y - 7$
6. $2x < 5 - y$
7. $y > \frac{1}{5}x - 8$
8. $2y - 5x \leq 8$
9. $-2x + 5 \leq \frac{2}{3}y$
10. $3x + 2y \geq 0$
11. $x \leq 2$
12. $\frac{y}{2} \leq x - 1$
13. $y - 3 < 5$
14. $y \geq -|x|$
15. $|x| \leq y + 3$
16. $y > |5x - 3|$
17. $y \leq |8 - x|$
18. $y < |x + 3| - 1$
19. $y + |2x| \geq 4$
20. $y \geq |2x - 1| + 5$
21. $y < \left|\frac{2x}{3}\right| - 1$

## Lesson 3-1

(pages 116–122)

**Solve each system of equations by graphing or by completing a table.**

1. $x + 3y = 18$; $-x + 2y = 7$
2. $x - y = 2$; $2x - 2y = 10$
3. $2x + 6y = 6$; $\frac{1}{3}x + y = 1$
4. $x + 3y = 0$; $2x + 6y = 5$
5. $2x - y = 7$; $\frac{2}{5}x - \frac{4}{3}y = -2$
6. $y = \frac{1}{3}x + 1$; $y = 4x + 1$

**Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

7. $2x + 3y = 5$; $-6x - 9y = -15$
8. $x - 2y = 4$; $y = x - 2$
9. $y = 0.5x$; $2y = x + 4$
10. $9x - 5 = 7y$; $4.5x - 3.5y = 2.5$
11. $\frac{3}{4}x - y = 0$; $\frac{1}{3}y + \frac{1}{2}x = 6$
12. $\frac{2}{3}x = \frac{5}{3}y$; $2x - 5y = 0$

## Lesson 3-2

(pages 123–129)

**Solve each system of equations by using substitution.**

1. $2x + 3y = 10$; $x + 6y = 32$
2. $x = 4y - 10$; $5x + 3y = -4$
3. $3x - 4y = -27$; $2x + y = -7$

**Solve each system of equations by using elimination.**

4. $7x + y = 9$; $5x - y = 15$
5. $r + 5s = -17$; $2r - 6s = -2$
6. $6p + 8q = 20$; $5p - 4q = -26$

**Solve each system of equations by using either substitution or elimination.**

7. $2x - 3y = 7$; $3x + 6y = 42$
8. $2a + 5b = -13$; $3a - 4b = 38$
9. $3c + 4d = -1$; $6c - 2d = 3$
10. $7x - y = 35$; $y = 5x - 19$
11. $3m + 4n = 28$; $5m - 3n = -21$
12. $x = 2y - 1$; $4x - 3y = 21$
13. $2.5x + 1.5y = -2$; $3.5x - 0.5y = 18$
14. $\frac{5}{2}x + \frac{1}{3}y = 13$; $\frac{1}{2}x - y = -7$
15. $\frac{2}{7}c - \frac{4}{3}d = 16$; $\frac{4}{7}c + \frac{8}{3}d = -16$

Extra Practice

Extra Practice

## Lesson 3-3

(pages 130–135)

**Solve each system of inequalities.**

1. $x \leq 5$
   $y \geq -3$
2. $y < 3$
   $y - x \geq -1$
3. $x + y < 5$
   $x < 2$
4. $y + x < 2$
   $y \geq x$
5. $x + y \leq 2$
   $y - x \leq 4$
6. $y \leq x + 4$
   $y - x \geq 1$
7. $y < \frac{1}{3}x + 5$
   $y > 2x + 1$
8. $y + x \geq 1$
   $y - x \geq -1$
9. $|x| > 2$
   $|y| \leq 5$
10. $|x - 3| \leq 3$
    $4y - 2x \leq 6$
11. $4x + 3y \geq 12$
    $2y - x \geq -1$
12. $y \leq -1$
    $3x - 2y \geq 6$

**Find the coordinates of the vertices of the figure formed by each system of inequalities.**

13. $y \leq 3$
    $x \leq 2$
    $y \geq -\frac{3}{2}x + 3$
14. $y \geq -1$
    $y \leq x$
    $y \leq -x + 4$
15. $y \leq \frac{1}{3}x + \frac{7}{3}$
    $4x - y \leq 5$
    $y \geq -\frac{3}{2}x + \frac{1}{2}$

## Lesson 3-4

(pages 138–144)

**A feasible region has vertices at (−3, 2), (1, 3), (6, 1), and (2, −2). Find the maximum and minimum values of each function.**

1. $f(x, y) = 2x - y$
2. $f(x, y) = x + 5y$
3. $f(x, y) = y - 4x$
4. $f(x, y) = -x + 3y$
5. $f(x, y) = 3x - y$
6. $f(x, y) = 2y - 2x$

**Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.**

7. $4x - 5y \leq -10$
   $y \leq 6$
   $2x + y \geq 2$
   $f(x, y) = x + y$
8. $x \leq 5$
   $y \geq 2$
   $2x - 5y \geq -10$
   $f(x, y) = 3x + y$
9. $x - 2y \geq -7$
   $x + y \leq 8$
   $y \geq 5x + 8$
   $f(x, y) = 3x - 4y$
10. $y \leq 4x + 6$
    $x + 4y \geq 7$
    $2x + y \leq 7$
    $f(x, y) = 2x - y$
11. $y \geq 0$
    $y \leq 5$
    $y \leq -x + 7$
    $5x + 3y \geq 20$
    $f(x, y) = x + 2y$
12. $y \geq 0$
    $3x - 2y \geq 0$
    $x + 3y \leq 11$
    $2x + 3y \leq 16$
    $f(x, y) = 4x + y$

## Lesson 3-5

(pages 145–152)

**For each system of equations, an ordered triple is given. Determine whether or not it is a solution of the system.**

1. $4x + 2y - 6z = -38$
   $5x - 4y + z = -18$
   $x + 3y + 7z = 38$;
   $(-3, 2, 5)$
2. $u + 3v + w = 14$
   $2u - v + 3w = -9$
   $4u - 5v - 2w = -2$;
   $(1, 5, -2)$
3. $x + y = -6$
   $x + z = -2$
   $y + z = 2$;
   $(-4, -2, 2)$

**Solve each system of equations.**

4. $5a = 5$
   $6b - 3c = 15$
   $2a + 7c = -5$
5. $s + 2t = 5$
   $7r - 3s + t = 20$
   $2t = 8$
6. $2u - 3v = 13$
   $3v + w = -3$
   $4u - w = 2$
7. $4a + 2b - c = 5$
   $2a + b - 5c = -11$
   $a - 2b + 3c = 6$
8. $x + 2y - z = 1$
   $x + 3y + 2z = 7$
   $2x + 6y + z = 8$
9. $2x + y - z = 7$
   $3x - y + 2z = 15$
   $x - 4y + z = 2$

## Lesson 4-1

(pages 162–167)

**Solve each matrix equation.**

**1.** $[2x \quad 3y \quad -z] = [2y \quad -z \quad 15]$

**2.** $\begin{bmatrix} x+y \\ 4x-3y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

**3.** $-2\begin{bmatrix} w+5 & x-z \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 6 & 2x+8z \end{bmatrix}$

**4.** $y\begin{bmatrix} 2 & x \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -10 \\ 10 & 2z \end{bmatrix}$

**5.** $\begin{bmatrix} 2x \\ -y \\ 3z \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ -21 \end{bmatrix}$

**6.** $\begin{bmatrix} x-3y \\ 4y-3x \end{bmatrix} = -5\begin{bmatrix} 2 \\ x \end{bmatrix}$

**7.** $\begin{bmatrix} x^2+4 & y+6 \\ x-y & 2-y \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 0 & 1 \end{bmatrix}$

**8.** $\begin{bmatrix} x+y & 3 \\ y & 6 \end{bmatrix} = \begin{bmatrix} 0 & 2y-x \\ z & 4-2x \end{bmatrix}$

## Lesson 4-2

(pages 169–176)

**Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.**

**1.** $\begin{bmatrix} 3 & 5 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 8 & -1 \end{bmatrix}$

**2.** $[0 \quad -1 \quad 3] + \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$

**3.** $\begin{bmatrix} 45 & 36 & 18 \\ 63 & 29 & 5 \end{bmatrix} - \begin{bmatrix} 45 & -2 & 36 \\ 18 & 9 & -10 \end{bmatrix}$

**4.** $4[-8 \quad 2 \quad 9] - 3[2 \quad -7 \quad 6]$

**5.** $5\begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} - 2\begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} + 4\begin{bmatrix} 7 & -6 \\ -4 & 2 \end{bmatrix}$

**6.** $1.3\begin{bmatrix} 3.7 \\ -5.4 \end{bmatrix} + 4.1\begin{bmatrix} 6.4 \\ -3.7 \end{bmatrix} - 6.2\begin{bmatrix} -0.8 \\ 7.4 \end{bmatrix}$

**Use matrices *A, B, C, D,* and *E* to find the following.**

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix}, D = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}, E = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$

**7.** $A + B$ **8.** $C + D$ **9.** $A - B$ **10.** $4B$

**11.** $D - C$ **12.** $E + 2A$ **13.** $D - 2B$ **14.** $2A + 3E - D$

## Lesson 4-3

(pages 177–184)

**Find each product, if possible.**

**1.** $[-3 \quad 4] \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

**2.** $\begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix}$

**3.** $\begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix}$

**4.** $\begin{bmatrix} 3 & 2 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 15 \end{bmatrix}$

**5.** $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 6 & 1 \\ 2 & -4 & 0 \end{bmatrix}$

**6.** $\begin{bmatrix} 0 & 1 & -2 \\ 5 & 3 & -4 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix}$

**7.** $\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**8.** $\begin{bmatrix} -1 & 0 & 2 \\ -6 & 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$

## Lesson 4-4

(pages 185–192)

**For Exercises 1–3, use the following information.**

The vertices of quadrilateral $ABCD$ are $A(1, 1)$, $B(-2, 3)$, $C(-4, -1)$, and $D(2, -3)$. The quadrilateral is dilated so that its perimeter is 2 times the original perimeter.

1. Write the coordinates for $ABCD$ in a vertex matrix.
2. Find the coordinates of the image $A'B'C'D'$.
3. Graph $ABCD$ and $A'B'C'D'$.

**For Exercises 4–10, use the following information.**

The vertices of $\triangle MQN$ are $M(2, 4)$, $Q(3, -5)$, and $N(1, -1)$.

4. Write the coordinates of $\triangle MQN$ in a vertex matrix.
5. Write the reflection matrix for reflecting over the line $y = x$.
6. Find the coordinates of $\triangle M'Q'N'$ after the reflection.
7. Graph $\triangle MQN$ and $\triangle M'Q'N'$.
8. Write a rotation matrix for rotating $\triangle MQN$ 90° counterclockwise about the origin.
9. Find the coordinates of $\triangle M'Q'N'$ after the rotation.
10. Graph $\triangle MQN$ and $\triangle M'Q'N'$.

## Lesson 4-5

(pages 194–200)

**Evaluate each determinant using expansion by minors.**

1. $\begin{vmatrix} 2 & -3 & 5 \\ 1 & -2 & -7 \\ -1 & 4 & -3 \end{vmatrix}$
2. $\begin{vmatrix} 0 & -1 & 2 \\ -2 & 1 & 0 \\ 2 & 0 & -1 \end{vmatrix}$
3. $\begin{vmatrix} 4 & 3 & -2 \\ 2 & 5 & -8 \\ 6 & 4 & -1 \end{vmatrix}$
4. $\begin{vmatrix} -3 & 0 & 2 \\ 1 & -2 & -1 \\ 0 & 5 & 0 \end{vmatrix}$

**Evaluate each determinant using diagonals.**

5. $\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 3 \end{vmatrix}$
6. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
7. $\begin{vmatrix} 6 & 4 & -1 \\ 2 & 5 & -8 \\ 4 & 3 & -2 \end{vmatrix}$
8. $\begin{vmatrix} 6 & 12 & 15 \\ 9 & 3 & 14 \\ 5 & 6 & 3 \end{vmatrix}$

## Lesson 4-6

(pages 201–207)

**Use Cramer's Rule to solve each system of equations.**

1. $5x - 3y = 19$
   $7x + 2y = 8$
2. $4p - 3q = 22$
   $2p + 8q = 30$
3. $-x + y = 5$
   $2x + 4y = 38$
4. $\frac{1}{3}x - \frac{1}{2}y = -8$
   $\frac{3}{5}x + \frac{5}{6}y = -4$
5. $\frac{1}{4}c + \frac{2}{3}d = 6$
   $\frac{3}{4}c - \frac{5}{3}d = -4$
6. $0.3a + 1.6b = 0.44$
   $0.4a + 2.5b = 0.66$
7. $x + y + z = 6$
   $2x - y - z = -3$
   $3x + y - 2z = -1$
8. $2a + b - c = -6$
   $a - 2b + c = 8$
   $-a - 3b + 2c = 14$
9. $r + 2s - t = 10$
   $-2r + 3s + t = 6$
   $3r - 2s + 2t = -19$

## Lesson 4-7

(pages 208–215)

**Determine whether each pair of matrices are inverses.**

1. $A = \begin{bmatrix} -7 & -6 \\ 8 & 7 \end{bmatrix}, B = \begin{bmatrix} -7 & -6 \\ 8 & 7 \end{bmatrix}$
2. $C = \begin{bmatrix} -3 & 4 \\ 2 & -2 \end{bmatrix}, D = \begin{bmatrix} -2 & -2 \\ -4 & -3 \end{bmatrix}$
3. $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
4. $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, M = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

**Find the inverse of each matrix, if it exists.**

5. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$
6. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$
7. $\begin{bmatrix} 3 & 8 \\ 0 & -1 \end{bmatrix}$
8. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$
9. $\begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}$
10. $\begin{bmatrix} 8 & -5 \\ -6 & 4 \end{bmatrix}$
11. $\begin{bmatrix} 10 & 3 \\ 5 & -2 \end{bmatrix}$
12. $\begin{bmatrix} -3 & 4 \\ -4 & 8 \end{bmatrix}$

## Lesson 4-8

(pages 216–222)

**Write a matrix equation for each system of equations.**

1. $5a + 3b = 6$
   $2a - b = 9$
2. $3x + 4y = -8$
   $2x - 3y = 6$
3. $m + 3n = 1$
   $4m - n = -22$
4. $4c - 3d = -1$
   $5c - 2d = 39$
5. $x + 2y - z = 6$
   $-2x + 3y + z = 1$
   $x + y + 3z = 8$
6. $2a - 3b - c = 4$
   $4a + b + c = 15$
   $a - b - c = -2$

**Solve each matrix equation or system of equations.**

7. $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33 \\ -1 \end{bmatrix}$
8. $\begin{bmatrix} -1 & 1 \\ 7 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$
9. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -29 \\ 52 \end{bmatrix}$
10. $5x - y = 7$
    $8x + 2y = 4$
11. $3m + n = 4$
    $2m + 2n = 3$
12. $6c + 5d = 7$
    $3c - 10d = -4$
13. $3a - 5b = 1$
    $a + 3b = 5$
14. $2r - 7s = 24$
    $-r + 8s = -21$
15. $x + y = -3$
    $3x - 10y = 43$
16. $2m - 3n = 3$
    $-4m + 9n = -8$
17. $x + y = 1$
    $2x - 2y = -12$

## Lesson 5-1

(pages 236–244)

**For Exercises 1–12, complete parts a–c for each quadratic function.**

a. **Find the $y$-intercept, the equation of the axis of symmetry, and the $x$-coordinate of the vertex.**
b. **Make a table of values that includes the vertex.**
c. **Use this information to graph the function.**

1. $f(x) = 6x^2$
2. $f(x) = -x^2$
3. $f(x) = x^2 + 5$
4. $f(x) = -x^2 - 2$
5. $f(x) = 2x^2 + 1$
6. $f(x) = -3x^2 + 6x$
7. $f(x) = x^2 + 6x - 3$
8. $f(x) = x^2 - 2x - 8$
9. $f(x) = -3x^2 - 6x + 12$
10. $f(x) = x^2 + 5x - 6$
11. $f(x) = 2x^2 + 7x - 4$
12. $f(x) = -5x^2 + 10x + 1$

**Determine whether each function has a maximum or a minimum value and find the maximum or minimum value. Then state the domain and range of the function.**

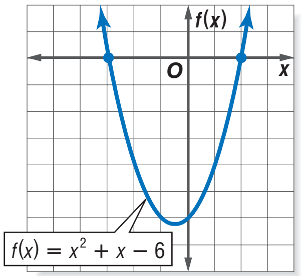
13. $f(x) = 9x^2$
14. $f(x) = 9 - x^2$
15. $f(x) = x^2 - 5x + 6$
16. $f(x) = 2 + 7x - 6x^2$
17. $f(x) = 4x^2 - 9$
18. $f(x) = x^2 + 2x + 1$
19. $f(x) = 8 - 3x - 4x^2$
20. $f(x) = x^2 - x + \frac{5}{4}$
21. $f(x) = -x^2 + \frac{14}{3}x + \frac{5}{3}$

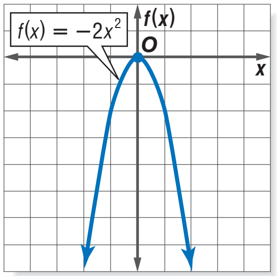
## Lesson 5-2

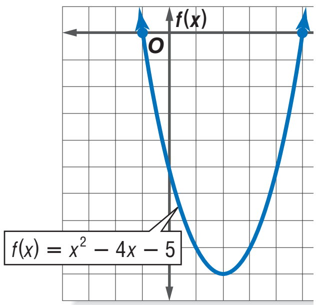
(pages 246–251)

**Use the related graph of each equation to determine its solutions.**

1. $x^2 + x - 6 = 0$
2. $-2x^2 = 0$
3. $x^2 - 4x - 5 = 0$




**Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

4. $x^2 - 2x = 0$
5. $x^2 + 8x - 20 = 0$
6. $-2x^2 + 10x - 5 = 0$
7. $-5x + 2x^2 - 3 = 0$
8. $3x^2 - x + 8 = 0$
9. $-x^2 + 2 = 7x$
10. $4x^2 - 4x + 1 = 0$
11. $4x + 1 = 3x^2$
12. $x^2 = -9x$
13. $x^2 + 6x - 27 = 0$
14. $0.4x^2 + 1 = 0$
15. $0.5x^2 + 3x - 2 = 0$

## Lesson 5-3

(pages 253–258)

**Solve each equation by factoring.**

1. $x^2 + 7x + 10 = 0$
2. $3x^2 = 75x$
3. $2x^2 + 7x = 9$
4. $8x^2 = 48 - 40x$
5. $5x^2 = 20x$
6. $16x^2 - 64 = 0$
7. $24x^2 - 15 = 2x$
8. $x^2 = 72 - x$
9. $4x^2 + 9 = 12x$
10. $2x^2 - 8x = 0$
11. $8x^2 + 10x = 3$
12. $12x^2 - 5x = 3$
13. $x^2 + 9x + 14 = 0$
14. $9x^2 + 1 = 6x$
15. $6x^2 + 7x = 3$
16. $x^2 - 4x = 21$

**Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are integers.**

17. $2, 1$
18. $-3, 4$
19. $-1, -7$
20. $-1, \frac{1}{2}$
21. $-5, \frac{1}{4}$
22. $-\frac{1}{3}, -\frac{1}{2}$

## Lesson 5-4

(pages 259–266)

**Simplify.**

1. $\sqrt{-289}$
2. $\sqrt{-\frac{25}{121}}$
3. $\sqrt{-625b^8}$
4. $\sqrt{-\frac{28t^6}{27s^5}}$
5. $(7i)^2$
6. $(6i)(-2i)(11i)$
7. $(\sqrt{-8})(\sqrt{-12})$
8. $-i^{22}$
9. $i^{17} \cdot i^{12} \cdot i^{26}$
10. $(14 - 5i) + (-8 + 19i)$
11. $(7i) - (2 + 3i)$
12. $(2 + 2i) - (5 + i)$
13. $(7 + 3i)(7 - 3i)$
14. $(8 - 2i)(5 + i)$
15. $(6 + 8i)^2$
16. $\frac{3}{6 - 2i}$
17. $\frac{5i}{3 + 4i}$
18. $\frac{3 - 7i}{5 + 4i}$

**Solve each equation.**

19. $x^2 + 8 = 3$
20. $\frac{4x^2}{49} + 6 = 3$
21. $8x^2 + 5 = 1$
22. $12 - 9x^2 = 38$
23. $9x^2 + 7 = 4$
24. $\frac{1}{2}x^2 + 1 = 0$

## Lesson 5-5

(pages 268–275)

**Find the value of $c$ that makes each trinomial a perfect square. Then write the trinomial as a perfect square.**

1. $x^2 - 4x + c$
2. $x^2 + 20x + c$
3. $x^2 - 11x + c$
4. $x^2 - \frac{2}{3}x + c$
5. $x^2 + 30x + c$
6. $x^2 + \frac{3}{8}x + c$
7. $x^2 - \frac{2}{5}x + c$
8. $x^2 - 3x + c$

**Solve each equation by completing the square.**

9. $x^2 + 3x - 4 = 0$
10. $x^2 + 5x = 0$
11. $x^2 + 2x - 63 = 0$
12. $3x^2 - 16x - 35 = 0$
13. $x^2 + 7x + 13 = 0$
14. $5x^2 - 8x + 2 = 0$
15. $x^2 - 6x + 11 = 0$
16. $x^2 - 12x + 36 = 0$
17. $8x^2 + 13x - 4 = 0$
18. $3x^2 + 5x + 6 = 0$
19. $x^2 + 14x - 1 = 0$
20. $4x^2 - 32x + 15 = 0$
21. $3x^2 - 11x - 4 = 0$
22. $x^2 + 8x - 84 = 0$
23. $x^2 - 7x + 5 = 0$
24. $x^2 + 3x - 8 = 0$
25. $x^2 - 5x - 10 = 0$
26. $3x^2 - 12x + 4 = 0$
27. $x^2 + 20x + 75 = 0$
28. $x^2 - 5x - 24 = 0$
29. $2x^2 + x - 21 = 0$

## Lesson 5-6

(pages 276–283)

**For Exercises 1–16, complete parts a–c for each quadratic equation.**

a. **Find the value of the discriminant.**
b. **Describe the number and type of roots.**
c. **Find the exact solutions by using the Quadratic Formula.**

1. $x^2 + 7x + 13 = 0$
2. $6x^2 + 6x - 21 = 0$
3. $5x^2 - 5x + 4 = 0$
4. $9x^2 + 42x + 49 = 0$
5. $4x^2 - 16x + 3 = 0$
6. $2x^2 = 5x + 3$
7. $x^2 + 81 = 18x$
8. $3x^2 - 30x + 75 = 0$
9. $24x^2 + 10x = 43$
10. $9x^2 + 4 = 2x$
11. $7x = 8x^2$
12. $18x^2 = 9x + 45$
13. $x^2 - 4x + 4 = 0$
14. $4x^2 + 16x + 15 = 0$
15. $x^2 - 6x + 13 = 0$

**Solve each equation by using the method of your choice. Find exact solutions.**

16. $x^2 + 4x + 29 = 0$
17. $4x^2 + 3x - 2 = 0$
18. $2x^2 + 5x = 9$
19. $x^2 = 8x - 16$
20. $7x^2 = 4x$
21. $2x^2 + 6x + 5 = 0$
22. $9x^2 - 30x + 25 = 0$
23. $3x^2 - 4x + 2 = 0$
24. $3x^2 = 108x$

## Lesson 5-7

(pages 286–292)

**Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.**

1. $y = (x + 6)^2 - 1$
2. $y = 2(x - 8)^2 - 5$
3. $y = -(x + 1)^2 + 7$
4. $y = -9(x - 7)^2 + 3$
5. $y = -x^2 + 10x - 3$
6. $y = -2x^2 + 16x + 7$

**Graph each function.**

7. $y = x^2 - 2x + 4$
8. $y = -3x^2 + 18x$
9. $y = -2x^2 - 4x + 1$
10. $y = 2x^2 - 8x + 9$
11. $y = \frac{1}{3}x^2 + 2x + 7$
12. $y = x^2 + 6x + 9$
13. $y = x^2 + 3x + 6$
14. $y = -0.5x^2 + 4x - 3$
15. $y = -2x^2 - 8x - 1$

## Lesson 5-8

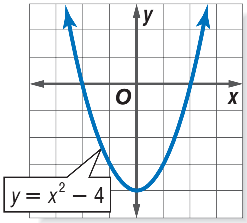
(pages 294–301)

**Graph each inequality.**

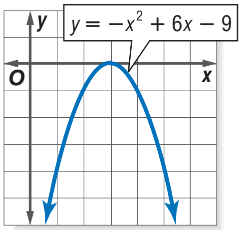
1. $y \leq 5x^2 + 3x - 2$
2. $y > -3x^2 + 2$
3. $y \geq x^2 - 8x$
4. $y \geq -x^2 - x + 3$
5. $y \leq 3x^2 + 4x - 8$
6. $y \leq -5x^2 + 2x - 3$
7. $y > 4x^2 + x$
8. $y \geq -x^2 - 3$

**Use the graph of the related function of each inequality to write its solutions.**

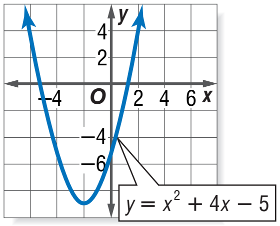
9. $x^2 - 4 \leq 0$




10. $-x^2 + 6x - 9 \geq 0$




11. $x^2 + 4x - 5 < 0$




**Solve each inequality algebraically.**

12. $x^2 - 1 < 0$
13. $10x^2 - x - 2 \geq 0$
14. $-x^2 - 5x - 6 > 0$
15. $-3x^2 \geq 5$
16. $x^2 - 2x - 8 \leq 0$
17. $2x^2 \geq 5x + 12$
18. $x^2 + 3x - 4 > 0$
19. $2x - x^2 \leq -15$

## Lesson 6-1

(pages 312–318)

**Simplify. Assume that no variable equals 0.**

1. $x^7 \cdot x^3 \cdot x$
2. $m^8 \cdot m \cdot m^{10}$
3. $7^5 \cdot 7^2$
4. $(-3)^4(-3)$
5. $\dfrac{t^{12}}{t}$
6. $-\dfrac{16x^8}{8x^2}$
7. $\dfrac{6^5}{6^3}$
8. $\dfrac{p^5q^7}{p^2q^5}$
9. $-(m^3)^8$
10. $(3^5)^7$
11. $-3^4$
12. $(abc)^3$
13. $(x^2)^5$
14. $(b^4)^6$
15. $(-2y^5)^2$
16. $3x^0$
17. $(5x^4)^{-2}$
18. $(-3)^{-2}$
19. $-3^{-2}$
20. $\dfrac{x}{x^7}$
21. $-\left(\dfrac{x}{5}\right)^2$
22. $\left(\dfrac{5a^7}{2b^5c}\right)^3$
23. $\dfrac{1}{x^{-3}}$
24. $\dfrac{5^6a^{x+y}}{5^4a^{x-y}}$

**Evaluate. Express the result in scientific notation.**

25. $(8.95 \times 10^9)(1.82 \times 10^7)$
26. $(3.1 \times 10^5)(7.9 \times 10^{-8})$
27. $\dfrac{(2.38 \times 10^{13})(7.56 \times 10^{-5})}{(4.2 \times 10^{18})}$

## Lesson 6-2

(pages 320–324)

**Simplify.**

1. $(4x^3 + 5x - 7x^2) + (-2x^3 + 5x^2 - 7y^2)$
2. $(2x^2 - 3x + 11) + (7x^2 + 2x - 8)$
3. $(-3x^2 + 7x + 23) + (-8x^2 - 5x + 13)$
4. $(-3x^2 + 7x + 23) - (-8x^2 - 5x + 13)$
5. $\dfrac{7}{uw}\left(4u^2w^3 - 5uw + \dfrac{w}{7u}\right)$
6. $-4x^5(-3x^4 - x^3 + x + 7)$
7. $(2x - 3)(4x + 7)$
8. $(3x - 5)(-2x - 1)$
9. $(3x - 5)(2x - 1)$
10. $(2x + 5)(2x - 5)$
11. $(3x - 7)(3x + 7)$
12. $(5 + 2w)(5 - 2w)$
13. $(2a^2 + 8)(2a^2 - 8)$
14. $(-5x + 10)(-5x - 10)$
15. $(4x - 3)^2$
16. $(5x + 6)^2$
17. $(-x + 1)^2$
18. $\dfrac{3}{4}x(x^2 + 4x + 14)$
19. $-\dfrac{1}{2}a^2(a^3 - 6a^2 + 5a)$

## Lesson 6-3

(pages 325–330)

**Find $p(5)$ and $p(-1)$ for each function.**

1. $p(x) = 7x - 3$
2. $p(x) = -3x^2 + 5x - 4$
3. $p(x) = 5x^4 + 2x^2 - 2x$
4. $p(x) = -13x^3 + 5x^2$
5. $p(x) = x^6 - 2$
6. $p(x) = \frac{2}{3}x^2 + 5x$
7. $p(x) = x^3 + x^2 - x + 1$
8. $p(x) = x^4 - x^2 - 1$
9. $p(x) = 1 - x^3$

**If $p(x) = -2x^2 + 5x + 1$ and $q(x) = x^3 - 1$, find each value.**

10. $q(n)$
11. $p(2b)$
12. $q(z^3)$
13. $p(3m^2)$
14. $q(x + 1)$
15. $p(3 - x)$
16. $q(a^2 - 2)$
17. $3q(h - 3)$
18. $5[p(c - 4)]$
19. $q(n - 2) + q(n^2)$
20. $-3p(4a) - p(a)$
21. $2[q(d^2 + 1)] + 3q(d)$

## Lesson 6-4

(pages 331–338)

**For Exercises 1–16, complete each of the following.**

a. **Graph each function by making a table of values.**

b. **Determine the values of $x$ between which the real zeros are located.**

c. **Estimate the $x$-coordinates at which the relative maxima and relative minima occur.**

1. $f(x) = x^3 + x^2 - 3x$
2. $f(x) = -x^4 + x^3 + 5$
3. $f(x) = x^3 - 3x^2 + 8x - 7$
4. $f(x) = 2x^5 + 3x^4 - 8x^2 + x + 4$
5. $f(x) = x^4 - 5x^3 + 6x^2 - x - 2$
6. $f(x) = 2x^6 + 5x^4 - 3x^2 - 5$
7. $f(x) = -x^3 - 8x^2 + 3x - 7$
8. $f(x) = -x^4 - 3x^3 + 5x$
9. $f(x) = x^5 - 7x^4 - 3x^3 + 2x^2 - 4x + 9$
10. $f(x) = x^4 - 5x^3 + x^2 - x - 3$
11. $f(x) = x^4 - 128x^2 + 960$
12. $f(x) = -x^5 + x^4 - 208x^2 + 145x + 9$
13. $f(x) = x^5 - x^3 - x + 1$
14. $f(x) = x^3 - 2x^2 - x + 5$
15. $f(x) = 2x^4 - x^3 + x^2 - x + 1$
16. $f(x) = -x^3 - x^2 - x - 1$

## Lesson 6-5

(pages 339–345)

**Factor completely. If the polynomial is not factorable, write *prime*.**

1. $14a^3b^3c - 21a^2b^4c + 7a^2b^3c$
2. $10ax - 2xy - 15ab + 3by$
3. $x^2 + x - 42$
4. $2x^2 + 5x + 3$
5. $6x^2 + 71x - 12$
6. $6x^4 - 12x^3 + 3x^2$
7. $x^2 - 6x + 2$
8. $x^2 - 2x - 15$
9. $6x^2 + 23x + 20$
10. $24x^2 - 76x + 40$
11. $6p^2 - 13pq - 28q^2$
12. $2x^2 - 6x + 3$
13. $x^2 + 49 - 14x$
14. $9x^2 - 64$
15. $36 - t^{10}$
16. $x^2 + 16$
17. $a^4 - 81b^4$
18. $3a^3 + 12a^2 - 63a$
19. $x^3 - 8x^2 + 15x$
20. $x^2 + 6x + 9$
21. $18x^3 - 8x$
22. $3x^2 - 42x + 40$
23. $2x^2 + 4x - 1$
24. $2x^3 + 6x^2 + x + 3$
25. $35ac - 3bd - 7ad + 15bc$
26. $5h^2 - 10hj + h - 2j$

**Simplify. Assume that no denominator is equal to 0.**

27. $\dfrac{x^2 + 8x + 15}{x^2 + 4x + 3}$
28. $\dfrac{x^2 + x - 2}{x^2 - 6x + 5}$
29. $\dfrac{x^2 - 15x + 56}{x^2 - 4x - 21}$
30. $\dfrac{x^2 + x - 6}{x^3 + 9x^2 + 27x + 27}$

Extra Practice

## Lesson 6-6

(pages 349–355)

**Simplify.**

1. $\dfrac{18r^3s^2 + 36r^2s^3}{9r^2s^2}$
2. $\dfrac{15v^3w^2 - 5v^4w^3}{-5v^4w^3}$
3. $\dfrac{x^2 - x + 1}{x}$
4. $(5bh + 5ch) \div (b + c)$
5. $(25c^4d + 10c^3d^2 - cd) \div 5cd$
6. $(16f^{18} + 20f^9 - 8f^6) \div 4f^3$
7. $(33m^5 + 55mn^5 - 11m^3)(11m)^{-1}$
8. $(8g^3 + 19g^2 - 12g + 9) \div (g + 3)$
9. $(p^{21} + 3p^{14} + p^7 - 2)(p^7 + 2)^{-1}$
10. $(8k^2 - 56k + 98) \div (2k - 7)$
11. $(2r^2 + 5r - 3) \div (r + 3)$
12. $(n^3 + 125) \div (n + 5)$
13. $(10y^4 + 3y^2 - 7) \div (2y^2 - 1)$
14. $(q^4 + 8q^3 + 3q + 17) \div (q + 8)$
15. $(15v^3 + 8v^2 - 21v + 6) \div (5v - 4)$
16. $(-2x^3 + 15x^2 - 10x + 3) \div (x + 3)$
17. $(5s^3 + s^2 - 7) \div (s + 1)$
18. $(t^4 - 2t^3 + t^2 - 3t + 2) \div (t - 2)$
19. $(z^4 - 3z^3 - z^2 - 11z - 4) \div (z - 4)$
20. $(3r^4 - 6r^3 - 2r^2 + r - 6) \div (r + 1)$
21. $(2b^3 - 11b^2 + 12b + 9) \div (b - 3)$

## Lesson 6-7

(pages 356–361)

**Use synthetic substitution to find $f(3)$ and $f(-4)$ for each function.**

1. $f(x) = x^2 - 6x + 2$
2. $f(x) = x^3 + 5x - 6$
3. $f(x) = x^3 - x^2 - 3x + 1$
4. $f(x) = -3x^3 + 5x^2 + 7x - 3$
5. $f(x) = 3x^5 - 5x^3 + 2x - 8$
6. $f(x) = 10x^3 + 2$

**Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.**

7. $(x^3 - x^2 + x + 14); (x + 2)$
8. $(5x^3 - 17x^2 + 6x); (x - 3)$
9. $(2x^3 + x^2 - 41x + 20); (x - 4)$
10. $(x^3 - 8); (x - 2)$
11. $(x^2 + 6x + 5); (x + 1)$
12. $(x^4 + x^3 + x^2 + x); (x + 1)$
13. $(x^3 - 8x^2 + x + 42); (x - 7)$
14. $(x^4 + 5x^3 - 27x - 135); (x - 3)$
15. $(2x^3 - 15x^2 - 2x + 120); (2x + 5)$
16. $(6x^3 - 17x^2 + 6x + 8); (3x - 4)$
17. $(10x^3 + x^2 - 46x + 35); (5x - 7)$
18. $(x^3 + 9x^2 + 23x + 15); (x + 1)$

## Lesson 6-8

(pages 362–368)

**Solve each equation. State the number and type of roots.**

1. $-5x - 7 = 0$
2. $3x^2 + 10 = 0$
3. $x^4 - 2x^3 = 23x^2 - 60x$

**State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.**

4. $f(x) = 5x^8 - x^6 + 7x^4 - 8x^2 - 3$
5. $f(x) = 6x^5 - 7x^2 + 5$
6. $f(x) = -2x^6 - 5x^5 + 8x^2 - 3x + 1$
7. $f(x) = 4x^3 + x^2 - 38x + 56$
8. $f(x) = 3x^4 - 5x^3 + 2x^2 - 7x + 5$
9. $f(x) = x^5 - x^4 + 7x^3 - 25x^2 + 8x - 13$

**Find all of the zeros of the function.**

10. $f(x) = x^3 - 7x^2 + 16x - 10$
11. $f(x) = 10x^3 + 7x^2 - 82x + 56$
12. $f(x) = x^3 - 16x^2 + 79x - 114$
13. $f(x) = -3x^3 + 6x^2 + 5x - 8$
14. $f(x) = 24x^3 + 64x^2 + 6x - 10$
15. $f(x) = 2x^3 + 2x^2 - 34x + 30$

## Lesson 6-9

(pages 369–373)

**List all of the possible rational zeros for each function.**

1. $f(x) = 3x^5 - 7x^3 - 8x + 6$
2. $f(x) = 4x^3 + 2x^2 - 5x + 8$
3. $f(x) = 6x^9 - 7$

**Find all of the rational zeros for each function.**

4. $f(x) = x^4 + 3x^3 - 7x^2 - 27x - 18$
5. $f(x) = 6x^4 - 31x^3 - 119x^2 + 214x + 560$
6. $f(x) = 20x^4 - 16x^3 + 11x^2 - 12x - 3$
7. $f(x) = 2x^4 - 30x^3 + 117x^2 - 75x + 280$
8. $f(x) = 3x^4 + 8x^3 + 9x^2 + 32x - 12$
9. $f(x) = x^5 - x^4 + x^3 + 3x^2 - x$

**Find all of the zeros of each function.**

10. $f(x) = x^4 + 8x^2 - 9$
11. $f(x) = 3x^4 - 9x^2 - 12$
12. $f(x) = 4x^4 + 19x^2 - 63$

## Lesson 7-1

(pages 384–390)

**Find $(f + g)(x)$, $(f - g)(x)$, $(f \bullet g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.**

1. $f(x) = 3x + 5$, $g(x) = x - 3$
2. $f(x) = \sqrt{x}$, $g(x) = x^2$
3. $f(x) = x^2 - 5$, $g(x) = x^2 + 5$
4. $f(x) = x^2 + 1$, $g(x) = x + 1$

**For each set of ordered pairs, find $f \circ g$ and $g \circ f$, if they exist.**

5. $f = \{(-1, 1), (2, -1), (-3, 5)\}$, $g = \{(1, -1), (-1, 2), (5, -3)\}$
6. $f = \{(0, 6), (5, -8), (-9, 2)\}$, $g = \{(-8, 3), (6, 4), (2, 1)\}$
7. $f = \{(8, 2), (6, 5), (-3, 4), (1, 0)\}$, $g = \{(2, 8), (5, 6), (4, -3), (0, 1)\}$
8. $f = \{(10, 4), (-1, 2), (5, 6), (-1, 0)\}$, $g = \{(-4, 10), (2, -9), (-7, 5), (-2, -1)\}$

**Find $[g \circ h](x)$ and $[h \circ g](x)$.**

9. $g(x) = 8 - 2x$, $h(x) = 3x$
10. $g(x) = x^2 - 7$, $h(x) = 3x + 2$
11. $g(x) = 2x + 7$, $h(x) = \frac{x - 7}{2}$
12. $g(x) = 3x + 2$, $h(x) = 5 - 3x$

**If $f(x) = x^2 + 1$, $g(x) = 2x$, and $h(x) = x - 1$, find each value.**

13. $g[f(1)]$
14. $[f \circ h](3)$
15. $[h \circ f](3)$
16. $[g \circ f](-2)$
17. $g[h(-20)]$
18. $f[h(-3)]$
19. $g[f(a)]$
20. $[f \circ (g \circ f)](c)$

## Lesson 7-2

(pages 391–396)

**Find the inverse of each relation.**

1. $\{(-2, 7), (3, 0), (5, -8)\}$
2. $\{(-3, 9), (-2, 4), (3, 9), (-1, 1)\}$

**Find the inverse of each function. Then graph the function and its inverse.**

3. $f(x) = x - 7$
4. $y = 2x + 8$
5. $g(x) = 3x - 8$
6. $y = -5x - 6$
7. $y = -2$
8. $g(x) = 5 - 2x$
9. $h(x) = \frac{x}{5} + 1$
10. $h(x) = -\frac{2}{3}x$
11. $y = \frac{x - 5}{3}$
12. $y = \frac{1}{2}x - 1$
13. $f(x) = \frac{3x + 8}{4}$
14. $g(x) = \frac{2x - 1}{3}$

**Determine whether each pair of functions are inverse functions.**

15. $f(x) = \frac{2x - 3}{5}$, $g(x) = \frac{3x - 5}{3}$
16. $f(x) = 5x - 6$, $g(x) = \frac{x + 6}{5}$
17. $f(x) = 6 - 3x$, $g(x) = 2 - \frac{1}{3}x$
18. $f(x) = 3x - 7$, $g(x) = \frac{1}{3}x + 7$

Extra Practice

Extra Practice

## Lesson 7-3

(pages 397–401)

**Graph each function. State the domain and range of the function.**

1. $y = \sqrt{x - 4}$
2. $y = \sqrt{x + 3} - 1$
3. $y = \frac{1}{3}\sqrt{x + 2}$
4. $y = \sqrt{2x + 5}$
5. $y = -\sqrt{4x}$
6. $y = 2\sqrt{x}$
7. $y = -3\sqrt{x}$
8. $y = \sqrt{x} + 5$
9. $y = \sqrt{2x} - 1$
10. $y = 5\sqrt{x} + 1$
11. $y = \sqrt{x + 1} - 2$
12. $y = 6 - \sqrt{x + 3}$

**Graph each inequality.**

13. $y > \sqrt{2x}$
14. $y \leq \sqrt{-5x}$
15. $y \geq \sqrt{x + 6} + 6$
16. $y < \sqrt{3x + 1} + 2$
17. $y \geq \sqrt{8x - 3} + 1$
18. $y < \sqrt{5x - 1} + 3$

## Lesson 7-4

(pages 402–406)

**Use a calculator to approximate each value to three decimal places.**

1. $\sqrt{289}$
2. $\sqrt{7832}$
3. $\sqrt[4]{0.0625}$
4. $\sqrt[3]{-343}$
5. $\sqrt[10]{32^4}$
6. $\sqrt[3]{49}$
7. $\sqrt[5]{5}$
8. $-\sqrt[4]{25}$

**Simplify.**

9. $\sqrt{9h^{22}}$
10. $\sqrt[5]{0}$
11. $\sqrt{\frac{16}{9}}$
12. $\sqrt{\left(-\frac{2}{3}\right)^4}$
13. $\sqrt[5]{-32}$
14. $-\sqrt{-144}$
15. $\sqrt[4]{a^{16}b^8}$
16. $\pm\sqrt[4]{81x^4}$
17. $\sqrt[5]{\frac{1}{100{,}000}}$
18. $\sqrt[3]{-d^6}$
19. $\sqrt[5]{p^{25}q^{15}r^5s^{20}}$
20. $\sqrt[4]{(2x^2 - y^8)^8}$
21. $\pm\sqrt{16m^6n^2}$
22. $-\sqrt[3]{(2x - y)^3}$
23. $\sqrt[4]{(r + s)^4}$
24. $\sqrt{9a^2 + 6a + 1}$
25. $\sqrt{4y^2 + 12y + 9}$
26. $-\sqrt{x^2 - 2x + 1}$
27. $\pm\sqrt{x^2 + 2x + 1}$
28. $\sqrt[3]{a^3 + 6a^2 + 12a + 8}$

## Lesson 7-5

(pages 408–414)

**Simplify.**

1. $\sqrt{75}$
2. $7\sqrt{12}$
3. $\sqrt[3]{81}$
4. $\sqrt{5r^5}$
5. $\sqrt[4]{7^8x^5y^6}$
6. $3\sqrt{5} + 6\sqrt{5}$
7. $\sqrt{18} - \sqrt{50}$
8. $4\sqrt[3]{32} + \sqrt[3]{500}$
9. $\sqrt{12}\,\sqrt{27}$
10. $3\sqrt{12} + 2\sqrt{300}$
11. $\sqrt[3]{54} - \sqrt[3]{24}$
12. $\sqrt{10}(2 - \sqrt{5})$
13. $-\sqrt{3}(2\sqrt{6} - \sqrt{63})$
14. $(5 + \sqrt{2})(3 + \sqrt{3})$
15. $(2 + \sqrt{5})(2 - \sqrt{5})$
16. $(8 + \sqrt{11})^2$
17. $(\sqrt{3} + \sqrt{6})(\sqrt{3} - \sqrt{6})$
18. $(\sqrt{8} + \sqrt{13})^2$
19. $(1 - \sqrt{7})(4 + \sqrt{7})$
20. $(5 - 2\sqrt{7})^2$
21. $\sqrt{\frac{3m^3}{24n^5}}$
22. $\frac{\sqrt{18}}{\sqrt{32}}$
23. $2\sqrt[3]{\frac{r^5}{2s^2t}}$
24. $\sqrt[3]{\frac{4}{7}}$
25. $\sqrt[5]{\frac{32}{a^4}}$
26. $\sqrt{\frac{2}{3}} - \sqrt{\frac{3}{8}}$
27. $\frac{5}{3 - \sqrt{10}}$
28. $\frac{\sqrt{5}}{1 + \sqrt{3}}$
29. $\frac{-2 + \sqrt{7}}{2 + \sqrt{7}}$
30. $\frac{1 - \sqrt{3}}{1 + \sqrt{8}}$
31. $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$
32. $\frac{x + \sqrt{5}}{x - \sqrt{5}}$

## Lesson 7-6

(pages 415–421)

**Write each expression in radical form.**

1. $10^{\frac{1}{3}}$
2. $8^{\frac{1}{4}}$
3. $a^{\frac{2}{3}}$
4. $(b^2)^{\frac{3}{4}}$

**Write each radical using rational exponents.**

5. $\sqrt{35}$
6. $\sqrt[4]{32}$
7. $3\sqrt{27a^2x}$
8. $\sqrt[5]{25ab^3c^4}$

**Evaluate each expression.**

9. $2401^{\frac{1}{4}}$
10. $27^{\frac{4}{3}}$
11. $(-32)^{\frac{2}{5}}$
12. $-81^{\frac{3}{4}}$
13. $(-125)^{-\frac{2}{3}}$
14. $16^{\frac{5}{2}} \cdot 16^{\frac{1}{2}}$
15. $8^{-\frac{2}{3}} \cdot 64^{\frac{1}{6}}$
16. $\left(\frac{48}{1875}\right)^{-\frac{5}{4}}$

**Simplify each expression.**

17. $7^{\frac{5}{9}} \cdot 7^{\frac{4}{9}}$
18. $32^{\frac{2}{3}} \cdot 32^{\frac{3}{5}}$
19. $\left(k^{\frac{8}{5}}\right)^5$
20. $x^{\frac{2}{5}} \cdot x^{\frac{8}{5}}$
21. $m^{\frac{2}{5}} \cdot m^{\frac{4}{5}}$
22. $\left(p^{\frac{5}{4}} \cdot q^{\frac{7}{2}}\right)^{\frac{8}{3}}$
23. $\left(4^{\frac{9}{2}}c^{\frac{3}{2}}\right)^2$
24. $\dfrac{7^{\frac{3}{4}}}{7^{\frac{5}{3}}}$
25. $\dfrac{1}{t^{\frac{9}{5}}}$
26. $a^{-\frac{8}{7}}$
27. $\dfrac{r}{r^{\frac{7}{5}}}$
28. $\sqrt[4]{36}$
29. $\sqrt[4]{9a^2}$
30. $\sqrt[3]{\sqrt{81}}$
31. $\dfrac{v^{\frac{11}{7}} - v^{\frac{4}{7}}}{v^{\frac{4}{7}}}$
32. $\dfrac{1}{5^{\frac{1}{2}} + 3^{\frac{1}{2}}}$

## Lesson 7-7

(pages 422–427)

**Solve each equation or inequality.**

1. $\sqrt{x} = 16$
2. $\sqrt{z+3} = 7$
3. $\sqrt[3]{a+5} = 1$
4. $5\sqrt{s} - 8 = 3$
5. $\sqrt[4]{m+7} + 11 = 9$
6. $d + \sqrt{d^2 - 8} = 4$
7. $g\sqrt{5} + 4 = g + 4$
8. $\sqrt{x-8} = \sqrt{13+x}$
9. $\sqrt{3x+9} > 2$
10. $\sqrt{3n-1} \leq 5$
11. $2 - 4\sqrt{21-6c} < -6$
12. $\sqrt{5y+4} > 8$
13. $\sqrt{2w+3} + 5 \geq 7$
14. $\sqrt{2c+3} - 7 > 0$
15. $\sqrt{3z-5} - 3 = 1$
16. $\sqrt{5y+1} + 6 < 10$
17. $\sqrt{3n+1} - 2 \leq 6$
18. $\sqrt{y-5} - \sqrt{y} \geq 1$
19. $(5n-1)^{\frac{1}{2}} = 0$
20. $(7x-6)^{\frac{1}{3}} + 1 = 3$
21. $(6a-8)^{\frac{1}{4}} + 9 \geq 10$

## Lesson 8-1

(pages 442–449)

**Simplify each expression.**

1. $\dfrac{25xy^2}{15y}$
2. $\dfrac{-4a^2b^3}{28ab^4}$
3. $\dfrac{(-2cd^3)^2}{8c^2d^5}$
4. $\dfrac{3x^3}{-2} \cdot \dfrac{-4}{9x}$
5. $\dfrac{21x^2}{-5} \cdot \dfrac{10}{7x^3}$
6. $\dfrac{2u^2}{3} \div \dfrac{6u^3}{5}$
7. $\dfrac{15x^3}{14} \div \dfrac{18x}{7}$
8. $\dfrac{xy^2}{2} \cdot \dfrac{x^2}{2y} \cdot \dfrac{2}{x^2y}$
9. $axy \div \dfrac{ax}{y}$
10. $\dfrac{9u^2}{28v} \div \dfrac{27u^2}{8v^2}$
11. $\dfrac{x^2-4}{4x^2-1} \cdot \dfrac{2x-1}{x+2}$
12. $\dfrac{x^2-1}{2x^2-x-1} \div \dfrac{x^2-4}{2x^2-3x-2}$
13. $\dfrac{2x^2+x-1}{2x^2+3x-2} \div \dfrac{x^2-2x+1}{x^2+x-2}$
14. $\dfrac{\frac{(ab)^2}{c}}{\frac{xa^3b}{cx^2}}$
15. $\dfrac{x^4-y^4}{x^3+y^3} \div \dfrac{x^3-y^3}{x+y}$

Extra Practice

Extra Practice

## Lesson 8-2

(pages 450–456)

**Find the LCM of each set of polynomials.**

1. $2a^2b, 4ab^2, 20a$
2. $x^2 - 4x - 12, x^2 + 7x + 10$

**Simplify each expression.**

3. $\frac{12}{7d} - \frac{3}{14d}$
4. $\frac{x+1}{x} - \frac{x-1}{x^2}$
5. $\frac{2x+1}{4x^2} - \frac{x+3}{6x}$
6. $\frac{7x}{13y^2} + \frac{4y}{6x^2}$
7. $\frac{x}{x-1} + \frac{1}{1-x}$
8. $\frac{1}{3v^2} + \frac{1}{uv} + \frac{3}{4u^2}$
9. $\frac{1}{x^2-x} + \frac{1}{x^2+x}$
10. $\frac{1}{x^2-1} - \frac{1}{(x-1)^2}$
11. $\frac{5}{x} - \frac{3}{x+5}$
12. $y - 1 + \frac{1}{y-1}$
13. $3m + 1 - \frac{2m}{3m+1}$
14. $\frac{3x}{x-y} + \frac{4x}{y-x}$
15. $\frac{4}{a^2-4} - \frac{3}{a^2+4a+4}$
16. $\frac{4}{3-3z^2} - \frac{2}{z^2+5z+4}$
17. $\frac{2c}{c^2-9} - \frac{1}{c^2+6c+9}$
18. $\dfrac{\frac{1}{x+y}}{\frac{1}{x}+\frac{1}{y}}$
19. $\dfrac{1-\frac{1}{x+1}}{1+\frac{1}{x-1}}$
20. $\dfrac{4+\frac{1}{x-2}}{3-\frac{1}{x-2}}$

## Lesson 8-3

(pages 457–463)

**Determine the equations of any vertical asymptotes and the values of $x$ for any holes in the graph of each rational function.**

1. $f(x) = \frac{1}{x+4}$
2. $f(x) = \frac{x-2}{x+3}$
3. $f(x) = \frac{5}{(x+1)(x-8)}$
4. $f(x) = \frac{x}{x+2}$
5. $f(x) = \frac{x^2-4}{x+2}$
6. $f(x) = \frac{x^2+x-6}{x^2+8x+15}$

**Graph each rational function.**

7. $f(x) = \frac{1}{x-5}$
8. $f(x) = \frac{3x}{x+1}$
9. $f(x) = \frac{x^2-16}{x-4}$
10. $f(x) = \frac{x}{x-6}$
11. $f(x) = \frac{1}{(x-3)^2}$
12. $f(x) = \frac{2}{(x+3)(x-4)}$
13. $f(x) = \frac{x+4}{x^2-1}$
14. $f(x) = \frac{x+2}{x+3}$
15. $f(x) = \frac{x^2+5x-14}{x^2+9x+14}$

## Lesson 8-4

(pages 465–471)

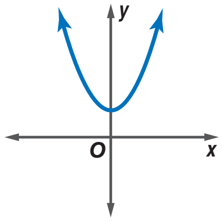
**State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.**

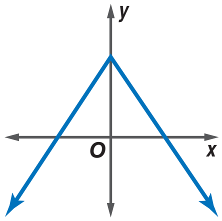
1. $xy = 10$
2. $\frac{x}{7} = y$
3. $\frac{x}{y} = -6$
4. $10x = y$
5. $x = \frac{2}{y}$
6. $A = \ell w$
7. $\frac{1}{4}b = -\frac{3}{5}c$
8. $D = rt$
9. If $y$ varies directly as $x$ and $y = 16$ when $x = 4$, find $y$ when $x = 12$.
10. If $x$ varies inversely as $y$ and $x = 12$ when $y = -3$, find $x$ when $y = -18$.
11. If $m$ varies directly as $w$ and $m = -15$ when $w = 2.5$, find $m$ when $w = 12.5$.
12. If $y$ varies jointly as $x$ and $z$ and $y = 10$ when $z = 4$ and $x = 5$, find $y$ when $x = 4$ and $z = 2$.
13. If $y$ varies inversely as $x$ and $y = \frac{1}{4}$ when $x = 24$, find $y$ when $x = \frac{3}{4}$.

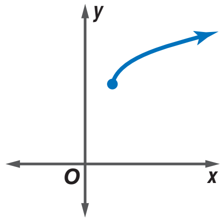
## Lesson 8-5

(pages 473–478)

**Identify the type of function represented by each graph.**

1.



2.



3.



**Identify the function represented by each equation. Then graph the equation.**

4. $y = \sqrt{5x}$
5. $y = \frac{3}{4}x$
6. $y = |x| + 3$
7. $y = x^2 - 2$
8. $y = \frac{2}{x}$
9. $y = 2[\![x]\!]$
10. $y = -2x^2 + 1$
11. $y = \frac{x^2 + 2x - 3}{x^2 + 7x + 12}$
12. $y = -3$

## Lesson 8-6

(pages 479–486)

**Solve each equation or inequality. Check your solutions.**

1. $\frac{x}{x-3} = \frac{1}{4}$
2. $\frac{5}{x} + \frac{3}{5} = \frac{2}{x}$
3. $\frac{5}{b-2} < 5$
4. $\frac{4}{a+3} > 2$
5. $\frac{x-2}{x} = \frac{x-4}{x-6}$
6. $-6 - \frac{8}{n} < n$
7. $\frac{2}{d} + \frac{1}{d-2} = 1$
8. $\frac{1}{2+3x} + \frac{2}{2-3x} = 0$
9. $\frac{1}{n+1} + \frac{1}{n-1} = \frac{2}{n^2-1}$
10. $\frac{p}{p+1} + \frac{3}{p-3} + 1 = 0$
11. $\frac{5z+2}{z^2-4} = \frac{-5z}{2-z} + \frac{2}{z+2}$
12. $\frac{1}{x-3} + \frac{2}{x^2-9} = \frac{5}{x+3}$
13. $\frac{1}{m^2-1} = \frac{2}{m^2+m-2}$
14. $\frac{12}{x^2-16} - \frac{24}{x-4} = 3$
15. $n + \frac{1}{n+3} = \frac{n^2}{n-1}$

## Lesson 9-1

(pages 498–506)

**Sketch the graph of each function. Then state the function's domain and range.**

1. $y = 3(5)^x$
2. $y = 0.5(2)^x$
3. $y = 3\left(\frac{1}{4}\right)^x$
4. $y = 2(1.5)^x$

**Determine whether each function represents exponential *growth* or *decay*.**

5. $y = 4(3)^x$
6. $y = 10^{-x}$
7. $y = 5\left(\frac{1}{2}\right)^x$
8. $y = 2\left(\frac{5}{4}\right)^x$

**Write an exponential function for the graph that passes through the given points.**

9. (0, 6) and (2, 54)
10. (0, –4) and (–4, –64)
11. (0, 1.5) and (3, 40.5)

**Solve each equation or inequality. Check your solution.**

12. $27^{2x-1} = 3$
13. $8^{2+x} \geq 2$
14. $4^{2x+5} < 8^{x+1}$
15. $6^{x+1} = 36^{x-1}$
16. $10^{x-1} > 100^{4-x}$
17. $\left(\frac{1}{5}\right)^{x-3} = 125$
18. $2^{x^2+1} = 32$
19. $36^x = 6^{x^2-3}$

## Lesson 9-2
(pages 509–517)

**Write each equation in logarithmic form.**

1. $3^5 = 243$
2. $10^3 = 1000$
3. $4^{-3} = \frac{1}{64}$

**Write each equation in exponential form.**

4. $\log_2 \frac{1}{8} = -3$
5. $\log_{25} 5 = \frac{1}{2}$
6. $\log_7 \frac{1}{7} = -1$

**Evaluate each expression.**

7. $\log_4 16$
8. $\log_{10} 10{,}000$
9. $\log_3 \frac{1}{9}$
10. $\log_2 1024$
11. $\log_6 6^5$
12. $\log_{\frac{1}{2}} 8$
13. $\log_{11} 121$
14. $5^{\log_5 10}$

**Solve each equation or inequality. Check your solutions.**

15. $\log_8 b = 2$
16. $\log_4 x < 3$
17. $\log_{\frac{1}{9}} n = -\frac{1}{2}$
18. $\log_x 7 = 1$
19. $\log_{\frac{2}{3}} a < 3$
20. $\log_2 (x^2 - 9) = 4$

## Lesson 9-3
(pages 520–526)

**Use $\log_3 5 \approx 1.4651$ and $\log_3 7 \approx 1.7712$ to approximate the value of each expression.**

1. $\log_3 \frac{7}{5}$
2. $\log_3 245$
3. $\log_3 35$

**Solve each equation. Check your solutions.**

4. $\log_2 x + \log_2 (x - 2) = \log_2 3$
5. $\log_3 x = 2 \log_3 3 + \log_3 5$
6. $\log_5 (x^2 + 7) = \frac{2}{3} \log_5 64$
7. $\log_2 (x^2 - 9) = 4$
8. $\log_3 (x + 2) + \log_3 6 = 3$
9. $\log_6 x + \log_6 (x - 5) = 2$
10. $\log_5 (x + 3) = \log_5 8 - \log_5 2$
11. $2 \log_3 x - \log_3 (x - 2) = 2$
12. $\log_6 x = \frac{3}{2} \log_6 9 + \log_6 2$
13. $\log_8 (x + 6) + \log_8 (x - 6) = 2$
14. $\log_3 14 + \log_3 x = \log_3 42$
15. $\log_{10} x = \frac{1}{2} \log_{10} 81$

## Lesson 9-4
(pages 528–533)

**Use a calculator to evaluate each expression to four decimal places.**

1. $\log 55$
2. $\log 6.7$
3. $\log 3.3$
4. $\log 0.08$
5. $\log 9.9$
6. $\log 0.6$

**Solve each equation or inequality. Round to four decimal places.**

7. $2^x = 15$
8. $4^{2a} > 45$
9. $7^{2x} = 35$
10. $11^{x+4} > 57$
11. $1.5^{a-7} = 9.6$
12. $3^{b^2} = 64$
13. $7^{3c} < 35^{2c-1}$
14. $5^{m^2+1} = 30$
15. $7^{3y-1} < 2^{2y+4}$
16. $9^{n-3} = 2^{n+3}$
17. $11^{t+1} \le 22^{t+3}$
18. $2^{3a-1} = 3^{a+2}$

**Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.**

19. $\log_3 21$
20. $\log_4 62$
21. $\log_5 28$
22. $\log_2 25$

## Lesson 9-5
(pages 536–542)

**Use a calculator to evaluate each expression to four decimal places.**

**1.** $e^3$ **2.** $e^{0.75}$ **3.** $e^{-4}$ **4.** $e^{-2.5}$

**5.** ln 5 **6.** ln 8 **7.** ln 8.4 **8.** ln 0.6

**Write an equivalent exponential or logarithmic equation.**

**9.** $e^x = 10$ **10.** $\ln x \approx 2.3026$ **11.** $e^3 = 9x$ **12.** $\ln 0.2 = x$

**Solve each equation or inequality.**

**13.** $25e^x = 1000$ **14.** $e^{0.075x} > 25$ **15.** $e^x < 3.8$

**16.** $-2e^x + 5 = 1$ **17.** $5 + 4e^{2x} = 17$ **18.** $e^{-3x} \leq 15$

**19.** $\ln 7x = 10$ **20.** $\ln 4x = 8$ **21.** $3 \ln 2x \geq 9$

**22.** $\ln (x + 2) = 4$ **23.** $\ln (2x + 3) > 0$ **24.** $\ln (3x - 1) = 5$

## Lesson 9-6
(pages 544–550)

**1.** **FARMING** Mr. Rogers purchased a combine for $175,000 for his farming operation. It is expected to depreciate at a rate of 18% per year. What will be the value of the combine in 3 years?

**2.** **REAL ESTATE** The Jacksons bought a house for $65,000 in 1992. Houses in the neighborhood have appreciated at the rate of 4.5% a year. How much is the house worth in 2003?

**3.** **POPULATION** In 1950, the population of a city was 50,000. Since then, the population has increased by 2.25% per year. If it continues to grow at this rate, what will the population be in 2005?

**4.** **BEARS** In a particular state, the population of black bears has been decreasing at the rate of 0.75% per year. In 1990, it was estimated that there were 400 black bears in the state. If the population continues to decline at the same rate, what will the population be in 2010?

## Lesson 10-1
(pages 562–566)

**Find the midpoint of the line segment with endpoints at the given coordinates.**

**1.** $(7, -3), (-11, 13)$ **2.** $(16, 29), (-7, 2)$ **3.** $(43, -18), (-78, -32)$

**4.** $(-7.54, 3.42), (4.89, -9.28)$ **5.** $\left(\frac{1}{2}, \frac{1}{4}\right), \left(\frac{2}{3}, \frac{3}{5}\right)$ **6.** $\left(-\frac{1}{4}, \frac{2}{3}\right), \left(-\frac{1}{2}, -\frac{1}{2}\right)$

**Find the distance between each pair of points with the given coordinates.**

**7.** $(5, 7), (3, 19)$ **8.** $(-2, -1), (5, 3)$

**9.** $(-3, 15), (7, -8)$ **10.** $(6, -3), (-4, -9)$

**11.** $(3.89, -0.38), (4.04, -0.18)$ **12.** $(5\sqrt{3}, 2\sqrt{2}), (-11\sqrt{3}, -4\sqrt{2})$

**13.** $\left(\frac{1}{4}, 0\right), \left(-\frac{2}{3}, \frac{1}{2}\right)$ **14.** $\left(4, -\frac{5}{6}\right), \left(-2, \frac{1}{6}\right)$

**15.** A circle has a radius with endpoints at $(-3, 1)$ and $(2, -5)$. Find the circumference and area of the circle. Write the answer in terms of $\pi$.

**16.** Triangle $ABC$ has vertices $A(0, 0)$, $B(-3, 4)$, and $C(2, 6)$. Find the perimeter of the triangle.

## Lesson 10-2
(pages 567–573)

**Write each equation in standard form.**

1. $y = x^2 - 4x + 7$
2. $y = 2x^2 + 12x + 17$
3. $x = 3y^2 - 6y + 5$

**Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.**

4. $y + 4 = x^2$
5. $y = 5(x + 2)^2$
6. $4(y + 2) = 3(x - 1)^2$
7. $5x + 3y^2 = 15$
8. $y = 2x^2 - 8x + 7$
9. $x = 2y^2 - 8y + 7$
10. $3(x - 8)^2 = 5(y + 3)$
11. $x = 3(y + 4)^2 + 1$
12. $8y + 5x^2 + 30x + 101 = 0$
13. $x = -\frac{1}{5}y^2 + \frac{8}{5}y - 7$
14. $6x = y^2 - 6y + 39$
15. $-8y = x^2$
16. $y = 4x^2 + 24x + 38$
17. $y = x^2 - 6x + 3$
18. $y = x^2 + 4x + 1$

**Write an equation for each parabola described below. Then graph.**

19. focus (1, 1), directrix $y = -1$
20. vertex (−1, 2), directrix $y = -4$

## Lesson 10-3
(pages 574–579)

**Write an equation for the circle that satisfies each set of conditions.**

1. center (3, 2), $r = 5$ units
2. center (−5, 8), $r = 3$ units
3. center (1, −6), $r = \frac{2}{3}$ units
4. center (0, 7), tangent to $x$-axis
5. center (−2, −4), tangent to $y$-axis
6. endpoints of a diameter at (−9, 0) and (2, −5)
7. endpoints of a diameter at (4, 1) and (−3, 2)
8. center (6, −10), passes through origin
9. center (0.8, 0.5), passes through (2, 2)

**Find the center and radius of the circle with the given equation. Then graph.**

10. $x^2 + y^2 = 36$
11. $(x - 5)^2 + (y + 4)^2 = 1$
12. $x^2 + 3x + y^2 - 5y = 0.5$
13. $x^2 + y^2 = 14x - 24$
14. $x^2 + y^2 = 2(y - x)$
15. $x^2 + 10x + \left(y - \sqrt{3}\right)^2 = 11$
16. $x^2 + y^2 = 4x + 9$
17. $x^2 + y^2 - 6x + 4y = 156$
18. $x^2 + y^2 - 2x + 7y = 1$

## Lesson 10-4
(pages 581–588)

**Write an equation for the ellipse that satisfies each set of conditions.**

1. endpoints of major axis at (−2, 7) and (4, 7), endpoints of minor axis at (1, 5) and (1, 9)
2. endpoints of minor axis at (1, −4) and (1, 5), endpoints of major axis at (−4, 0.5) and (6, 0.5)
3. major axis 24 units long and parallel to the $y$-axis, minor axis 4 units long, center at (0, 3)

**Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.**

4. $\frac{x^2}{36} + \frac{y^2}{81} = 1$
5. $\frac{x^2}{121} + \frac{(y - 5)^2}{16} = 1$
6. $\frac{(x + 2)^2}{12} + \frac{(y + 1)^2}{16} = 1$
7. $8x^2 + 2y^2 = 32$
8. $7x^2 + 3y^2 = 84$
9. $9x^2 + 16y^2 = 144$
10. $169x^2 - 338x + 169 + 25y^2 = 4225$
11. $x^2 + 4y^2 + 8x - 64y = -128$
12. $4x^2 + 5y^2 = 6(6x + 5y) + 658$
13. $9x^2 + 16y^2 - 54x + 64y + 1 = 0$

## Lesson 10-5
(pages 590–597)

**Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.**

1. $\frac{y^2}{25} - \frac{x^2}{9} = 1$
2. $\frac{x^2}{4} - \frac{y^2}{9} = 1$
3. $\frac{x^2}{81} - \frac{y^2}{36} = 1$
4. $\frac{(x-4)^2}{64} - \frac{(y+1)^2}{16} = 1$
5. $\frac{(y-7)^2}{2.25} - \frac{(x-3)^2}{4} = 1$
6. $(x+5)^2 - \frac{(y+3)^2}{48} = 1$
7. $x^2 - 9y^2 = 36$
8. $4x^2 - 9y^2 = 72$
9. $49x^2 - 16y^2 = 784$
10. $576y^2 = 49x^2 + 490x + 29{,}449$
11. $25(y+5)^2 - 20(x-1)^2 = 500$

**Write an equation for the hyperbola that satisfies each set of conditions.**

12. vertices $(-3, 0)$ and $(3, 0)$; conjugate axis of length 8 units
13. vertices $(0, -7)$ and $(0, 7)$; conjugate axis of length 25 units
14. center $(0, 0)$; horizontal transverse axis of length 12 units and a conjugate axis of length 10 units

## Lesson 10-6
(pages 598–602)

**Write each equation in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation.**

1. $9x^2 - 36x + 36 = 4y^2 + 24y + 72$
2. $x^2 + 4x + 2y^2 + 16y + 32 = 0$
3. $x^2 + 6x + y^2 - 6y + 9 = 0$
4. $9y^2 = 25x^2 + 400x + 1825$
5. $2y^2 + 12y - x + 6 = 0$
6. $x^2 + y^2 = 10x + 2y + 23$
7. $3x^2 + y = 12x - 17$
8. $9x^2 - 18x + 16y^2 + 160y = -265$
9. $x^2 + 10x + 5 = 4y^2 + 16$
10. $\frac{(y-5)^2}{4} - (x+1)^2 = 4$
11. $9x^2 + 49y^2 = 441$
12. $4x^2 - y^2 = 4$

**Without writing the equation in standard form, state whether the graph of each equation is a *parabola, circle, ellipse,* or *hyperbola*.**

13. $(x+3)^2 = 8(y+2)$
14. $x^2 + 4x + y^2 - 8y = 2$
15. $2x^2 - 13y^2 + 5 = 0$
16. $16(x-3)^2 + 81(y+4)^2 = 1296$

## Lesson 10-7
(pages 603–608)

**Solve each system of inequalities by graphing.**

1. $\frac{x^2}{16} - \frac{y^2}{1} \geq 1$
   $x^2 + y^2 \leq 49$
2. $\frac{x^2}{25} + \frac{y^2}{16} \leq 1$
   $y \leq x - 2$
3. $y \geq x + 3$
   $x^2 + y^2 < 25$
4. $4x^2 + (y-3)^2 \leq 16$
   $x + 2y \geq 4$

**Find the exact solution(s) of each system of equations.**

5. $\frac{x^2}{16} + \frac{y^2}{16} = 1$
   $y = x + 3$
6. $x = y^2$
   $(x+3)^2 + y^2 = 53$
7. $\frac{x^2}{3} - \frac{(y+2)^2}{4} = 1$
   $x^2 = y^2 + 11$
8. $\frac{(x-1)^2}{5} + \frac{y^2}{2} = 1$
   $y = x + 1$
9. $x^2 + y^2 = 13$
   $x^2 - y^2 = -5$
10. $\frac{x^2}{25} - \frac{y^2}{5} = 1$
    $y = x - 4$
11. $x^2 + y = 0$
    $x + y = -2$
12. $x^2 - 9y^2 = 36$
    $x = y$
13. $4x^2 + 6y^2 = 360$
    $y = x$

Extra Practice

## Lesson 11-1
(pages 622–628)

**Find the next four terms of each arithmetic sequence.**

1. 9, 7, 5, . . .
2. 3, 4.5, 6, . . .
3. 40, 35, 30, . . .
4. 2, 5, 8, . . .

**Find the first five terms of each arithmetic sequence described.**

5. $a_1 = 1, d = 7$
6. $a_1 = -5, d = 2$
7. $a_1 = 1.2, d = 3.7$
8. $a_1 = -\frac{5}{4}, d = -\frac{1}{2}$

**Find the indicated term of each arithmetic sequence.**

9. $a_1 = 4, d = 5, n = 10$
10. $a_1 = -30, d = -6, n = 5$
11. $a_1 = -3, d = 32, n = 8$

**Write an equation for the *n*th term of each arithmetic sequence.**

12. 3, 5, 7, 9, . . .
13. 2, −1, −4, −7, . . .
14. 20, 28, 36, 44, . . .

**Find the arithmetic means in each sequence.**

15. 2, __?__, __?__, __?__, 34
16. 0, __?__, __?__, __?__, −28
17. −10, __?__, __?__, __?__, 14

## Lesson 11-2
(pages 629–635)

**Find $S_n$ for each arithmetic series described.**

1. $a_1 = 3, a_n = 20, n = 6$
2. $a_1 = 90, a_n = -4, n = 10$
3. $a_1 = 16, a_n = 14, n = 12$
4. $a_1 = -1, d = 10, n = 30$
5. $a_1 = 4, d = -5, n = 11$
6. $a_1 = 5, d = -\frac{1}{2}, n = 17$

**Find the sum of each arithmetic series.**

7. $\sum_{n=1}^{6}(n + 2)$
8. $\sum_{n=5}^{10}(2n - 5)$
9. $\sum_{k=1}^{5}(40 - 2k)$
10. $\sum_{k=8}^{12}(6 - 3k)$
11. $\sum_{n=1}^{4}(10n + 2)$
12. $\sum_{n=6}^{10}(2 + 3n)$

**Find the first three terms of each arithmetic series described.**

13. $a_1 = 11, a_n = 38, S_n = 245$
14. $n = 12, a_n = 13, S_n = -42$
15. $n = 11, a_n = 5, S_n = 0$

## Lesson 11-3
(pages 636–641)

**Find the next two terms of each geometric sequence.**

1. 5, 15, 45, . . .
2. 2, 10, 50, . . .
3. 64, 16, 4, . . .
4. −9, 27, −81, . . .
5. 0.5, 0.75, 1.125, . . .
6. $\frac{1}{2}, -\frac{3}{8}, \frac{9}{32}, \ldots$

**Find the first five terms of each geometric sequence described.**

7. $a_1 = -2, r = 6$
8. $a_1 = 4, r = -5$
9. $a_1 = 0.8, r = 2.5$
10. $a_1 = -\frac{1}{3}, r = -\frac{3}{5}$

**Find the indicated term of each geometric sequence.**

11. $a_1 = 5, r = 7, n = 6$
12. $a_1 = 200, r = -\frac{1}{2}, n = 10$
13. $a_1 = 60, r = -2, n = 4$

**Write an equation for the *n*th term of each geometric sequence.**

14. 20, 40, 80, . . .
15. $-\frac{1}{2}, -\frac{1}{8}, -\frac{1}{32}, \ldots$

**Find the geometric means in each sequence.**

16. 1, __?__, __?__, __?__, 81
17. 5, __?__, __?__, __?__, 6480

Extra Practice

## Lesson 11-4
(pages 643–649)

**Find $S_n$ for each geometric series described.**

1. $a_1 = \frac{1}{81}, r = 3, n = 6$
2. $a_1 = 1, r = -2, n = 7$
3. $a_1 = 5, r = 4, n = 5$
4. $a_1 = -27, r = -\frac{1}{3}, n = 6$
5. $a_1 = 1000, r = \frac{1}{2}, n = 7$
6. $a_1 = 125, r = -\frac{2}{5}, n = 5$
7. $a_1 = 10, r = 3, n = 6$
8. $a_1 = 1250, r = -\frac{1}{5}, n = 5$
9. $a_1 = 1215, r = \frac{1}{3}, n = 5$
10. $a_1 = 16, r = \frac{3}{2}, n = 5$
11. $a_1 = 7, r = 2, n = 7$
12. $a_1 = -\frac{3}{2}, r = -\frac{1}{2}, n = 6$

**Find the sum of each geometric series.**

13. $\sum_{k=1}^{5} 2^k$
14. $\sum_{n=0}^{3} 3^{-n}$
15. $\sum_{n=0}^{3} 2(5^n)$
16. $\sum_{k=2}^{5} -(-3)^{k-1}$

**Find the indicated term for each geometric series described.**

17. $S_n = 300, a_n = 160, r = 2; a_1$
18. $S_n = -171, n = 9, r = -2; a_5$
19. $S_n = -4372, a_n = -2916, r = 3; a_4$

## Lesson 11-5
(pages 650–655)

**Find the sum of each infinite geometric series, if it exists.**

1. $a_1 = 54, r = \frac{1}{3}$
2. $a_1 = 2, r = -1$
3. $a_1 = 1000, r = -0.2$
4. $a_1 = 7, r = \frac{3}{7}$
5. $49 + 14 + 4 + \ldots$
6. $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \ldots$
7. $12 - 4 + \frac{4}{3} - \ldots$
8. $3 - 9 + 27 - \ldots$
9. $3 - 2 + \frac{4}{3} - \ldots$
10. $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^{n-1}$
11. $\sum_{n=1}^{\infty} 5\left(-\frac{1}{10}\right)^{n-1}$
12. $\sum_{n=1}^{\infty} -\frac{2}{3}\left(-\frac{3}{4}\right)^{n-1}$

**Write each repeating decimal as a fraction.**

13. $0.\overline{4}$
14. $0.\overline{27}$
15. $0.\overline{123}$
16. $0.\overline{645}$
17. $0.\overline{67}$
18. $0.8\overline{53}$

## Lesson 11-6
(pages 658–662)

**Find the first five terms of each sequence.**

1. $a_1 = 4, a_{n+1} = 2a_{n+1}$
2. $a_1 = 6, a_{n+1} = a_n + 7$
3. $a_1 = 16, a_{n+1} = a_n + (n + 4)$
4. $a_1 = 1, a_{n+1} = \frac{n}{n+2} \cdot a_n$
5. $a_1 = -\frac{1}{2}, a_{n+1} = 2a_n + \frac{1}{4}$
6. $a_1 = \frac{1}{3}, a_2 = \frac{1}{4}, a_{n+1} = a_n + a_{n-1}$

**Find the first three iterates of each function for the given initial value.**

7. $f(x) = 3x - 1, x_0 = 3$
8. $f(x) = 2x^2 - 8, x_0 = -1$
9. $f(x) = 4x + 5, x_0 = 0$
10. $f(x) = 3x^2 + 1, x_0 = 1$
11. $f(x) = x^2 + 4x + 4, x_0 = 1$
12. $f(x) = x^2 + 9, x_0 = 2$
13. $f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$
14. $f(x) = 3x^2 + 2x - 1, x_0 = \frac{2}{3}$

## Lesson 11-7
(pages 664–669)

**Evaluate each expression.**

1. $6!$
2. $4!$
3. $\frac{13!}{6!}$
4. $\frac{10!}{3!7!}$
5. $\frac{14!}{4!10!}$
6. $\frac{7!}{2!5!}$
7. $\frac{9!}{8!}$
8. $\frac{10!}{10!0!}$

**Expand each power.**

9. $(z - 3)^5$
10. $(m + 1)^4$
11. $(x + 6)^4$
12. $(z - y)^2$
13. $(m + n)^5$
14. $(a - b)^4$
15. $(2n + 1)^4$
16. $(3n - 4)^3$
17. $(2n - m)^0$
18. $(4x - a)^4$
19. $(3r - 4s)^5$
20. $\left(\frac{b}{2} - 1\right)^4$

**Find the indicated term of each expansion.**

21. sixth term of $(x + 3)^8$
22. fourth term of $(x - 2)^7$
23. fifth term of $(a + b)^6$
24. fourth term of $(x - y)^9$
25. sixth term of $(x + 4y)^7$
26. fifth term of $(3x + 5y)^{10}$

## Lesson 11-8
(pages 670–673)

**Prove that each statement is true for all positive integers.**

1. $2 + 4 + 6 + \ldots + 2n = n^2 + n$
2. $1^3 + 3^3 + 5^3 + \ldots + (2n - 1)^3 = n^2(2n^2 - 1)$
3. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{n(n + 2)} = \frac{n(3n + 5)}{4(n + 1)(n + 2)}$
4. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots + n(n + 2) = \frac{n(n + 1)(2n + 7)}{6}$
5. $\frac{5}{1 \cdot 2} \cdot \frac{1}{3} + \frac{7}{2 \cdot 3} \cdot \frac{1}{3^2} + \frac{9}{3 \cdot 4} \cdot \frac{1}{3^3} + \ldots + \frac{2n + 3}{n(n + 1)} \cdot \frac{1}{3^n} = 1 - \frac{1}{3^n(n + 1)}$

**Find a counterexample for each statement.**

6. $n^2 + 2n - 1$ is divisible by 2.
7. $2^n + 3^n$ is prime.
8. $2^{n-1} + n = 2^n + 2 - n$ for all integers $n \geq 2$
9. $3^n - 2n = 3^n - 2^n$ for all integers $n \geq 1$

## Lesson 12-1
(pages 684–689)

**For Exercises 1–5, state whether the events are *independent* or *dependent*.**

1. tossing a penny and rolling a number cube
2. choosing first and second place in an academic competition
3. choosing from three pairs of shoes if only two pairs are available
4. A comedy video and an action video are selected from the video store.
5. The numbers 1–10 are written on pieces of paper and are placed in a hat. Three of them are selected one after the other without replacement.
6. In how many different ways can a 10-question true-false test be answered?
7. A student council has 6 seniors, 5 juniors, and 1 sophomore as members. In how many ways can a 3-member council committee be formed that includes one member of each class?
8. How many license plates of 5 symbols can be made using a letter for the first symbol and digits for the remaining 4 symbols?

## Lesson 12-2

(pages 690–695)

**Evaluate each expression.**

1. $P(3, 2)$
2. $P(5, 2)$
3. $P(10, 6)$
4. $P(4, 3)$
5. $P(12, 2)$
6. $P(7, 2)$
7. $C(8, 6)$
8. $C(20, 17)$
9. $C(9, 4) \cdot C(5, 3)$
10. $C(6, 1) \cdot C(4, 1)$
11. $C(10, 5) \cdot C(8, 4)$
12. $C(7, 6) \cdot C(3, 1)$

**Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.**

13. choosing a team of 9 players from a group of 20
14. selecting the batting order of 9 players in a baseball game
15. arranging the order of 8 songs on a CD
16. finding the number of 5-card hands that include 4 diamonds and 1 club

## Lesson 12-3

(pages 697–702)

**A jar contains 3 red, 4 green, and 5 orange marbles. If three marbles are drawn at random and not replaced, find each probability.**

1. $P(\text{all green})$
2. $P(\text{1 red, then 2 not red})$

**Find the odds of an event occurring, given the probability of the event.**

3. $\frac{5}{9}$
4. $\frac{4}{8}$
5. $\frac{3}{10}$

**Find the probability of an event occurring, given the odds of the event.**

6. $\frac{2}{7}$
7. $\frac{6}{13}$
8. $\frac{1}{19}$

**The table shows the number of ways to achieve each product when two dice are tossed. Find each probability.**

| Product | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 | 15 | 16 | 18 | 20 | 24 | 25 | 30 | 36 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ways | 1 | 2 | 2 | 3 | 2 | 4 | 2 | 1 | 2 | 4 | 2 | 1 | 2 | 2 | 2 | 1 | 2 | 1 |

9. $P(6)$
10. $P(12)$
11. $P(\text{not } 36)$
12. $P(\text{not } 12)$

## Lesson 12-4

(pages 703–709)

**An octahedral die is rolled twice. The sides are numbered 1–8. Find each probability.**

1. $P(\text{1, then 8})$
2. $P(\text{two different numbers})$
3. $P(\text{8, then any number})$

**Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.**

4. $P(\text{jack, jack})$
5. $P(\text{heart, club})$
6. $P(\text{two diamonds})$
7. $P(\text{2 of hearts, diamond})$
8. $P(\text{2 red cards})$
9. $P(\text{2 black aces})$

**Determine whether the events are *independent* or *dependent*. Then find the probability.**

10. According to the weather reports, the probability of rain on a certain day is 70% in Yellow Falls and 50% in Copper Creek. What is the probability that it will rain in both cities?
11. The odds of winning a carnival game are 1 to 5. What is the probability that a player will win the game three consecutive times?

Extra Practice

## Lesson 12-5

(pages 710–715)

**An octahedral die is rolled. The sides are numbered 1–8. Find each probability.**

1. $P$(7 or 8)
2. $P$(less than 4)
3. $P$(greater than 6)
4. $P$(not prime)
5. $P$(odd or prime)
6. $P$(multiple of 5 or odd)

**Ten slips of paper are placed in a container. Each is labeled with a number from 1 through 10. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.**

7. $P$(1 or 10)
8. $P$(3 or odd)
9. $P$(6 or less than 7)

10. Two letters are chosen at random from the word GEESE and two are chosen at random from the word PLEASE. What is the probability that all four letters are Es or none of the letters is an E?
11. Three dice are rolled. What is the probability they all show the same number?
12. Two marbles are simultaneously drawn at random from a bag containing 3 red, 5 blue, and 6 green marbles. Find each probability.
    a. $P$(at least one red marble)
    b. $P$(at least one green marble)
    c. $P$(two marbles of the same color)
    d. $P$(two marbles of different colors)

## Lesson 12-6

(pages 717–723)

**Find the mean, median, mode, and standard deviation of each set of data. Round to the nearest hundredth, if necessary.**

1. [4, 1, 2, 1, 1]
2. [86, 71, 74, 65, 45, 42, 76]
3. [16, 20, 15, 14, 24, 23, 25, 10, 19]
4. [25.5, 26.7, 20.9, 23.4, 26.8, 24.0, 25.7]
5. [18, 24, 16, 24, 22, 24, 22, 22, 24, 13, 17, 18, 16, 20, 16, 7, 22, 5, 4, 24]
6. [55, 50, 50, 55, 65, 50, 45, 35, 50, 40, 70, 40, 70, 50, 90, 30, 35, 55, 55, 40, 75, 35, 40, 45, 65, 50, 60]
7. [364, 305, 217, 331, 305, 311, 352, 319, 272, 238, 311, 226, 220, 226, 215, 160, 123, 4, 24, 238, 99]

## Lesson 12-7

(pages 724–728)

**For Exercises 1–4, use the following information.**

The diameters of metal fittings made by a machine are normally distributed. The diameters have a mean of 7.5 centimeters and a standard deviation of 0.5 centimeters.

1. What percent of the fittings have diameters between 7.0 and 8.0 centimeters?
2. What percent of the fittings have diameters between 7.5 and 8.0 centimeters?
3. What percent of the fittings have diameters greater than 6.5 centimeters?
4. Of 100 fittings, how many will have a diameter between 6.0 and 8.5 centimeters?

**For Exercises 5–7, use the following information.**

A college entrance exam was administered at a state university. The scores were normally distributed with a mean of 510, and a standard deviation of 80.

5. What percent would you expect to score above 510?
6. What percent would you expect to score between 430 and 590?
7. What is the probability that a student chosen at random scored between 350 and 670?

## Lesson 12-8

(pages 729–733)

**HORSES For Exercises 1 and 2, use the following information.**

The average lifespan of a horse is 20 years.

1. What is the probability that a randomly selected horse will live more than 25 years?
2. What is the probability that a randomly selected horse will live less than 10 years?

**MINIATURE GOLF For Exercises 3 and 4, use the following information.**

The probability of reaching in a basket of golf balls at a miniature golf course and picking out a yellow golf ball is 0.25.

3. If 5 golf balls are drawn, what is the probability that at least 2 will be yellow?
4. What is the expected number of yellow golf balls if 8 golf balls are drawn?

## Lesson 12-9

(pages 735–739)

**Find each probability if a coin is tossed 5 times.**

1. $P$(0 heads)
2. $P$(exactly 4 heads)
3. $P$(exactly 3 tails)

**Ten percent of a batch of toothpaste is defective. Five tubes of toothpaste are selected at random from this batch. Find each probability.**

4. $P$(0 defective)
5. $P$(exactly one defective)
6. $P$(at least three defective)
7. $P$(less than three defective)

**On a 20-question true-false test, you guess at every question. Find each probability.**

8. $P$(all answers correct)
9. $P$(exactly 10 correct)

## Lesson 12-10

(pages 741–744)

**Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.**

1. finding the most often prescribed pain reliever by asking all of the doctors at a hospital
2. taking a poll of the most popular baby girl names this year by studying birth announcements in newspapers from different cities across the country
3. polling people who are leaving a pizza parlor about their favorite restaurant in the city

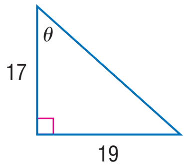
**For Exercises 4–6, find the margin of sampling error to the nearest percent.**

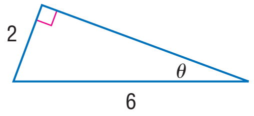
4. $p = 45\%, n = 125$
5. $p = 62\%, n = 240$
6. $p = 24\%, n = 600$
7. A poll conducted on the favorite breakfast choice of students in your school showed that 75% of the 2250 students asked indicated oatmeal as their favorite breakfast.

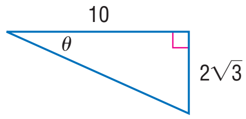
## Lesson 13-1

(pages 759–767)

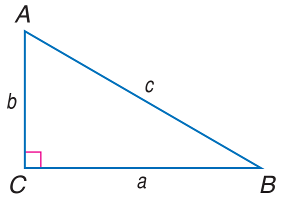
**Find the values of the six trigonometric functions for angle $\theta$.**

1. 


2. 


3. 


**Solve $\triangle ABC$ using the diagram at the right and the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**




4. $B = 42°, c = 30$
5. $A = 84°, a = 4$
6. $B = 19°, b = 34$
7. $A = 75°, c = 55$
8. $b = 24, c = 36$
9. $a = 51, c = 115$
10. $\cos B = \frac{2}{5}, a = 12$
11. $\tan A = \frac{3}{2}, b = 22$

## Lesson 13-2

(pages 768–774)

**Draw an angle with the given measure in standard position.**

1. $60°$
2. $250°$
3. $315°$
4. $150°$

**Rewrite each degree measure in radians and each radian measure in degrees.**

5. $-135°$
6. $-315°$
7. $45°$
8. $80°$
9. $24°$
10. $-54°$
11. $-\pi$
12. $\frac{9\pi}{4}$
13. $\frac{3\pi}{2}$
14. $-\frac{7\pi}{2}$
15. $\frac{9\pi}{10}$
16. $\frac{17\pi}{30}$
17. $\frac{7\pi}{12}$
18. $1$
19. $-2\frac{1}{3}$

**Find one angle with positive measure and one angle with negative measure coterminal with each angle.**

20. $50°$
21. $-75°$
22. $125°$
23. $-400°$
24. $550°$
25. $3\pi$
26. $-2\pi$
27. $\frac{2\pi}{3}$
28. $\frac{12\pi}{5}$
29. $0$

## Lesson 13-3

(pages 776–783)

**Find the exact values of the six trigonometric functions of $\theta$ if the terminal side of $\theta$ in standard position contains the given point.**

1. $P(3, -4)$
2. $P(1, \sqrt{3})$
3. $P(0, 24)$
4. $P(-5, -5)$
5. $P(\sqrt{2}, -\sqrt{2})$

**Find the exact value of each trigonometric function.**

6. $\cos 225°$
7. $\sin\left(-\frac{5\pi}{3}\right)$
8. $\tan \frac{7\pi}{6}$
9. $\tan(-300°)$
10. $\cos \frac{7\pi}{4}$

**Suppose $\theta$ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of $\theta$.**

11. $\cos \theta = -\frac{1}{3}$; Quadrant III
12. $\sec \theta = 2$; Quadrant IV
13. $\sin \theta = \frac{2}{3}$; Quadrant II
14. $\tan \theta = -4$; Quadrant IV
15. $\csc \theta = -5$; Quadrant III
16. $\cot \theta = -2$; Quadrant II
17. $\tan \theta = \frac{1}{3}$; Quadrant III
18. $\cos \theta = \frac{1}{4}$; Quadrant I
19. $\csc \theta = -\frac{5}{2}$; Quadrant IV

## Lesson 13-4

(pages 785–792)

**Find the area of $\triangle ABC$. Round to the nearest tenth.**

**1.** $a = 11$ m, $b = 13$ m, $C = 31°$ **2.** $a = 15$ ft, $b = 22$ ft, $C = 90°$ **3.** $a = 12$ cm, $b = 12$ cm, $C = 50°$

**Solve each triangle. Round to the nearest tenth.**

**4.** $A = 18°, B = 37°, a = 15$ **5.** $A = 60°, C = 25°, c = 3$ **6.** $B = 40°, C = 32°, b = 10$

**7.** $B = 10°, C = 23°, c = 8$ **8.** $A = 12°, B = 60°, b = 5$ **9.** $A = 35°, C = 45°, a = 30$

**Determine whether each triangle has no solution, one solution, or two solutions. Then solve each triangle. Round to the nearest tenth.**

**10.** $A = 40°, B = 60°, c = 20$ **11.** $B = 70°, C = 58°, a = 84$ **12.** $A = 40°, a = 5, b = 12$

**13.** $A = 58°, a = 26, b = 29$ **14.** $A = 38°, B = 63°, c = 15$ **15.** $A = 150°, a = 6, b = 8$

**16.** $A = 57°, a = 12, b = 19$ **17.** $A = 25°, a = 125, b = 150$ **18.** $C = 98°, a = 64, c = 90$

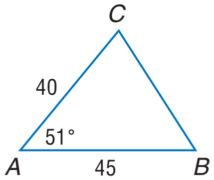
**19.** $A = 40°, B = 60°, c = 20$ **20.** $A = 132°, a = 33, b = 50$ **21.** $A = 5\ 45°, a = 83, b = 79$

## Lesson 13-5

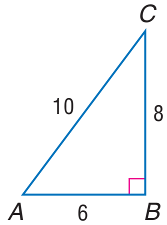
(pages 793–798)

**Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle.**

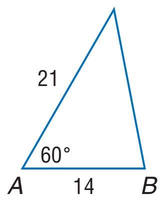
**1.**




**2.**




**3.**




**4.** $a = 14, b = 15, c = 16$ **5.** $B = 41°, C = 52°, c = 27$ **6.** $a = 19, b = 24.3, c = 21.8$

**7.** $A = 112°, a = 32, c = 20$ **8.** $b = 8, c = 7, A = 28°$ **9.** $a = 5, b = 6, c = 7$

**10.** $C = 25°, a = 12, b = 9$ **11.** $a = 8, A = 49°, B = 58°$ **12.** $A = 42°, b = 120, c = 160$

**13.** $c = 10, A = 35°, C = 65°$ **14.** $a = 10, b = 16, c = 19$ **15.** $B = 45°, a = 40, c = 48$

**16.** $B = 100°, a = 10, c = 8$ **17.** $A = 40°, B = 45°, c = 4$ **18.** $A = 20°, b = 100, c = 84$

## Lesson 13-6

(pages 799–805)

**The given point $P$ is located on the unit circle. Find $\sin\theta$ and $\cos\theta$.**

**1.** $P\left(\frac{4}{5}, \frac{3}{5}\right)$ **2.** $P\left(\frac{12}{3}, -\frac{5}{13}\right)$ **3.** $P\left(-\frac{8}{17}, -\frac{15}{17}\right)$ **4.** $P\left(\frac{3}{7}, \frac{2\sqrt{10}}{7}\right)$ **5.** $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

**Find the exact value of each function.**

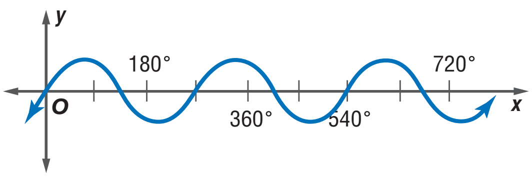
**6.** $\sin 210°$ **7.** $\cos 150°$ **8.** $\cos (2135°)$ **9.** $\cos$

**10.** $\sin 570°$ **11.** $\sin 390°$ **12.** $\sin \frac{4\pi}{3}$ **13.** $\cos -\frac{7\pi}{3}$

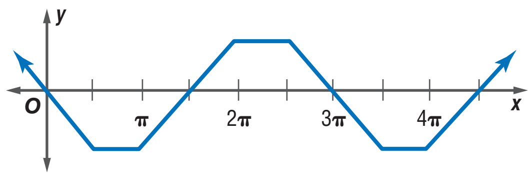
**14.** $\cos 30° + \cos 60°$ **15.** $5(\sin 45°)(\cos 45°)$ **16.** $\frac{\sin 210° + \cos 240°}{3}$ **17.** $\frac{6 \cos 120° + 4 \sin 150°}{5}$

**Determine the period of each function.**

**18.**




**19.**






Extra Practice

## Lesson 13–7

(pages 806–811)

**Write each equation in the form of an inverse function.**

1. $\text{Sin } m + n$
2. $\text{Tan } 45° = 1$
3. $\text{Cos } x = \frac{1}{2}$
4. $\text{Sin } 65° = a$
5. $\text{Tan } 60° = \sqrt{3}$
6. $\text{Sin } x = \frac{\sqrt{2}}{2}$

**Solve each equation.**

7. $y = \text{Sin}^{-1} -\frac{\sqrt{2}}{2}$
8. $\text{Tan}^{-1}(1) = x$
9. $a = \text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$
10. $\text{Arcsin}(0) = x$
11. $y = \text{Cos}^{-1}\frac{1}{2}$
12. $y = \text{Sin}^{-1}(1)$

**Find each value. Round to the nearest hundredth.**

13. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$
14. $\text{Sin}^{-1}(-1)$
15. $\cos\left[\text{Arcsin}\left(\frac{\sqrt{2}}{2}\right)\right]$
16. $\tan\left[\text{Sin}^{-1}\left(\frac{15}{13}\right)\right]$
17. $\sin\left[\text{Arccos}\frac{1}{2}\right]$
18. $\sin\left[\text{Arccos}\left(\frac{5}{17}\right)\right]$
19. $\sin\left[\text{Tan}^{-1}\left(\frac{5}{12}\right)\right]$
20. $\tan\left[\text{Arccos} -\left(\frac{\sqrt{3}}{2}\right)\right]$
21. $\sin^{-1}[\text{Cos}^{-1}(1) - 1]$
22. $\text{Cos}^{-1}\left[\tan\frac{\pi}{4}\right]$
23. $\cos\left[\text{Sin}^{-1}\frac{1}{2}\right]$
24. $\sin[\text{Cos}^{-1}(0)]$

## Lesson 14-1

(pages 822–828)

**Find the amplitude, if it exists, and period of each function. Then graph each function.**

1. $y = 2\cos\theta$
2. $y = \frac{1}{3}\sin\theta$
3. $y = \sin 3\theta$
4. $y = 3\sec\theta$
5. $y = \sec\frac{1}{3}\theta$
6. $y = 2\csc\theta$
7. $y = 3\tan\theta$
8. $y = 3\sin\frac{2}{3}\theta$
9. $y = 2\sin\frac{1}{5}\theta$
10. $y = 3\sin 2\theta$
11. $y = \frac{1}{2}\cos\frac{3}{4}\theta$
12. $y = 5\csc 3\theta$
13. $y = 2\cot 6\theta$
14. $y = 2\csc 6\theta$
15. $y = 3\tan\frac{1}{3}\theta$

## Lesson 14-2

(pages 829–836)

**State the phase shift for each function. Then graph the function.**

1. $y = \sin(\theta + 60°)$
2. $y = \cos(\theta - 90°)$
3. $y = \tan\left(\theta + \frac{\pi}{2}\right)$
4. $y = \sin\theta + \frac{\pi}{6}$

**State the vertical shift and the equation of the midline for each function. Then graph the function.**

5. $y = \cos\theta + 3$
6. $y = \sin\theta - 2$
7. $y = \sec\theta + 5$
8. $y = \csc\theta - 6$
9. $y = 2\sin\theta - 4$
10. $y = \frac{1}{3}\sin\theta + 7$

**State the vertical shift, amplitude, period, and phase shift of each function. Then graph the function.**

11. $y = 3\cos[2(\theta + 30°)] + 4$
12. $y = 2\tan[3(\theta - 60°)] - 2$
13. $y = \frac{1}{2}\sin[4(\theta + 45°)] + 1$
14. $y = \frac{2}{5}\cos[6(\theta + 45°)] - 5$
15. $y = 6 - 2\sin\left[3\left(\theta + \frac{\pi}{2}\right)\right]$
16. $y = 3 + 3\cos\left[2\left(\theta - \frac{\pi}{3}\right)\right]$

## Lesson 14-3

(pages 837–841)

**Find the value of each expression.**

1. $\sin\theta$, if $\cos\theta = \frac{4}{5}$; $0° \le \theta \le 90°$
2. $\tan\theta$, if $\sin\theta = \frac{1}{2}$; $0° \le \theta \le 90°$
3. $\csc\theta$, if $\sin\theta = \frac{3}{4}$; $90° \le \theta \le 180°$
4. $\cos\theta$, if $\tan\theta = 24$; $90° \le \theta \le 180°$
5. $\sec\theta$, if $\tan\theta = 24$; $90° \le \theta \le 180°$
6. $\sin\theta$, if $\cot\theta = -\frac{1}{4}$; $270° \le \theta \le 360°$
7. $\tan\theta$, if $\sec\theta = 23$; $90° \le \theta \le 180°$
8. $\sin\theta$, if $\cos\theta = \frac{3}{5}$; $270° \le \theta \le 360°$
9. $\cos\theta$, if $\sin\theta = -\frac{1}{2}$; $270° \le \theta \le 360°$
10. $\csc\theta$, if $\cot\theta = -\frac{1}{4}$; $90° \le \theta \le 180°$
11. $\csc\theta$, if $\sec\theta = -\frac{5}{3}$; $180° \le \theta \le 270°$
12. $\cos\theta$, if $\tan\theta = 5$; $180° \le \theta \le 270°$

**Simplify each expression.**

13. $\csc^2\theta - \cot^2\theta$
14. $\sin\theta\tan\theta\csc\theta$
15. $\tan\theta\csc\theta$
16. $\sec\theta\cot\theta\cos\theta$
17. $\cos\theta\,(1 - \cos^2\theta)$
18. $\dfrac{1 - \sin^2\theta}{\cos^2\theta}$
19. $\dfrac{\sin^2\theta + \cos^2\theta}{\cos^2}$
20. $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta}$
21. $\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta}$

## Lesson 14-4

(pages 842–846)

**Verify that each of the following is an identity.**

1. $\sin^2\theta + \cos^2\theta + \tan^2\theta = \sec^2\theta$
2. $\dfrac{\tan\theta}{\sin\theta} = \sec\theta$
3. $\dfrac{\tan\theta}{\cot\theta} = \tan^2\theta$
4. $\csc^2\theta\,(1 - \cos^2\theta) = 1$
5. $1 - \cot^4\theta = 2\csc^2\theta - \csc^4\theta$
6. $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$
7. $\sin^2\theta + \cot^2\theta\sin^2\theta = 1$
8. $\dfrac{\cos\theta}{\csc\theta} - \dfrac{\csc\theta}{\sec\theta} = -\dfrac{\cos^3\theta}{\sin\theta}$
9. $\dfrac{\cos\theta}{\sec\theta} - \dfrac{1 + \cos\theta}{\sec\theta + 1} = 2\cot^2\theta$
10. $\dfrac{1 + \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1 - \cos\theta}$
11. $\sec\theta + \tan\theta = \dfrac{\cos\theta}{1 - \sin\theta}$
12. $\tan\theta + \cot\theta = \csc\theta\sec\theta$
13. $\dfrac{\cot^2\theta}{1 + \cot^2\theta} = 1 - \sin^2\theta$
14. $\dfrac{\tan\theta\,2\sin\theta}{\sec\theta} = \dfrac{\sin^3\theta}{1 + \cos\theta}$
15. $\sin^2\theta(1 - \cos^2\theta) = \sin 4\theta$
16. $\sin^2\theta + \sin^2\theta\tan^2\theta = \tan^2\theta$
17. $\dfrac{\sec\theta - 1}{\sec\theta + 1} + \dfrac{\cos\theta - 1}{\cos\theta + 1} = 0$
18. $\tan^2\theta(1 - \sin^2\theta) = \sin^2\theta$
19. $\tan\theta + \dfrac{\cos\theta}{1 + \sin\theta} = \sec\theta$
20. $\dfrac{\tan\theta}{\sec\theta + 1} = \dfrac{1 - \cos\theta}{\sin\theta}$
21. $\csc\theta - \dfrac{\sin\theta}{1 + \cos\theta} = \cot\theta$

## Lesson 14-5

(pages 848–852)

**Find the exact value of each expression.**

1. $\sin 195°$
2. $\cos 285°$
3. $\sin 255°$
4. $\sin 105°$
5. $\cos 15°$
6. $\sin 15°$
7. $\cos 375°$
8. $\sin 165°$
9. $\sin(-225°)$
10. $\cos(-210°)$
11. $\cos(-225°)$
12. $\sin(-30°)$
13. $\sin 120°$
14. $\sin 225°$
15. $\cos(-30°)$

**Verify that each of the following is an identity.**

16. $\sin(90° + \theta) = \cos\theta$
17. $\cos(180° - \theta) = -\cos\theta$
18. $\sin(p + \theta) = -\sin\theta$
19. $\sin(\theta + 30°) + \sin(\theta + 60°) = \sqrt{3} + \frac{1}{2}(\sin\theta + \cos\theta)$
20. $\cos(30° - \theta) + \cos(30° + \theta) = \sqrt{3}\cos\theta$

Extra Practice

## Lesson 14-6

(pages 853–859)

**Find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.**

1. $\cos \theta = \frac{7}{25}$; $0 < \theta < 90°$
2. $\sin \theta = \frac{2}{7}$; $0 < \theta < 90°$
3. $\cos \theta = -\frac{1}{8}$; $180° < \theta < 270°$
4. $\sin \theta = -\frac{5}{13}$; $270° < \theta < 360°$
5. $\sin \theta = \frac{\sqrt{35}}{6}$; $0° < \theta < 90°$
6. $\cos \theta = -\frac{17}{18}$; $90° < \theta < 180°$

**Find the exact value of each expression by using the half-angle formulas.**

7. $\sin 75°$
8. $\cos 75°$
9. $\sin \frac{\pi}{8}$
10. $\cos \frac{13\pi}{12}$
11. $\cos 22.5°$
12. $\cos \frac{\pi}{4}$

**Verify that each of the following is an identity.**

13. $\frac{\sin 2\theta}{2 \sin^2 \theta} = \cot \theta$
14. $1 + \cos 2\theta = \frac{2}{1 + \tan^2 \theta}$
15. $\csc \theta \sec \theta = 2 \csc 2\theta$
16. $\sin 2\theta (\cot \theta + \tan \theta) = 2$
17. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$
18. $\frac{\csc \theta + \sin \theta}{\csc \theta - \sin \theta} = \frac{1 + \sin 2\theta}{\cos 2\theta}$

## Lesson 14-7

(pages 861–866)

**Find all the solutions for each equation for $0° \leq \theta < 360°$.**

1. $\cos \theta = -\frac{\sqrt{3}}{2}$
2. $\sin 2\theta = -\frac{\sqrt{3}}{2}$
3. $\cos 2\theta = 8 - 15 \sin \theta$
4. $\sin \theta + \cos \theta = 1$
5. $2 \sin^2 \theta + \sin \theta = 0$
6. $\sin 2\theta = \cos \theta$

**Solve each equation for all values of $\theta$ if $\theta$ is measured in radians.**

7. $\cos 2\theta \sin \theta = 1$
8. $\sin \frac{\theta}{2} + \cos \frac{\theta}{2} = \sqrt{2}$
9. $\cos 2\theta + 4 \cos \theta = -3$
10. $\sin \frac{\theta}{2} + \cos \theta = 1$
11. $3 \tan^2 \theta - \sqrt{3} \tan \theta = 0$
12. $4 \sin \theta \cos \theta = -\sqrt{3}$

**Solve each equation for all values of $\theta$ if $\theta$ is measured in degrees.**

13. $2 \sin^2 \theta - 1 = 0$
14. $\cos \theta - 2 \cos \theta \sin \theta = 0$
15. $\cos 2\theta \sin \theta = 1$
16. $(\tan \theta - 1)(2 \cos \theta + 1) = 0$
17. $2 \cos^2 \theta = 0.5$
18. $\sin \theta \tan \theta - \tan \theta = 0$

**Solve each equation for all values of $\theta$.**

19. $\tan \theta = 1$
20. $\cos 8\theta = 1$
21. $\sin \theta + 1 = \cos 2\theta$
22. $8 \sin \theta \cos \theta = 2\sqrt{3}$
23. $\cos \theta = 1 + \sin \theta$
24. $2 \cos^2 \theta = \cos \theta$

# Mixed Problem Solving

## Chapter 1 Equations and Inequalities

(pp. 4–55)

**GEOMETRY For Exercises 1 and 2, use the following information.**

The formula for the surface area of a sphere is $SA = 4\pi r^2$, and the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. (Lesson 1-1)

1. Find the volume and surface area of a sphere with radius 2 inches. Write your answer in terms of $\pi$.
2. Is it possible for a sphere to have the same numerical value for the surface area and volume? If so, find the radius of such a sphere.

3. **CONSTRUCTION** The Birtic family is building a family room on their house. The dimensions of the room are 26 feet by 28 feet. Show how to use the Distributive Property to mentally calculate the area of the room. (Lesson 1-2)

**GEOMETRY For Exercises 4–6, use the following information.**

The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. (Lesson 1-2)

4. Use the Distributive Property to rewrite the formula by factoring out the greatest common factor of the two terms.
5. Find the surface area for a cylinder with radius 3 centimeters and height 10 centimeters using both formulas. Leave the answer in terms of $\pi$.
6. Which formula do you prefer? Explain your reasoning.

**POPULATION For Exercises 7 and 8, use the following information.**

In 2004, the population of Bay City was 19,611. For each of the next five years, the population decreased by an average of 715 people per year. (Lesson 1-3)

7. What was the population in 2009?
8. If the population continues to decline at the same rate as from 2004 to 2009, what would you expect the population to be in 2020?

**ASTRONOMY For Exercises 9 and 10, use the following information.**

The planets in our solar system travel in orbits that are not circular. For example, Pluto's farthest distance from the Sun is 4539 million miles, and its closest distance is 2756 million miles. (Lesson 1-4)

9. What is the average of the two distances ?
10. Write an equation that can be solved to find the minimum and maximum distances from the Sun to Pluto.

**HEALTH For Exercises 11 and 12, use the following information.**

The National Heart Association recommends that less than 30% of a person's total daily Caloric intake come from fat. One gram of fat yields nine Calories. Jason is a healthy 21-year-old male whose average daily Caloric intake is between 2500 and 3300 Calories. (Lesson 1-5)

11. Write an inequality that represents the suggested fat intake for Jason.
12. What is the greatest suggested fat intake for Jason?

**TRAVEL For Exercises 13 and 14, use the following information.**

Bonnie is planning a 5-day trip to a convention. She wants to spend no more than \$1000. The plane ticket is \$375, and the hotel is \$85 per night. (Lesson 1-5)

13. Let $f$ represent the cost of food for one day. Write an inequality to represent this situation.
14. Solve the inequality and interpret the solution.

15. **PAINTING** Phil owns and operates a home remodeling business. He estimates that he will need 12–15 gallons of paint for a particular project. If each gallon of paint costs \$18.99, write and solve a compound inequality to determine what the cost $c$ of the paint could be. (Lesson 1-6)

Mixed Problem Solving

**AGRICULTURE For Exercises 1–3, use the following information.**

The table shows the average prices received by farmers for a bushel of corn. (Lesson 2-1)

| Year | Price | Year | Price |
|---|---|---|---|
| 1940 | \$0.62 | 1980 | \$3.11 |
| 1950 | \$1.52 | 1990 | \$2.28 |
| 1960 | \$1.00 | 2000 | \$1.85 |
| 1970 | \$1.33 | | |

**Source:** *The World Almanac*

1. Write a relation to represent the data.
2. Graph the relation.
3. Is the relation a function? Explain.

**MEASUREMENT For Exercises 4 and 5, use the following information.**

The equation $y = 0.3937x$ can be used to convert any number of centimeters $x$ to inches $y$. (Lesson 2-2)

4. Find the number of inches in 100 centimeters.
5. Find the number of centimeters in 12 inches.

**POPULATION For Exercises 6 and 7, use the following information.**

The table shows the growth in the population of Miami, Florida. (Lesson 2-3)

| Year | Population | Year | Population |
|---|---|---|---|
| 1950 | 249,276 | 1990 | 358,648 |
| 1970 | 334,859 | 2000 | 362,437 |
| 1980 | 346,681 | 2003 | 376,815 |

**Source:** *The World Almanac*

6. Graph the data in the table.
7. Find the average rate of change.

**HEALTH For Exercises 8–10, use the following information.**

In 1985, 39% of people in the United States age 12 and over reported using cigarettes. The percent of people using cigarettes has decreased about 1.7% per year following 1985.
**Source:** *The World Almanac* (Lesson 2-4)

8. Write an equation that represents how many people use cigarettes in $x$ years.
9. If the percent of people using cigarettes continues to decrease at the same rate, what percent of people would you predict to be using cigarettes in 2005?
10. If the trend continues, when would you predict there to be no people using cigarettes in the U.S.? How accurate is your prediction?

**EMPLOYMENT For Exercises 11–15, use the table that shows unemployment statistics for 1993 to 1999.** (Lesson 2-5)

| Year | Number Unemployed | Percent Unemployed |
|---|---|---|
| 1993 | 8,940,000 | 6.9 |
| 1994 | 7,996,000 | 6.1 |
| 1995 | 7,404,000 | 5.6 |
| 1996 | 7,236,000 | 5.4 |
| 1997 | 6,739,000 | 4.9 |
| 1998 | 6,210,000 | 4.5 |
| 1999 | 5,880,000 | 4.2 |

**Source:** *The World Almanac*

11. Draw two scatter plots of the data. Let $x$ represent the year.
12. Use two ordered pairs to write an equation for each scatter plot.
13. Compare the two equations.
14. Predict the percent of people that will be unemployed in 2005.
15. In 1999, what was the total number of people in the United States?
16. **EDUCATION** At Madison Elementary, each classroom can have at most 25 students. Draw a graph of a step function that shows the relationship between the number of students $x$ and the number of classrooms $y$ that are needed. (Lesson 2-6)

**CRAFTS For Exercises 17–19, use the following information.**

Priscilla sells stuffed animals at a local craft show. She charges \$10 for the small and \$15 for the large ones. To cover her expenses, she needs to sell at least \$350. (Lesson 2-7)

17. Write an inequality for this situation.
18. Graph the inequality.
19. If she sells 10 small and 15 large animals, will she cover her expenses?

**EXERCISE For Exercises 1–4, use the following information.**

At Everybody's Gym, you have two options for becoming a member. You can pay $400 per year or you can pay $150 per year plus $5 per visit. (Lesson 3-1)

1. For each option, write an equation that represents the cost of belonging to the gym.
2. Graph the equations. Estimate the break-even point for the gym memberships.
3. Explain what the break-even point means.
4. If you plan to visit the gym at least once per week during the year, which option should you choose?

5. **GEOMETRY** Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines whose equations are $y = 3$, $y = 7$, $y = 2x$, and $y = 2x - 13$. (Lesson 3-2)

**EDUCATION For Exercises 6–9, use the following information.**

Mr. Gunlikson needs to purchase equipment for his physical education classes. His budget for the year is $4250. He decides to purchase cross-country ski equipment. He is able to find skis for $75 per pair and boots for $40 per pair. He knows that he should buy more boots than skis because the skis are adjustable to several sizes of boots. (Lesson 3-3)

6. Let $y$ be the number of pairs of boots and $x$ be the number of pairs of skis. Write a system of inequalities for this situation. (Remember that the number of pairs of boots and skis must be positive.)
7. Graph the region that shows how many pairs of boots and skis he can buy.
8. Give an example of three different purchases that Mr. Gunlikson can make.
9. Suppose Mr. Gunlikson wants to spend all of the money. What combination of skis and boots should he buy? Explain.

**MANUFACTURING For Exercises 10–14, use the following information.**

A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one, 1 hour in step two, and produces a profit of $20. Each pair of indoor shoes requires 1 hour in step one, 3 hours in step two, and produces a profit of $15. The company has 40 hours of labor per day available for step one and 60 hours available for step two. (Lesson 3-4)

10. Let $x$ represent the number of pairs of outdoor shoes and let $y$ represent the number of indoor shoes that can be produced per day. Write a system of inequalities to represent the number of pairs of outdoor and indoor soccer shoes that can be produced in one day.
11. Draw the graph showing the feasible region.
12. List the coordinates of the vertices of the feasible region.
13. Write a function for the total profit.
14. What is the maximum profit? What is the combination of shoes for this profit?

**GEOMETRY For Exercises 15–17, use the following information.**

An isosceles trapezoid has shorter base of measure $a$, longer base of measure $c$, and congruent legs of measure $b$. The perimeter of the trapezoid is 58 inches. The average of the bases is 19 inches and the longer base is twice the leg plus 7. (Lesson 3-5)

15. Write a system of three equations that represents this situation.
16. Find the lengths of the sides of the trapezoid.
17. Find the area of the trapezoid.

18. **EDUCATION** The three American universities with the greatest endowments in 2000 were Harvard, Yale, and Stanford. Their combined endowments are $38.1 billion. Harvard had $0.1 billion more in endowments than Yale and Stanford together. Stanford's endowments trailed Harvard's by $10.2 billion. What were the endowments of each of these universities? (Lesson 3-5)

**AGRICULTURE** **For Exercises 1 and 2, use the following information.**

In 2003, the United States produced 63,590,000 metric tons of wheat, 9,034,000 metric tons of rice, and 256,905,000 metric tons of corn. In that same year, Russia produced 34,062,000 metric tons of wheat, 450,000 metric tons of rice, and 2,113,000 metric tons of corn.

**Source:** *The World Almanac* (Lesson 4-1)

1. Organize the data in two matrices.
2. What are the dimensions of the matrices?

**LIFE EXPECTANCY** **For Exercises 3–5, use the life expectancy table.** (Lesson 4-2)

| Year | 1910 | 1930 | 1950 | 1970 | 1990 |
|---|---|---|---|---|---|
| Male | 48.4 | 58.1 | 65.6 | 67.1 | 71.8 |
| Female | 51.8 | 61.6 | 71.1 | 74.7 | 78.8 |

**Source:** *The World Almanac*

3. Organize all the data in a matrix.
4. Show how to organize the data in two matrices in such a way that you can find the difference between the life expectancies of males and females for the given years. Then find the difference.
5. Does addition of any two of the matrices make sense? Explain.

**CRAFTS** **For Exercises 6 and 7, use the following information.**

Mrs. Long is selling crocheted items. She sells large afghans for \$60, baby blankets for \$40, doilies for \$25, and pot holders for \$5. She takes the following number of items to the fair: 12 afghans, 25 baby blankets, 45 doilies, and 50 pot holders. (Lesson 4-3)

6. Write an inventory matrix for the number of each item and a cost matrix for the price of each item.
7. Suppose Mrs. Long sells all of the items. Find her total income as a matrix.

**GEOMETRY** **For Exercises 8–11, use the following information.**

A trapezoid has vertices $T(3, 3)$, $R(-1, 3)$, $A(-2, -4)$, and $P(5, -4)$. (Lesson 4-4)

8. Show how to use a reflection matrix to find the vertices of *TRAP* after a reflection over the $x$-axis.
9. The area of a trapezoid is found by multiplying one-half the sum of the bases by the height. Find the areas of *TRAP* and $T'R'A'P'$. How do they compare?
10. Show how to use a matrix and scalar multiplication to find the vertices of *TRAP* after a dilation that triples its perimeter.
11. Find the areas of *TRAP* and $T'R'A'P'$ in Exercise 10. How do they compare?

**AGRICULTURE** **For Exercises 12 and 13, use the following information.**

A farm has a triangular plot defined by the coordinates $\left(-\frac{1}{2}, -\frac{1}{4}\right)$, $\left(\frac{1}{3}, \frac{1}{2}\right)$, and $\left(\frac{2}{3}, -\frac{1}{2}\right)$, where units are in square miles. (Lesson 4-5)

12. Find the area of the region in square miles.
13. One square mile equals 640 acres. To the nearest acre, how many acres are in the triangular plot?

**ART** **For Exercises 14 and 15, use the following information.**

Small beads sell for \$5.80 per pound, and large beads sell for \$4.60 per pound. Bernadette bought a bag of beads for \$33 that contained 3 times as many pounds of the small beads as the large beads. (Lesson 4-6)

14. Write a system of equations using the information given.
15. How many pounds of small and large beads did Bernadette buy?

**MATRICES** **For Exercises 16 and 17, use the following information.**

Two $2 \times 2$ inverse matrices have a sum of $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$. The value of each entry is no less than $-3$ and no greater than 2. (Lesson 4-7)

16. Find the two matrices that satisfy the conditions.
17. Explain your method.
18. **CONSTRUCTION** Alan charges \$750 to build a small deck and \$1250 to build a large deck. During the spring and summer, he built 5 more small decks than large decks. If he earned \$11,750, how many of each type of deck did he build? (Lesson 4-8)

Mixed Problem Solving

**PHYSICS For Exercises 1–3, use the following information.**

A model rocket is shot straight up from the top of a 100-foot building at a velocity of 800 feet per second. (Lesson 5-1)

1. The height $h(t)$ of the model rocket $t$ seconds after firing is given by $h(t) = -16t^2 + at + b$, where $a$ is the initial velocity in feet per second and $b$ is the initial height of the rocket above the ground. Write an equation for the rocket.
2. Find the maximum height of the rocket and the time that the height is reached.
3. Suppose a rocket is fired from the ground (initial height is 0). Find values for $a$, initial velocity, and $t$, time, if the rocket reaches a height of 32,000 feet at time $t$.

**RIDES For Exercises 4 and 5, use the following information.**

An amusement park ride carries riders to the top of a 225-foot tower. The riders then free-fall in their seats until they reach 30 feet above the ground. (Lesson 5-2)

4. Use the formula $h(t) = -16t^2 + h_0$, where the time $t$ is in seconds and the initial height $h_0$ is in feet, to find how long the riders are in free-fall.
5. Suppose the designer of the ride wants the riders to experience free-fall for 5 seconds before stopping 30 feet above the ground. What should be the height of the tower?

**CONSTRUCTION For Exercises 6 and 7, use the following information.**

Nicole's new house has a small deck that measures 6 feet by 12 feet. She would like to build a larger deck. (Lesson 5-3)

6. By what amount must each dimension be increased to triple the area of the deck?
7. What are the new dimensions of the deck?

**NUMBER THEORY For Exercises 8 and 9, use the following information.**

Two complex conjugate numbers have a sum of 12 and a product of 40. (Lesson 5-4)

8. Find the two numbers.
9. Explain the method you used.

**CONSTRUCTION For Exercises 10 and 11, use the following information.**

A contractor wants to construct a rectangular pool with a length that is twice the width. He plans to build a two-meter-wide walkway around the pool. He wants the area of the walkway to equal the surface area of the pool. (Lesson 5-5)

10. Find the dimensions of the pool to the nearest tenth of a meter.
11. What is the surface area of the pool to the nearest square meter?

**PHYSICS For Exercises 12–14, use the following information.**

A ball is thrown into the air vertically with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground $t$ seconds after release is modeled by the equation $h(t) = -16t^2 + 112t + 6$. (Lesson 5-6)

12. When will the ball reach 130 feet?
13. Will the ball ever reach 250 feet? Explain.
14. In how many seconds after its release will the ball hit the ground?

**WEATHER For Exercises 15–17, use the following information.**

The normal monthly high temperatures for Albany, New York, are 21, 24, 34, 46, 58, 67, 72, 70, 61, 50, 40, and 27 degrees Fahrenheit, respectively. **Source:** *The World Almanac* (Lesson 5-7)

15. Suppose January = 1, February = 2, and so on. A graphing calculator gave the following function as a model for the data: $y = -1.5x^2 + 21.2x - 8.5$. Graph the points in the table and the function on the same coordinate plane.
16. Identify the vertex, axis of symmetry, and direction of opening for this function.
17. Discuss how well you think the function models the actual temperature data.
18. **MODELS** John is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could the width of the table be? (Lesson 5-8)

1. **EDUCATION** In 2002 in the United States, there were 3,034,065 classroom teachers and 48,201,550 students. An average of \$7731 was spent per student. Find the total amount of money spent for students in 2002. Write the answer in both scientific and standard notation. **Source:** *The World Almanac* (Lesson 6-1)

**POPULATION For Exercises 2–4, use the following information.**

In 2000, the population of Mexico City was 18,131,000, and the population of Bombay was 18,066,000. It is projected that, until the year 2015, the population of Mexico City will increase at the rate of 0.4% per year and the population of Bombay will increase at the rate of 3% per year. **Source:** *The World Almanac* (Lesson 6-2)

2. Let $r$ represent the rate of increase in population for each city. Write a polynomial to represent the population of each city in 2002.
3. Predict the population of each city in 2015.
4. If the projected rates are accurate, in what year will the two cities have approximately the same population?

**POPULATION For Exercises 5–8, use the following information.**

The table shows the percent of the U.S. population that was foreign-born during various years. The $x$-values are years since 1900 and the $y$-values are the percent of the population. **Source:** *The World Almanac* (Lesson 6-3 and 6-4)

**U.S. Foreign-Born Population**

| x | y | x | y |
|---|---|---|---|
| 0 | 13.6 | 60 | 5.4 |
| 10 | 14.7 | 70 | 4.7 |
| 20 | 13.2 | 80 | 6.2 |
| 30 | 11.6 | 90 | 8.0 |
| 40 | 8.8 | 100 | 10.4 |
| 50 | 6.9 | | |

5. Graph the function.
6. Describe the turning points of the graph and its end behavior.
7. What do the relative maxima and minima represent?
8. If this graph were modeled by a polynomial equation, what is the least degree the equation could have?

**GEOMETRY For Exercises 9 and 10, use the following information.**

Hero's formula for the area of a triangle is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $a$, $b$, and $c$ are the lengths of the sides of the triangle and $s = 0.5(a + b + c)$. (Lesson 6-5)

9. Find the lengths of the sides of the triangle given in this application of Hero's formula: $A = \sqrt{s^4 - 12s^3 + 47s^2 - 60s}$.
10. What type of triangle is this?

**GEOMETRY For Exercises 11 and 12, use the following information.**

The volume of a rectangular box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. (Lesson 6-6)

11. What are the width and length of the box?
12. Will the ratio of the dimensions of the box always be the same regardless of the value of $x$? Explain .

**SALES For Exercises 13 and 14, use the following information.**

The sales of items related to information technology can be modeled by $S(x) = -1.7x^3 + 18x^2 + 26.4x + 678$, where $x$ is the number of years since 1996 and $y$ is billions of dollars. **Source:** *The World Almanac* (Lesson 6-7)

13. Use synthetic substitution to estimate the sales for 2003 and 2006.
14. Do you think this model is useful in estimating future sales? Explain.
15. **MANUFACTURING** A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. How much should be added to each dimension? (Lesson 6-8)
16. **CONSTRUCTION** A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet? (Lesson 6-9)

Mixed Problem Solving

**EMPLOYMENT For Exercises 1 and 2, use the following information.**

From 1994 to 1999, the number of employed women and men in the United States, age 16 and over, can be modeled by the following equations, where $x$ is the number of years since 1994 and $y$ is the number of people in thousands. Source: *The World Almanac* (Lesson 7-1)

women: $y = 1086.4x + 56{,}610$

men: $y = 999.2x + 66{,}450$

1. Write a function that models the total number of men and women employed in the United States during this time.
2. If $f$ is the function for the number of men, and $g$ is the function for the number of women, what does $(f - g)(x)$ represent?

3. **HEALTH** The average weight of a baby born at a certain hospital is $7\frac{1}{2}$ pounds, and the average length is 19.5 inches. One kilogram is about 2.2 pounds, and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters. (Lesson 7-2)

**SAFETY For Exercises 4 and 5, use the following information.**

The table shows the total stopping distance $x$, in meters, of a vehicle and the speed $y$, in meters per second. (Lesson 7-3)

| Distance | 92 | 68 | 49 | 32 | 18 |
|---|---|---|---|---|---|
| Speed | 29 | 25 | 20 | 16 | 11 |

4. Graph the data in the table.
5. Graph the function $y = 2\sqrt{2x}$ on the same coordinate plane. How well do you think this function models the given data? Explain.

6. **PHYSICS** The speed of sound in a liquid is $s = \sqrt{\frac{B}{d}}$, where $B$ is known as the bulk modulus of the liquid and $d$ is the density of the liquid. For water, $B = 2.1 \cdot 10^9$ N/m$^2$ and $d = 10^3$ kg/m$^3$. Find the speed of sound in water to the nearest meter per second. (Lesson 7-4)

**LAW ENFORCEMENT For Exercises 7 and 8, use the following information.**

The approximate speed $s$ in miles per hour that a car was traveling if it skidded $d$ feet is given by the formula $s = 5.5\sqrt{kd}$, where $k$ is the coefficient of friction. (Lesson 7-5)

7. For a dry concrete road, $k = 0.8$. If a car skids 110 feet on a dry concrete road, find its speed in miles per hour to the nearest whole number.
8. Another formula using the same variables is $s = 2\sqrt{5kd}$. Compare the results using the two formulas. Explain any variations in the answers.

**PHYSICS For Exercises 9–11, use the following information.**

Kepler's Third Law of planetary motion states that the square of the orbital period of any planet, in Earth years, is equal to the cube of the planet's distance from the Sun in astronomical units (AU). Source: *The World Almanac* (Lesson 7-6)

9. The orbital period of Mercury is 87.97 Earth days. What is Mercury's distance from the Sun in AU?
10. Pluto's period of revolution is 247.66 Earth years. What is Pluto's distance from the Sun?
11. What is Earth's distance from the Sun in AU? Explain your result.

**PHYSICS For Exercises 12 and 13, use the following information.**

The time $T$ in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where $L$ is the length of the pendulum in feet and $g$ is the acceleration due to gravity, 32 feet per second squared. (Lesson 7-7)

12. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?
Source: *The Guinness Book of Records*
13. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

**MANUFACTURING For Exercises 1–3, use the following information.**

The volume of a shipping container in the shape of a rectangular prism can be represented by the polynomial $6x^3 + 11x^2 + 4x$, where the height is $x$. (Lesson 8-1)

1. Find the length and width of the container.
2. Find the ratio of the three dimensions of the container when $x = 2$.
3. Will the ratio of the three dimensions be the same for all values of $x$?

**PHOTOGRAPHY For Exercises 4–6, use the following information.**

The formula $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ can be used to determine how far the film should be placed from the lens of a camera. The variable $q$ represents the distance from the lens to the film, $f$ represents the focal length of the lens, and $p$ represents the distance from the object to the lens. (Lesson 8-2)

4. Solve the formula for $\frac{1}{p}$.
5. Write the expression containing $f$ and $q$ as a single rational expression.
6. If a camera has a focal length of 8 centimeters and the lens is 10 centimeters from the film, how far should an object be from the lens so that the picture will be in focus?

**PHYSICS For Exercises 7 and 8, use the following information.**

The Inverse Square Law states that the relationship between two variables is related to the equation $y = \frac{1}{x^2}$. (Lesson 8-3)

7. Graph $y = \frac{1}{x^2}$.
8. Give the equations of any asymptotes.

**PHYSICS For Exercises 9 and 10, use the following information.**

The formula for finding the gravitational force between two objects is $F = G\frac{m_A m_B}{d^2}$, where $F$ is the gravitational force between the objects, $G$ is the universal constant, $m_A$ is the mass of the first object, $m_B$ is the mass of the second object, and $d$ is the distance between the centers of the objects. (Lesson 8-4)

9. If the mass of object A is constant, does Newton's formula represent a *direct* or *inverse* variation between the mass of object B and the distance?
10. The value of $G$ is $6.67 \times 10^{-11}$ N • m$^2$/kg$^2$. If two objects each weighing 5 kilograms are placed so that their centers are 0.5 meter apart, what is the gravitational force between the two objects?

**EDUCATION For Exercises 11–13, use the table that shows the average number of students per computer in United States public schools for various years.** (Lesson 8-5)

| Year | Students | Year | Students |
|---|---|---|---|
| 1988 | 32 | 1996 | 10 |
| 1989 | 25 | 1997 | 7.8 |
| 1990 | 22 | 1998 | 6.1 |
| 1991 | 20 | 1999 | 5.7 |
| 1992 | 18 | 2000 | 5.4 |
| 1993 | 16 | 2001 | 5.0 |
| 1994 | 14 | 2002 | 4.9 |
| 1995 | 10.5 | 2003 | 4.9 |

**Source:** *The World Almanac*

11. Let $x$ represent years where 1988 = 1, 1989 = 2, and so on. Let $y$ represent the number of students. Graph the data.
12. What type of function does the graph most closely resemble?
13. Use a graphing calculator to find an equation that models the data.

**TRAVEL For Exercises 14 and 15, use the following information.**

A trip between two towns takes 4 hours under ideal conditions. The first 150 miles of the trip is on an interstate, and the last 130 miles is on a highway with a speed limit that is 10 miles per hour less than on the interstate. (Lesson 8-6)

14. If $x$ represents the speed limit on the interstate, write expressions for the time spent at that speed and for the time spent on the other highway.
15. Write and solve an equation to find the speeds limits on the two highways.

**POPULATION For Exercises 1–4, use the following information.**

In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion. **Source:** *The World Almanac* (Lesson 9-1)

1. Write an exponential function of the form $y = ab^x$ that could be used to model the world population $y$ in billions for 1950 to 1980. Write the equation in terms of $x$, the number of years since 1950. (Round the value of $b$ to the nearest ten-thousandth.)
2. Suppose the population continued to grow at that rate. Estimate the population in 2000.
3. In 2000, the population of the world was about 6.08 billion. Compare your estimate to the actual population.
4. Use the equation you wrote in Exercise 1 to estimate the world population in the year 2020. How accurate do you think the estimate is? Explain your reasoning.

**EARTHQUAKES For Exercises 5–8, use the following information.**

The table shows the Richter scale that measures earthquake intensity. Column 2 shows the increase in intensity between each number. For example, an earthquake that measures 7 is 10 times more intense than one measuring 6. (Lesson 9-2)

| Richter Number $x$ | Increase in Magnitude $y$ |
|---|---|
| 1 | 1 |
| 2 | 10 |
| 3 | 100 |
| 4 | 1000 |
| 5 | 10,000 |
| 6 | 100,000 |
| 7 | 1,000,000 |
| 8 | 10,000,000 |

**Source:** The New York Public Library

5. Graph this function.
6. Write an equation of the form $y = b^{x-c}$ for the function in Exercise 5. (*Hint:* Write the values in the second column as powers of 10 to see a pattern and find the value of $c$.)
7. Graph the inverse of the function in Exercise 6.
8. Write an equation of the form $y = \log_{10} x + c$ for the function in Exercise 7.

**EARTHQUAKES For Exercises 9 and 10, use the table showing the magnitude of some major earthquakes.** (Lesson 9-3)

| Year/Location | Magnitude |
|---|---|
| 1939/Turkey | 8.0 |
| 1963/Yugoslavia | 6.0 |
| 1970/Peru | 7.8 |
| 1988/Armenia | 7.0 |
| 2004/Morocco | 6.4 |

**Source:** *The World Almanac*

9. For which two earthquakes was the intensity of one 10 times that of the other? For which two was the intensity of one 100 times that of the other?
10. What would be the magnitude of an earthquake that is 1000 times as intense as the 1963 earthquake in Yugoslavia?

11. Suppose you know that $\log_7 2 \approx 0.3562$ and $\log_7 3 \approx 0.5646$. Describe two different methods that you could use to approximate $\log_7 2.5$. (You can use a calculator, of course.) Then describe how you can check your result. (Lesson 9-4)

**WEATHER For Exercises 12 and 13, use the following information.**

The atmospheric pressure $P$, in bars, of a given height on Earth is given by using the formula $P = s \cdot e^{-\frac{k}{H}}$. In the formula, $s$ is the surface pressure on Earth, which is approximately 1 bar, $h$ is the altitude for which you want to find the pressure in kilometers, and $H$ is always 7 kilometers. (Lesson 9-5)

12. Find the pressure for 2, 4, and 7 kilometers.
13. What do you notice about the pressure as altitude increases?

**AGRICULTURE For Exercises 14–16, use the following information.**

An equation that models the decline in the number of U.S. farms is $y = 3{,}962{,}520(0.98)^x$, where $x$ is years since 1960 and $y$ is the number of farms. **Source:** *Wall Street Journal* (Lesson 9-6)

14. How can you tell that the number is declining?
15. By what annual rate is the number declining?
16. Predict when the number of farms will be less than 1.5 million.

**GEOMETRY For Exercises 1–4, use the following information.**

Triangle $ABC$ has vertices $A(2, 1)$, $B(-6, 5)$, and $C(-2, -3)$. (Lesson 10-1)

1. An isosceles triangle has two sides with equal length. Is $\triangle ABC$ isosceles? Explain.
2. An equilateral triangle has three sides of equal length. Is $\triangle ABC$ equilateral? Explain.
3. Triangle $EFG$ is formed by joining the midpoints of the sides of $\triangle ABC$. What type of triangle is $\triangle EFG$? Explain.
4. Describe any relationship between the lengths of the sides of the two triangles.

**ENERGY For Exercises 5–8, use the following information.**

A parabolic mirror can be used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The latus rectum of a particular mirror is 40 feet long. (Lesson 10-2)

5. Write an equation for the parabola formed by the mirror if the vertex of the mirror is 9.75 feet below the origin.
6. One foot is exactly 0.3048 meter. Rewrite the equation in terms of meters.
7. Graph one of the equations for the mirror.
8. Which equation did you choose to graph? Explain.

**COMMUNICATION For Exercises 9–11, use the following information.**

The radio tower for KCGM has a circular radius for broadcasting of 65 miles. The radio tower for KVCK has a circular radius for broadcasting of 85 miles. (Lesson 10-3)

9. Let the radio tower for KCGM be located at the origin. Write an equation for the set of points at the maximum broadcast distance from the tower.
10. The radio tower for KVCK is 50 miles south and 15 miles west of the KCGM tower. Let each mile represent one unit. Write an equation for the set of points at the maximum broadcast distance from the KVCK tower.
11. Graph the two equations and show the area where the radio signals overlap.

**ASTRONOMY For Exercises 12–14, use the table that shows the closest and farthest distances of Venus and Jupiter from the Sun in millions of miles.** (Lesson 10-4)

| Planet | Closest | Farthest |
|---|---|---|
| Venus | 66.8 | 67.7 |
| Jupiter | 460.1 | 507.4 |

**Source:** *The World Almanac*

12. Write an equation for the orbit of each planet, assuming that the center of the orbit is the origin, the center of the Sun is a focus, and the Sun lies on the $x$-axis.
13. Find the eccentricity for each planet.
14. Which planet has an orbit that is closer to a circle? Explain your reasoning.
15. A comet follows a path that is one branch of a hyperbola. Suppose Earth is the center of the hyperbolic curve and has coordinates (0, 0). Write an equation for the path of the comet if $c = 5{,}225{,}000$ miles and $a = 2{,}500{,}000$ miles. Let the $x$-axis be the transverse axis. (Lesson 10-5)

**AVIATION For Exercises 16–18, use the following information.**

The path of a military jet during an air show can be modeled by a conic section with equation $24x^2 + 1000y - 31{,}680x - 45{,}600 = 0$, where distances are in feet. (Lesson 10-6)

16. Identify the shape of the path of the jet. Write the equation in standard form.
17. If the jet begins its ascent at (0, 0), what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?
18. What is the maximum height of the jet?

**SATELLITES For Exercises 19 and 20, use the following information.**

The equations of the orbits of two satellites are $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$ and $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$, where distances are in km and Earth is the center of each curve. (Lesson 10-7)

19. Solve each equation for $y$.
20. Use a graphing calculator to estimate the intersection points of the two orbits.

**CLUBS For Exercises 1 and 2, use the following information.**

A quilting club consists of 9 members. Every week, each member must bring one completed quilt square. (Lesson 11-1)

1. Find the first eight terms of the sequence that describes the total number of squares that have been made after each meeting.
2. One particular quilt measures 72 inches by 84 inches and is being designed with 4-inch squares. After how many meetings will the quilt be complete?

**ART For Exercises 3 and 4, use the following information.**

Alberta is making a beadwork design consisting of rows of colored beads. The first row consists of 10 beads, and each consecutive row will have 15 more beads than the previous row. (Lesson 11-2)

3. Write an equation for the number of beads in the $n$th row.
4. Find the number of beads in the design if it contains 25 rows.

**GAMES For Exercises 5 and 6, use the following information.**

An audition is held for a TV game show. At the end of each round, one half of the prospective contestants are eliminated from the competition. On a particular day, 524 contestants begin the audition. (Lesson 11-3)

5. Write an equation for finding the number of contestants that are left after $n$ rounds.
6. Using this method, will the number of contestants that are to be eliminated always be a whole number? Explain.

**SPORTS For Exercises 7–9, use the following information.**

Caitlin is training for a marathon (about 26 miles). She begins by running 2 miles. Then, when she runs every other day, she runs one and a half times the distance she ran the time before. (Lesson 11-4)

7. Write the first five terms of a sequence describing her training schedule.
8. When will she exceed 26 miles in one run?
9. When will she have run 100 total miles?

**GEOMETRY For Exercises 10–12, use a square of paper at least 8 inches on a side.** (Lesson 11-5)

10. Let the square be one unit. Cut away one half of the square. Call this piece Term 1. Next, cut away one half of the remaining sheet of paper. Call this piece Term 2. Continue cutting the remaining paper in half and labeling the pieces with a term number as long as possible. List the fractions represented by the pieces.
11. If you could cut squares indefinitely, you would have an infinite series. Find the sum of the series.
12. How does the sum of the series relate to the original square of paper?

**BIOLOGY For Exercises 13–15, use the following information.**

In a particular forest, scientists are interested in how the population of wolves will change over the next two years. One model for animal population is the Verhulst population model, $p_{n+1} = p_n + rp_n(1 - p_n)$, where $n$ represents the number of time periods that have passed, $p_n$ represents the percent of the maximum sustainable population that exists at time $n$, and $r$ is the growth factor. (Lesson 11-6)

13. To find the population of the wolves after one year, evaluate $p_1 = 0.45 + 1.5(0.45)(1 - 0.45)$.
14. Explain what each number in the expression in Exercise 13 represents.
15. The current population of wolves is 165. Find the new population by multiplying 165 by the value in Exercise 13.
16. **PASCAL'S TRIANGLE** Study the first eight rows of Pascal's triangle. Write the sum of the terms in each row as a list. Make a conjecture about the sums of the rows of Pascal's triangle. (Lesson 11-7)
17. **NUMBER THEORY** Two statements that can be proved using mathematical induction are $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^n} = \frac{1}{2}\left(1 - \frac{1}{3^n}\right)$ and $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^n} = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$.

    Write and prove a conjecture involving $\frac{1}{5}$ that is similar to the statements. (Lesson 11-8)

**NUMBER THEORY For Exercises 1–3, use the following information.**

According to the Rational Zero Theorem, if $\frac{p}{q}$ is a rational root, then $p$ is a factor of the constant of the polynomial, and $q$ is a factor of the leading coefficient. (Lesson 12-1)

1. What is the maximum number of possible rational roots that you may need to check for the polynomial $3x^4 - 5x^3 + 2x^2 - 7x + 10$? Explain your answer.
2. Why may you not need to check the maximum number of possible roots?
3. Are choosing the numerator and the denominator for a possible rational root independent or dependent events?

4. **GARDENING** A gardener is selecting plants for a special display. There are 15 varieties of pansies from which to choose. The gardener can only use 9 varieties in the display. How many ways can 9 varieties be chosen from the 15 varieties? (Lesson 12-2)

**SPEED LIMITS For Exercises 5 and 6, use the following information.**

| Speed Limit | Number of States |
|---|---|
| 60 | 1 |
| 65 | 20 |
| 70 | 16 |
| 75 | 13 |

**Source:** *The World Almanac*

The table shows the number of states having each maximum speed limit for their rural interstates. (Lesson 12-3)

5. If a state is randomly selected, what is the probability that its speed limit is 75? 60?
6. If a state is randomly selected, what is the probability that its speed limit is 60 or greater?

7. **LOTTERIES** A lottery number for a particular state has seven digits, which can be any digit from 0 to 9. It is advertised that the odds of winning the lottery are 1 to 10,000,000. Is this statement about the odds correct? Explain your reasoning. (Lesson 12-4)

**For Exercises 8 and 9, use the table that shows the most popular colors for luxury cars in 2003.** (Lesson 12-5)

| Color | % of cars | Color | % of cars |
|---|---|---|---|
| gray | 23.3 | red | 3.9 |
| silver | 18.8 | blue | 3.8 |
| wh. metallic | 17.8 | gold | 3.6 |
| white | 12.6 | lt. blue | 3.1 |
| black | 10.9 | other | 2.2 |

**Source:** *The World Almanac*

8. If a car is randomly selected, what is the probability that it is gray or silver?
9. In a parking lot of 1000 cars sold in 2003, how many cars would you expect to be white or black?

**EDUCATION For Exercises 10–12, use the following information.**

The list shows the average scores for each state for the ACT for 2003–2004. (Lesson 12-6)

20.2, 21.3, 21.5, 20.4, 21.6, 20.3, 22.5, 21.5, 17.8, 20.5, 20.0, 21.7, 21.3, 20.3, 21.6, 22.0, 21.6, 20.3, 19.8, 22.6, 20.8, 22.4, 21.4, 22.2, 18.8, 21.5, 21.7, 21.7, 21.2, 22.5, 21.2, 20.1, 22.3, 20.3, 21.2, 21.4, 20.6, 22.5, 21.8, 21.9, 19.3, 21.5, 20.5, 20.3, 21.5, 22.7, 20.9, 22.5, 22.2, 21.4

10. Compare the mean and median of the data.
11. Find the standard deviation of the data. Round to the nearest hundredth.
12. Suppose the state with an average score of 20.0 incorrectly reported the results. The score for the state is actually 22.5. How are the mean and median of the data affected by this data change?

13. **HEALTH** The heights of students at Madison High School are normally distributed with a mean of 66 inches and a standard deviation of 2 inches. Of the 1080 students in the school, how many would you expect to be less than 62 inches tall? (Lesson 12-7)

14. **SURVEY** A poll of 1750 people shows that 78% enjoy travel. Find the margin of the sampling error for the survey. (Lesson 12-9)

**CABLE CARS For Exercises 1 and 2, use the following information.**

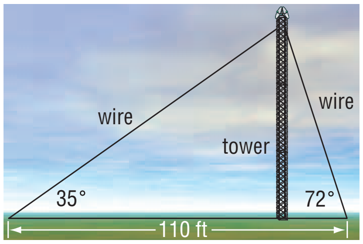
The longest cable car route in the world begins at an altitude of 5379 feet and ends at an altitude of 15,629 feet. The ride is 8-miles long. Source: The Guinness Book of Records (Lesson 13-1)

1. Draw a diagram to represent this situation.
2. To the nearest degree, what is the average angle of elevation of the cable car ride?

**RIDES For Exercises 3 and 4, use the following information.**

In 2000, a gigantic Ferris wheel, the London Eye, opened in England. The wheel has 32 cars evenly spaced around the circumference. (Lesson 13-2)

3. What is the measure, in degrees, of the angle between any two consecutive cars?
4. If a car is located such that the measure in standard position is 260°, what are the measures of one angle with positive measure and one angle with negative measure coterminal with the angle of this car?
5. **BASKETBALL** A person is selected to try to make a shot at a distance of 12 feet from the basket. The formula $R = \dfrac{V_0^2 \sin 2\theta}{32}$ gives the distance of a basketball shot with an initial velocity of $V_0$ feet per second at an angle of $u$ with the ground. If the basketball was shot with an initial velocity of 24 feet per second at an angle of 75°, how far will the basketball travel? (Lesson 13-3)
6. **COMMUNICATIONS** A telecommunications tower needs to be supported by two wires. The angle between the ground and the tower on one side must be 35° and the angle between the ground and the second tower must be 72°. The distance between the two wires is 110 feet.




To the nearest foot, what should be the lengths of the two wires? (Lesson 13-4)

**SURVEYING For Exercises 7 and 8, use the following information.**

A triangular plot of farm land measures 0.9 by 0.5 by 1.25 miles. (Lesson 13-5)

7. If the plot of land is fenced on the border, what will be the angles at which the fences of the three sides meet? Round to the nearest degree.
8. What is the area of the plot of land? (*Hint:* Use the area formula in Lesson 13-4.)
9. **WEATHER** The monthly normal temperatures, in degrees Fahrenheit, for New York City are given in the table. January is assigned a value of 1, February a value of 2, and so on. (Lesson 13-6)

| Month | Temperature | Month | Temperature |
|---|---|---|---|
| 1 | 32 | 7 | 77 |
| 2 | 34 | 8 | 76 |
| 3 | 42 | 9 | 68 |
| 4 | 53 | 10 | 58 |
| 5 | 63 | 11 | 48 |
| 6 | 72 | 12 | 37 |

A trigonometric model for the temperature T in degrees Fahrenheit of New York City at t months is given by $T = 22.5 \sin \left(\frac{\pi}{6x} - 2.25\right) + 54.3$. A quadratic model for the same situation is $T = -1.34x^2 + 18.84x + 5$. Which model do you think best fits the data? Explain your reasoning.

**PHYSICS For Exercises 10–12, use the following information.**

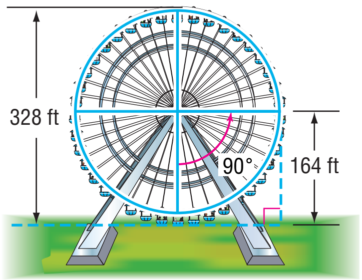
When light passes from one substance to another, it may be reflected and refracted. Snell's law can be used to find the angle of refraction as a beam of light passes from one substance to another. One form of the formula for Snell's law is $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where $n_1$ and $n_2$ are the indices of refraction for the two substances and $\theta_1$ and $\theta_2$ are the angles of the light rays passing through the two substances. (Lesson 13-7)

10. Solve the equation for $\sin \theta_1$.
11. Write an equation in the form of an inverse function that allows you to find $\theta_1$.
12. If a light beam in air with index of refraction of 1.00 hits a diamond with index of 2.42 at an angle of 30°, find the angle of refraction.

1. **TIDES** The world's record for the hightest tide is held by the Minas Basin in Nova Scotia, Canada, with a tidal range of 54.6 feet. A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. Write an equation to represent the height $h$ of the tide. Assume that the tide is at equilibrium at $t = 0$, that the high tide is beginning, and that the tide completes one cycle in 12 hours. (Lesson 14-1)

**RIDES For Exercises 2 and 3, use the following information.**

The Cosmoclock 21 is a huge Ferris wheel in Yokohama City, Japan. The diameter is 328 feet. Suppose that a rider enters the ride at 0 feet and then rotates in 90° increments counterclockwise. The table shows the angle measures of rotation and the height above the ground of the rider. (Lesson 14-2)




| Angle | Height | Angle | Height |
|---|---|---|---|
| 0° | 0 | 450° | 164 |
| 90° | 164 | 540° | 328 |
| 180° | 328 | 630° | 164 |
| 270° | 164 | 720° | 0 |
| 360° | 0 | | |

2. A function that models the data is $y = 164 \cdot (\sin (x - 90°)) + 164$. Identify the vertical shift, amplitude, period, and phase shift of the graph.

3. Write an equation using the sine that models the position of a rider on the Vienna Giant Ferris Wheel in Vienna, Austria, with a diameter of 200 feet. Check your equation by plotting the points and the equation with a graphing calculator.

4. **TRIGONOMETRY** Using the exact values for the sine and cosine functions, show that the identity $\cos^2 \theta + \sin^2 \theta = 1$ is true for angles of measure 30°, 45°, 60°, 90°, and 180°. (Lesson 14-3)

5. **ROCKETS** In the formula $h = \frac{v^2 \sin^2 \theta}{2g} = h$ is the maximum height reached by a rocket, $\theta$ is the angle between the ground and the initial path of the object, $v$ is the rocket's initial velocity, and $g$ is the acceleration due to gravity. Verify the identity $\frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \cos^2 \theta}{2g \cot^2 \theta}$. (Lesson 14-4)

**WEATHER For Exercises 6 and 7, use the following information.**

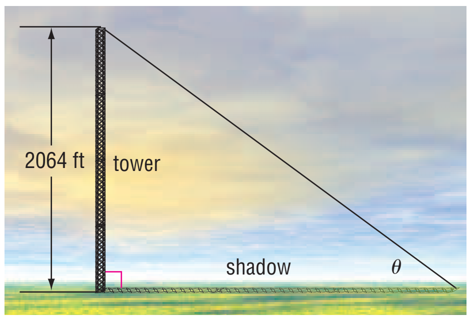
The monthly high temperatures for Minneapolis, Minnesota, can be modeled by the equation $y = 31.65 \sin \left(\frac{\pi}{6x} - 2.09\right) + 52.35$, where the months $x$ are January = 1, February = 2, and so on. The monthly low temperatures for Minneapolis can be modeled by the equation $y = 30.15 \sin \left(\frac{\pi}{6x} - 2.09\right) + 32.95$. (Lesson 14-5)

6. Write a new function by adding the expressions on the right side of each equation and dividing the result by 2.

7. What is the meaning of the function you wrote in Exercise 6?

8. Begin with one of the Pythagorean Identities. Perform equivalent operations on each side to create a new trigonometric identity. Then show that the identity is true. (Lesson 14-6)

9. **TELEVISION** The tallest structure in the world is a television transmitting tower located near Fargo, North Dakota, with a height of 2064 feet.




What is the measure of $\theta$ if the length of the shadow is 1 mile? **Source:** *The Guinness Book of Records* (Lesson 14-7)

# Preparing for Standardized Tests

## Becoming a Better Test-Taker



At some time in your life, you will probably have to take a standardized test. Sometimes this test may determine if you go on to the next grade or course, or even if you will graduate from high school. This section of your textbook is dedicated to making you a better test-taker.

**TYPES OF TEST QUESTIONS** In the following pages, you will see examples of four types of questions commonly seen on standardized tests. A description of each type of question is shown in the table below.

| Type of Question | Description | See Pages |
|---|---|---|
| multiple choice | Four or five possible answer choices are given from which you choose the best answer. | 942–943 |
| gridded response | You solve the problem. Then you enter the answer in a special grid and shade in the corresponding circles. | 944–947 |
| short response | You solve the problem, showing your work and/or explaining your reasoning. | 948–951 |
| extended response | You solve a multipart problem, showing your work and/or explaining your reasoning. | 952–956 |

**PRACTICE** After being introduced to each type of question, you can practice that type of question. Each set of practice questions is divided into five sections that represent the concepts most commonly assessed on standardized tests.

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

**USING A CALCULATOR** On some tests, you are permitted to use a calculator. You should check with your teacher to determine if calculator use is permitted on the test you will be taking, and if so, what type of calculator can be used.

**TEST-TAKING TIPS** In addition to Test-Taking Tips like the one shown at the right, here are some additional thoughts that might help you.

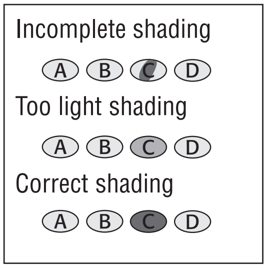
- Get a good night's rest before the test. Cramming the night before does not improve your results.
- Budget your time when taking a test. Don't dwell on problems that you cannot solve. Just make sure to leave that question blank on your answer sheet.
- Watch for key words like NOT and EXCEPT. Also look for order words like LEAST, GREATEST, FIRST, and LAST.

**TEST-TAKING TIP**

If you are allowed to use a calculator, make sure you are familiar with how it works so that you won't waste time trying to figure out the calculator when taking the test.

# Multiple-Choice Questions

Multiple-choice questions are the most common type of questions on standardized tests. These questions are sometimes called *selected-response questions*. You are asked to choose the best answer from four or five possible answers. To record a multiple-choice answer, you may be asked to shade in a bubble that is a circle or an oval or to just write the letter of your choice. Always make sure that your shading is dark enough and completely covers the bubble.




The answer to a multiple-choice question is usually not immediately obvious from the choices, but you may be able to eliminate some of the possibilities by using your knowledge of mathematics. Another answer choice might be that the correct answer is not given.

## EXAMPLE

**1** **White chocolate pieces sell for \$3.25 per pound while dark chocolate pieces sell for \$2.50 per pound. How many pounds of white chocolate are needed to produce a 10-pound mixture of both kinds that sells for \$2.80 per pound?**

**A** 2 lb  **B** 4 lb  **C** 6 lb  **D** 10 lb

**STRATEGY**

**Reasonableness** Check to see that your answer is reasonable with the given information.

The question asks you to find the number of pounds of the white chocolate. Let $w$ be the number of pounds of white chocolate and let $d$ be the number of pounds of dark chocolate. Write a system of equations.

$w + d = 10$ There is a total of 10 pounds of chocolate.

$3.25w + 2.50d = 2.80(10)$ The price is \$2.80 × 10 for the mixed chocolate.

Use substitution to solve.

$3.25w + 2.50d = 2.80(10)$ Original equation

$3.25w + 2.50(10 - w) = 28$ Solve the first equation for $d$ and substitute.

$3.25w + 25 - 2.5w = 28$ Distributive Property

$0.75w = 3$ Simplify.

$w = 4$ Divide each side by 0.75.

The answer is B.

## EXAMPLE

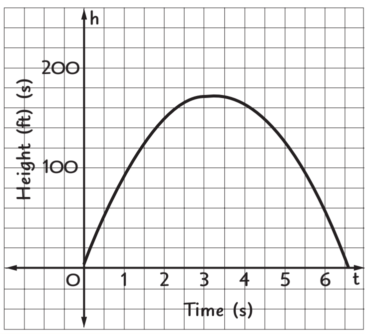
**2** **Josh throws a baseball upward at a velocity of 105 feet per second, releasing the baseball when it is 5 feet above the ground. The height of the baseball $t$ seconds after being thrown is given by the formula $h(t) = -16t^2 + 105t + 5$. Find the time at which the baseball reaches its maximum height. Round to the nearest tenth of a second.**

**F** 1.0 s  **G** 3.3 s  **H** 6.6 s  **J** 177.3 s

**STRATEGY**

**Diagrams** Drawing a diagram for a situation may help you to answer the question.

Graph the equation. The graph is a parabola. Make sure to label the horizontal axis as $t$ (time in seconds) and the vertical axis as $h$ for height in feet. The ball is at its maximum height at the vertex of the graph.




The graph indicates that the maximum height is achieved between 3 and 4 seconds after launch.

The answer is G.

# Multiple-Choice Practice

**Choose the best answer.**

## Number and Operations

**1.** In 2002, $1.8123 \times 10^8$ people in the United States and Canada used the Internet, while $5.442 \times 10^8$ people worldwide used the Internet. What percentage of users were from the United States and Canada?

**A** 33% **B** 35% **C** 37% **D** 50%

**2.** Serena has 6 plants to put in her garden. How many different ways can she arrange the plants?

**F** 21 **G** 30 **H** 360 **J** 720

## Algebra

**3.** The sum of Kevin's, Anna's, and Tia's ages is 40. Anna is 1 year more than twice as old as Tia. Kevin is 3 years older than Anna. How old is Anna?

**A** 7 **B** 14 **C** 15 **D** 18

**4.** Rafael's Theatre Company sells tickets for \$10. At this price, they sell 400 tickets. Rafael estimates that they would sell 40 fewer tickets for each \$2 price increase. What charge would give the most income?

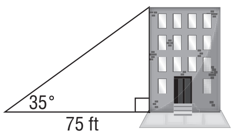
**F** 10 **G** 13 **H** 15 **J** 20

## Geometry

**5.** Hai stands 75 feet from the base of a building and sights the top at a 35° angle. What is the height of the building to the nearest tenth of a foot?

**A** 0.0 ft **C** 52.5 ft

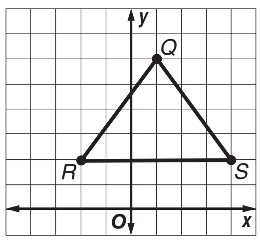
**B** 43.0 ft **D** 61.4 ft




**6.** Samone draws $\triangle QRS$ on grid paper to use for a design in her art class. She needs to rotate the triangle 180° counterclockwise. What will be the $y$-coordinate of the image of $S$?

**F** −6 **H** −1

**G** −2 **J** 2



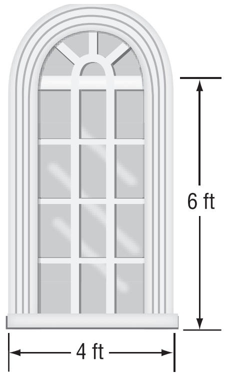

## Measurement

**7.** Lakeisha is teaching a summer art class for children. For one project, she estimates that she will need $\frac{2}{3}$ yard of string for each 3 students. How many yards will she need for 16 students?

**A** 3 yd **C** $10\frac{2}{3}$ yd

**B** $3\frac{5}{9}$ yd **D** 16 yd

**8.** Kari works at night so she needs to make her room as dark as possible during the day to sleep. How much black paper will she need to cover the window in her room, which is shaped as shown. Use 3.14 for $\pi$. Round to the nearest tenth of a square foot.




**F** 24.0 ft$^2$ **H** 30.3 ft$^2$

**G** 24.6 ft$^2$ **J** 36.6 ft$^2$

## Data Analysis and Probability

**9.** A card is drawn from a standard deck of 52 cards. If one card is drawn, what is the probability that it is a heart or a 2?

**A** $\frac{1}{52}$ **B** $\frac{1}{13}$ **C** $\frac{1}{4}$ **D** $\frac{4}{13}$

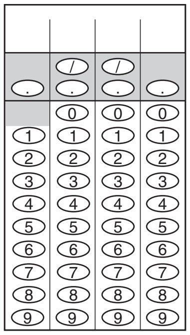
**10.** The weight of candy in boxes is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce. About what percent of the time will you get a box that weighs over 12.5 ounces?

**F** 13.5% **G** 16% **H** 50% **J** 68%

### TEST-TAKING TIP

**Question 8** Many standardized tests include a reference sheet with common formulas that you may use during the test. If it is available before the test, familiarize yourself with the reference sheet for quick reference during the test.

# Gridded-Response Questions



Gridded-response questions are another type of question on standardized tests. These questions are sometimes called *student-produced response griddable*, or *grid-in*, because you must create the answer yourself, not just choose from four or five possible answers.

For gridded response, you must mark your answer on a grid printed on an answer sheet. The grid contains a row of four or five boxes at the top, two rows of ovals or circles with decimal and fraction symbols, and four or five columns of ovals, numbered 0–9. At the right is an example of a grid from an answer sheet.

## EXAMPLE

**1 Find the $x$-coordinate for the solution of the given system of equations.**

$$4x - y = 14$$

$$-3x + y = -11$$

*What value do you need to find?*

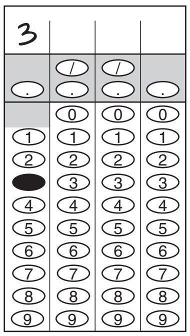
You need to find only the $x$-coordinate of the point where the graphs of the two equations intersect. You could graph the system, but that takes time. The easiest method is probably the substitution method since the second equation can be solved easily for $y$.

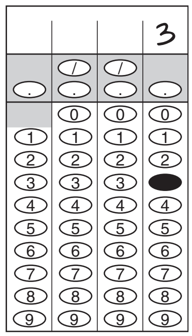
| | |
|---|---|
| $-3x + y = -11$ | Second equation |
| $y = -11 + 3x$ | Solve the second equation for $y$. |
| $4x - y = 14$ | First equation |
| $4x - (-11 + 3x) = 14$ | Substitute for $y$. |
| $4x + 11 - 3x = 14$ | Distributive Property |
| $x = 3$ | Simplify. |

The answer to be filled in on the grid is 3.

*How do you fill in the grid for the answer?*

- Write your answer in the answer boxes.
- Write only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.
- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.




Many gridded response questions result in an answer that is a fraction or a decimal. These values can also be filled in on the grid.

## EXAMPLE

**2** **Zuri is solving a problem about the area of a room. The equation she needs to solve is $4x^2 + 11x - 3 = 0$. Since the answer will be a length, she only needs to find the positive root. What is the solution?**

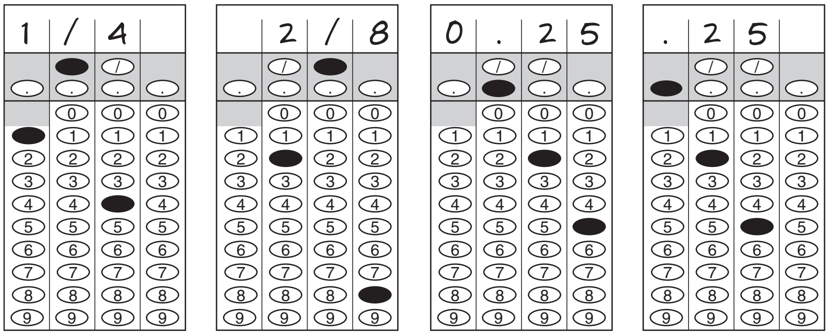
Since you can see the equation is not easily factorable, use the Quadratic Formula.

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-11 \pm \sqrt{11^2 - 4(4)(-3)}}{2(4)} \\
&= \frac{-11 + 13}{8} \\
&= \frac{2}{8} \text{ or } \frac{1}{4}
\end{aligned}
$$

There are two roots, but you only need the positive one.

*How do you grid the answer?*

You can either grid the fraction $\frac{1}{4}$, or rewrite it as 0.25 and grid the decimal. Be sure to write the decimal point or fraction bar in the answer box. The following are acceptable answer responses that represent $\frac{1}{4}$ and 0.25.

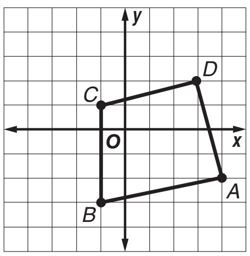


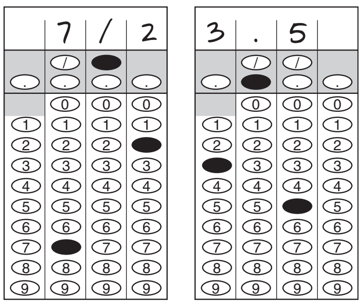

Do not leave a blank answer box in the middle of an answer.

Some problems may  result in an answer that is a mixed number. Before filling in the grid, change the mixed number to an equivalent improper fraction or decimal. For example, if the answer is $1\frac{1}{2}$, do not enter 1 1/2, as this will be interpreted as $\frac{11}{2}$. Instead, either enter 3/2 or 1.5.

## EXAMPLE

**3** **José is using this figure for a computer graphics design. He wants to dilate the figure by a scale factor of $\frac{7}{4}$. What will be the $y$-coordinate of the image of $D$?**




To find the $y$-coordinate of the image of $D$, multiply the $x$-coordinate by the scale factor of $\frac{7}{4}$.

$2 \cdot \frac{7}{4} = \frac{7}{2}$

Grid in 7/2 or 3.5. Do not grid 3 1/2.

# Gridded-Response Practice

**Solve each problem. Then copy and complete a grid.**

## Number and Operations

**1.** Rewrite $16^3$ as a power of 2. What is the value of the exponent for 2?

**2.** Wolf 359 is the fourth closest star to Earth. It is 45,531,250 million miles from Earth. A light-year, the distance light travels in a year, is $5.88 \times 10^{12}$ miles. What is the distance from Earth to Wolf 359 in light-years? Round to the nearest tenth.

**3.** A store received a shipment of coats. The coats were marked up 50% to sell to the customers. At the end of the season, the coats were discounted 60%. Find the ratio of the discounted price of a coat to the original cost of the coat to the store.

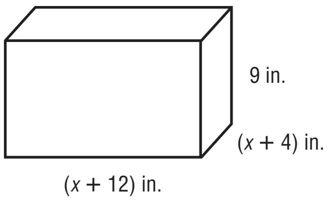
**4.** Find the value of the determinant $\begin{vmatrix} -1 & 4 \\ -3 & 0 \end{vmatrix}$.

**5.** Kendra is displaying eight sweaters in a store window. There are four identical red sweaters, three identical brown sweaters, and one white sweater. How many different arrangements of the eight sweaters are possible?

**6.** An electronics store reduced the price of a DVD player by 10% because it was used as a display model. If the reduced price was \$107.10, what was the cost in dollars before it was reduced? Round to the nearest cent if necessary.

## Algebra

**7.** The box shown can be purchased to ship merchandise at the Pack 'n Ship Store. The volume of the box is 945 cubic inches. What is the measure of the greatest dimension of the box in inches?



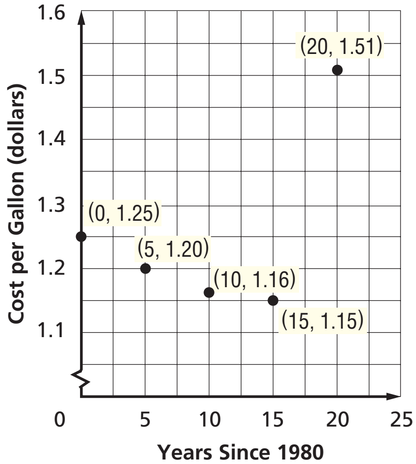

**8.** If $f(x) = 2x^2 - 3x + 10$, find $f(-1)$.

**TEST-TAKING TIP**

**Question 3** Fractions do not have to be written in lowest terms. Any equivalent fraction that fits the grid is correct.

**9.** The graph shows the retail price per gallon of unleaded gasoline in the U.S. from 1980 to 2000. What is the slope if a line is drawn through the points for 15 and 20 years since 1980?

**Retail Price of Unleaded Gasoline**

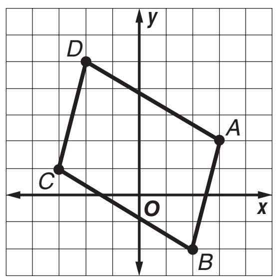



**Source:** U.S. Dept. of Energy

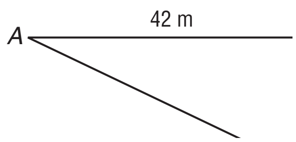
**10.** Solve $\sqrt{x + 11} - 9 = \sqrt{x} - 8$.

## Geometry

**11.** Polygon $DABC$ is rotated 90° counterclockwise and then reflected over the line $y = x$. What is the $x$-coordinate of the final image of $A$?

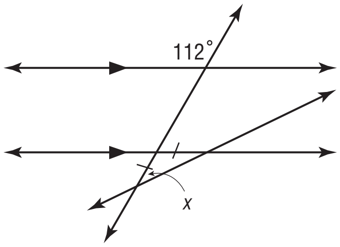



**12.** A garden is shaped as shown below. What is the measure of $\angle A$ to the nearest degree?




**13.** A circle of radius $r$ is circumscribed about a square. What is the ratio of the area of the circle to the area of the square? Express the ratio as a decimal rounded to the nearest hundredth.

**15.** Find the value of $x$



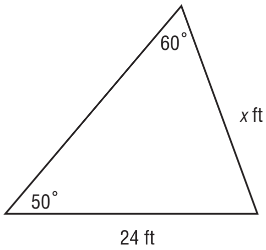

**16.** Each angle of a regular polygon measures 150°. How many sides does the polygon have?

## Measurement

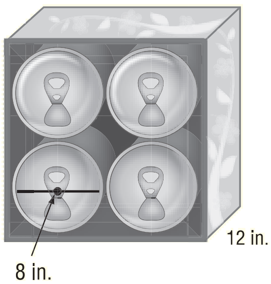
**17.** The Pascal (P) is a measure of pressure that is equivalent to 1 Newton per square meter. The typical pressure in an automobile tire is 2 × 105 P while typical blood pressure is 1.6 × 104 P. How many times greater is the pressure in a tire than typical blood pressure?

**18.** A circular ride at an amusement park rotated $\frac{7\pi}{4}$ radians while loading riders. What is the degree measure of the rotation?

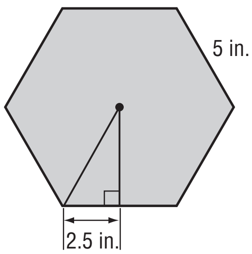
**19.** Find the value of $x$ in the triangle. Round to the nearest tenth of a foot.




**20.** Four equal-sized cylindrical juice cans are packed tightly in the box shown. What is the volume of space in the box that is not occupied by the cans in cubic inches? Use 3.14 for π and round to the nearest cubic inch.




**21.** Caroline is making a quilt. The diagram shows a piece of cloth she will cut for a portion of the pattern. Find the area of the entire hexagonal piece to the nearest tenth of a square inch.




## Data Analysis and Probability

**22.** Often girls on a team, three have blue eyes. If two girls are chosen at random, what is the probability that neither has blue eyes?

**23.** In order to win a game, Miguel needs to advance his game piece 4 spaces. What is the probability that the sum of the numbers on the two dice he rolls will be 4?

**24.** The table shows the number of televisions owned per 1000 people in each country. What is the absolute value of the difference between the mean and the median of the data?

| Country | Televisions |
|---|---|
| United States | 844 |
| Latvia | 741 |
| Japan | 719 |
| Canada | 715 |
| Australia | 706 |
| United Kingdom | 652 |
| Norway | 648 |
| Finland | 643 |
| France | 623 |

**Source:** International Telecommunication Union

**25.** The table shows the amount of breakfast cereal eaten per person each year by the ten countries that eat the most. Find the standard deviation of the data set. Round to the nearest tenth of a pound.

| Country | Cereal (lb) |
|---|---|
| Sweden | 23 |
| Canada | 17 |
| Australia | 16 |
| United Kingdom | 15 |
| Nauru | 14 |
| New Zealand | 14 |
| Ireland | 12 |
| United States | 11 |
| Finland | 10 |
| Denmark | 7 |

**Source:** Euromonitor

**26.** Two number cubes are rolled. If the two numbers appearing on the faces of the number cubes are different, find the probability that the sum is 6. Round to the nearest hundredth.

# Short-Response Questions

Short-response questions require you to provide a solution to the problem, as well as any method, explanation, and/or justification you used to arrive at the solution. These are-sometimes called *constructed-response, open-response, open-ended, free-response,* or *student-produced questions*. The following is a sample rubric, or scoring guide, for scoring short-response questions.

| Criteria | Score |
|---|---|
| **Full credit:** The answer is correct and a full explanation is provided that shows each step in arriving at the final answer. | 2 |
| **Partial credit:** There are two different ways to receive partial credit.<br>• The answer is correct, but the explanation provided is incomplete or incorrect.<br>• The answer is incorrect, but the explanation and method of solving the problem is correct. | 1 |
| **No credit:** Either an answer is not provided or the answer does not make sense. | 0 |

On some standardized tests, no credit is given for a correct answer if your work is not shown.

## EXAMPLE

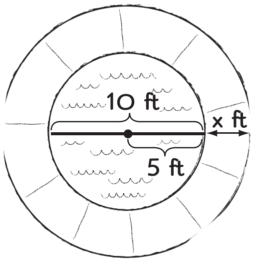
**1 Mr. Youngblood has a fish pond in his backyard. It is circular with a diameter of 10 feet. He wants to build a walkway of equal width around the pond. He wants the total area of the pond and walkway to be about 201 square feet. To the nearest foot, what should be the width of the walkway?**

### Full Credit Solution

**STRATEGY**

**Diagrams**
Draw a diagram of the pond and the walkway. Label important information.

First draw a diagram to represent the situation.




Since the diameter of the pond is 10 feet, the radius is 5 ft. Let the width of the walkway be $x$ feet.

$A = \pi r^2$

$201 = \pi(x+5)^2$ $\quad r = x+5,\ A = 201$

$201 = \pi(x^2+10x+25)$ $\quad$ Multiply.

$\frac{201}{\pi} = \frac{\pi(x^2+10x+25)}{\pi}$ $\quad$ Divide by $\pi$, using 3.14 for $\pi$.

$64 = x^2 + 10x + 25$ or $x^2 + 10x - 39 = 0$

Use the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= -10 \pm \frac{\sqrt{(-10)^2 - 4(1)(-39)}}{2(1)}$ $\quad a = 1, b = 10, c = -39$

$= \frac{-10 + \sqrt{256}}{2}$ or 3

The width of the walkway should be 3 feet.

The steps, calculations, and reasoning are clearly stated.

Before taking a standardized test, memorize common formulas, like the Quadratic Formula, to save time.

Since length must be positive, eliminate the negative solution.

## Partial Credit Solution

In this sample solution, the equation that can be used to solve the problem is correct. However, there is no justification for any of the calculations.

There is no explanation of how the quadratic equation was found.

$x^2 + 10x - 39 = 0$

$$x = \frac{-10 + \sqrt{256}}{2}$$
$$= -13 \text{ or } 3$$

The walkway should be 3 feet wide.

## Partial Credit Solution

In this sample solution, the answer is incorrect because the wrong root was chosen.

Since the diameter of the pond is 10 feet, the radius is 5 ft. Let the width of the walkway be x feet. Use the formula for the area of a circle.

$$A = \pi r^2$$
$$201 = \pi(x + 5)^2$$
$$201 = \pi(x^2 + 10x + 25)$$
$$\frac{201}{\pi} = \frac{\pi(x^2 + 10x + 25)}{\pi}$$
$$64 = x^2 \text{ a } 10x \text{ a } 25 \text{ or } x^2 \text{ a } 10x - 39 = 0$$

Use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= -13 \text{ or } 3$$

The walkway should be 13 feet wide.

The negative root was chosen as the solution.

## No Credit Solution

Use the formula for the area of a circle.

$$A = \pi r^2 x$$
$$201 = \pi(5)^2 x$$
$$201 = 3.14(25)x$$
$$201 = 78.5x$$
$$x = 2.56$$

Build the walkway 3 feet wide.

The width of the walkway $x$ is used incorrectly in the area formula for a circle. However, when the student rounds the value for the width of the walkway, the answer is correct. No credit is given for an answer achieved using faulty reasoning.

Preparing for Standardized Tests

# Short-Response Practice

**Solve each problem. Show all your work.**

## Number and Operations

**1.** An earthquake that measures a value of 1 on the Richter scale releases the same amount of energy as 170 grams of TNT, while one that measures 4 on the scale releases the energy of 5 metric tons of TNT. One metric ton is 1000 kilograms and 1000 grams is 1 kilogram. How many times more energy is released by an earthquake measuring 4 than one measuring 1?

**2.** In 2000, Cook County, Illinois was the second largest county in the U.S. with a population of about 5,377,000. This was about 43.3% of the population of Illinois. What was the approximate population of Illinois in 2000?

**3.** Show why $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix for multiplication for $2 \times 2$ matrices.

**4.** The total volume of the oceans on Earth is $3.24 \times 10^8$ cubic miles. The total surface area of the water of the oceans is 139.8 million square miles. What is the average depth of the oceans?

**5.** At the Blaine County Fair, there are 12 finalists in the technology project competition. How many ways can 1st, 2nd, 3rd, and 4th place be awarded?

## Algebra

**6.** Factor $3x^2a^2 + 3x^2b^2$. Explain each step.

**7.** Solve and graph $7 - 2a < \frac{15 - 2a}{6}$.

**8.** Solve the system of equations.

$x^2 + 9y^2 = 25 \quad y - x = -5$

**9.** The table shows what Miranda Richards charges for landscaping services for various numbers of hours. Write an equation to find the charge for any amount of time, where $y$ is the total charge in dollars and $x$ is the amount of time in hours. Explain the meaning of the slope and $y$-intercept of the graph of the equation.

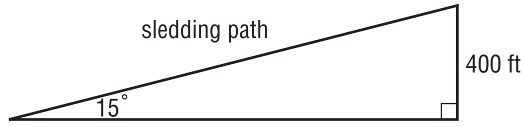
| Hours | Charge (dollars) | Hours | Charge (dollars) |
|---|---|---|---|
| 0 | 17.50 | 3 | 64.00 |
| 1 | 33.00 | 4 | 79.50 |
| 2 | 48.50 | 5 | 95.00 |

**10.** Write an equation that fits the data in the table.

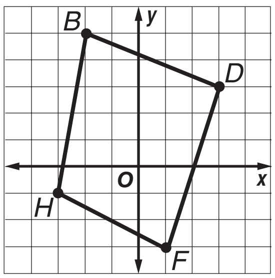
| $x$ | $y$ |
|---|---|
| −3 | 12 |
| −1 | 4 |
| 0 | 3 |
| 2 | 7 |
| 4 | 19 |

## Geometry

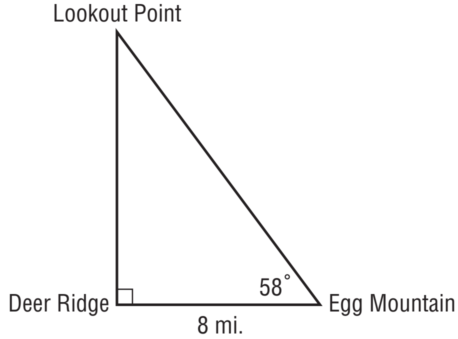
**11.** A sledding hill at the local park has an angle of elevation of 15°. Its vertical drop is 400 feet. What is the length of the sledding path?




**12.** Polygon $BDFH$ is transformed using the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Graph $B'D'F'H'$ and identify the type of transformation.




**13.** The map shows the trails that connect three hiking destinations. If Amparo hikes from Deer Ridge to Egg Mountain to Lookout Point and back to Deer Ridge, what is the distance she will have traveled?




**14.** Mr Washington is making a cement table for his backyard. The tabletop will be circular with a diameter of 6 feet and a depth of 6 inches. How much cement will Mr. Washington need to make the top of the table? Use 3.14 for $\pi$ and round to the nearest cubic foot.

**15.** A triangular garden is plotted on grid paper, where each unit is 1 meter. Its sides are segments that are parts of the lines with equations $y = -\frac{5}{4}x + 2$, $2y - 5x = 4$, and $y = -3$. Graph the triangle and find its area.

## Measurement

**16.** Dylan is flying a kite. He wants to know how high above the ground it is. He knows that he has let out 75 feet of string and that it is flying directly over a nearby fence post. If he is 50 feet from the fence post, how high is the kite? Round to the nearest tenth of a foot.

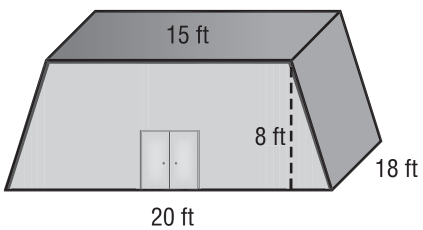
**17.** The temperature of the Sun can reach 27,000,000°F. The relationship between Fahrenheit F and Celsius C temperatures is given by the equation $F = 1.8C + 32$. Find the temperature of the Sun in degrees Celsius.

**18.** In 2003, Monaco was the most densely populated country in the world. There were about 32,130 people occupying the country at the rate of 16,477 people per square kilometer. What is the area of Monaco?

**19.** A box containing laundry soap is a cylinder with a diameter of 10.5 inches and a height of 16 inches. What is the surface area of the box?

**20.** Light travels at 186,291 miles per second or 299,792 kilometers per second. What is the relationship between miles and kilometers?

**21.** Home Place Hardware sells storage buildings for your backyard. The front of the building is a trapezoid as shown. The store manager wants to advertise the total volume of the building. Find the volume in cubic feet.




**TEST-TAKING TIP**

**Questions 15, 16, and 22** Be sure to read the instructions of each problem carefully. Some questions ask for more than one solution, specify how to round answers, or require an explanation.

## Data Analysis and Probability

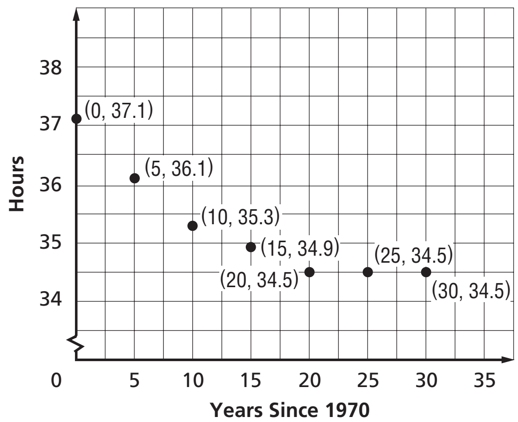
**22.** The table shows the 2000 populations of the six largest cities in Tennessee. Which measure, mean or median, do you think best represents the data? Explain your answer.

| City | Population |
|---|---|
| Chattanooga | 155,404 |
| Clarksville | 105,898 |
| Jackson | 60,635 |
| Knoxville | 173,661 |
| Memphis | 648,882 |
| Nashville-Davidson | 545,915 |

**Source:** International Communication Union

**23.** The scatter plot shows the number of hours worked per week for U.S. production workers from 1970 through 2000. Let $y$ be the hours worked per week and $x$ be the years since 1970. Write an equation that you think best models the data.

**Average Hours Worked per Week for Production Workers**




**Source:** Bureau of Labor Statistics

**24.** A day camp has 240 participants. Children can sign up for various activities. Suppose 135 children take swimming, 160 take soccer, and 75 take both swimming and soccer. What is the probability that a child selected at random takes swimming or soccer?

**25.** In how many different ways can seven members of a student government committee sit around a circular table?

**26.** Illinois residents can choose to buy an environmental license plate to support Illinois parks. Each environmental license plate displays 3 or 4 letters followed by a number 1 thru 99. How many different environmental license plates can be issued?

# Extended-Response Questions

Extended-response questions are often called open-ended or constructed-response questions. Most extended-response questions have multiple parts. You must answer all parts to receive full credit.

Extended-response questions are similar to short-response questions in that you must show all of your work in solving the problem and a rubric is used to determine whether you receive full, partial, or no credit. The following is a sample rubric for scoring extended-response questions.

| Criteria | Credit |
|---|---|
| **Full credit:** The answer is correct and a full explanation is provided that shows each step in arriving at the final answer. | 2 |
| **Partial credit:** There are two different ways to receive partial credit.<br>• The answer is correct, but the explanation provided is incomplete or incorrect.<br>• The answer is incorrect, but the explanation and method of solving the problem is correct. | 1 |
| **No credit:** Either an answer is not provided or the answer does not make sense. | 0 |

On some standardized tests, no credit is given for a correct answer if your work is not shown.

Make sure that when the problem says to *Show your work,* you show every aspect of your solution including figures, sketches of graphing calculator screens, or reasoning behind computations.

## EXAMPLE

**1 Libby throws a ball into the air with a velocity of 64 feet per second. She releases the ball 5 feet above the ground. The height of the ball above the ground $t$ seconds after release is modeled by an equation of the form $h(t) = -16t^2 + v_o t + h_o$ where $v_o$ is the initial velocity in feet per second and $h_o$ is the height at which the ball is released.**

**a.** Write an equation for the flight of the ball. Sketch the graph of the equation.

**b.** Find the maximum height that the ball reaches and the time that this height is reached.

**c.** Change only the speed of the release of the ball such that the ball will reach a maximum height greater than 100 feet. Write an equation for the flight of the ball.

### Full Credit Solution

**Part a** A complete graph includes appropriate scales and labels for the axes, and points that are correctly graphed.

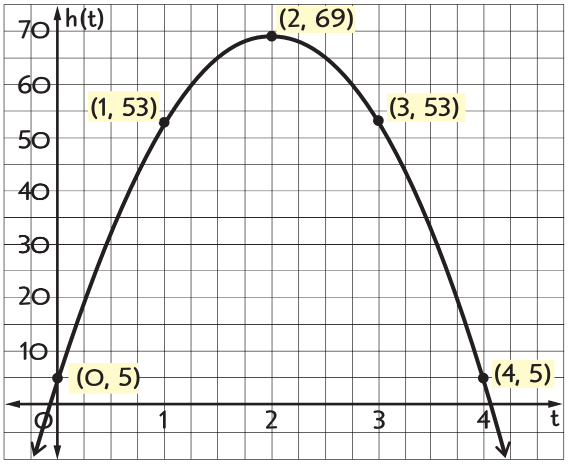
- A complete graph also shows the basic characteristics of the graph. The student should realize that the graph of this equation is a parabola opening downward with a maximum point reached at the vertex.
- The student should choose appropriate points to show the important characteristics of the graph.
- Students should realize that $t$ and $x$ and $h(t)$ and $y$ are interchangeable on a graph on the coordinate plane.

To write the equation for the ball, I substituted $v_o = 64$ and $h_o = 5$ into the equation $h(t) = -16t^2 + v_o t + h_o$, so the equation is $h(t) = -16t^2 + 64t + 5$. To graph the equation, I found the equation of the axis of symmetry and the vertex.

$$x = -\frac{b}{2a}$$

$$= -\frac{64}{2(-16)} \text{ or } 2$$

The equation of the axis of symmetry is $x = 2$, so the $x$-coordinate of the vertex is 2. You let $t = x$ and then $h(t) = -16t^2 + 64t + 5 = -16(2)^2 + 64(2) + 5$ or 69. The vertex is (2, 69). I found some other points and sketched the graph. I graphed points $(t, h(t))$ as $(x, y)$.




**Part b**

The maximum height of the ball is reached at the vertex of the parabola. So, the maximum height is 69 feet and the time it takes to reach the maximum height is 2 seconds.

**Part c**

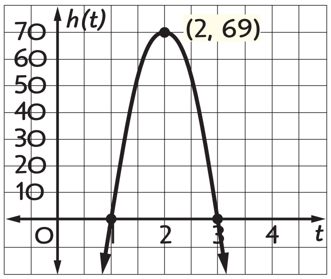
In part c, any equation whose graph has a vertex with $y$-coordinate greater than 100 would be a correct answer earning full credit.

Since I have a graphing calculator, I changed the value of $v_o$ until I found a graph in which the $y$ or $h(t)$ coordinate was greater than 100. The equation I used was $h(t) = -16t^2 + 80t + 5$.

## Partial Credit Solution

**Part a** This sample answer includes no labels for the graph or the axes and one of the points is not graphed correctly.

$h(t) = -16t^2 + 64t + 5$; (2, 69)



**Part b** Full credit is given because the vertex is correct and is interpreted correctly.

> The vertex shows the maximum height of the ball. The time it takes to reach the maximum height of 69 feet is 2 seconds.

**Part c** Partial credit is given for part c since no explanation is given for using this equation. The student did not mention that the vertex would have a *y*-coordinate greater than 100.

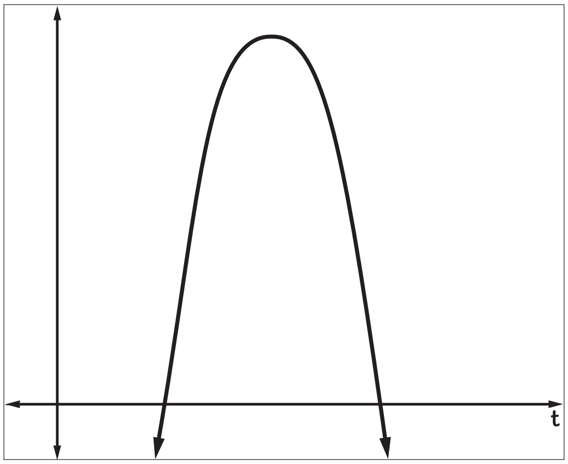
> I will write the equation $h(t) = -16t^2 + 100t + 5$.

This sample answer might have received a score of 2 or 1, depending on the judgment of the scorer. Had the student sketched a more accurate graph and given more complete explanations for Parts a and c, the score would probably have been a 3.

## No Credit Solution

**Part a** No credit is given because the equation is incorrect with no explanation and the sketch of the graph has no labels, making it impossible to determine whether the student understands the relationship between the equation for a parabola and the graph.

> $h(t) = -16t^2 + 5t + 64$




**Part b**

> It reaches about 10 feet.

**Part c**

> A good equation for the ball is $h(t) = -16t^2 + 5t + 100$.

In this sample answer, the student does not understand how to substitute the given information into the equation, graph a parabola, or interpret the vertex of a parabola.

# Extended Response Practice

**Solve each problem. Show your work.**

## Number and Operations

**1.** Mrs. Ebbrect is assigning identification (ID) numbers to freshman students. She plans to use only the digits 2, 3, 5, 6, 7, and 9. The ID numbers will consist of three digits with no repetitions.

**a.** How many 3-digit ID numbers can be formed?

**b.** How many more ID numbers can Mrs. Ebbrect make if she allows repetitions?

**c.** What type of system could Mrs. Ebbrect use to choose the numbers if there are at least 400 students who need ID numbers?

**2.** Use these four matrices.

$A = \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 4 & -6 & 5 \\ 0 & 1 & -3 \end{bmatrix}$

$C = \begin{bmatrix} -7 & 3 \\ -6 & 2 \end{bmatrix}$ $\quad D = \begin{bmatrix} -3 & 1 \end{bmatrix}$

**a.** Find $A + C$.

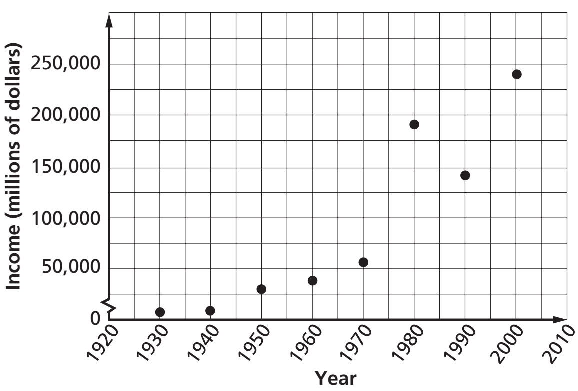
**b.** Compare the dimensions of $AB$ and $DB$.

**c.** Compare the matrices $BC$ and $CB$.

## Algebra

**3.** Roger is using the graph showing the gross cash income for all farms in the U.S. from 1930 through 2000 to make some predictions for the future.

**Farm Income**




**Source:** U.S. Department of Agriculture

**a.** Write an equation in slope-intercept form for the line passing through the point for 1930 and the point for 1970.

**b.** Write an equation in slope-intercept form for the line passing through the point for 1980 and the point for 2000. Compare the slope of this line to the slope of the line in part a.

**c.** Which equation, if any, do you think Roger should use to model the data? Explain.

**d.** Suggest an equation that is not linear for Roger to use.

**4.** Brad is coaching the bantam age division (8 years old and younger) swim team. On the first day of practice, he has the team swim 4 laps of the 25-meter pool. For each of the next practices, he increases the laps by 3. In other words, the children swim 4 laps the first day, 7 laps the second day, 10 laps the third day, and so on.

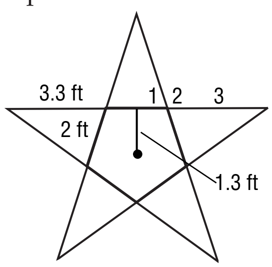
**a.** Write a formula for the $n$th term of the sequence of the number of laps each day. Explain how you found the formula.

**b.** How many laps will the children swim on the 10th day?

**c.** Brad's goal is to have the children swim at least one mile during practice on the 20th day. If one mile is approximately 1.6 kilometers, will Brad reach his goal?

## Geometry

**5.** Alejandra is planning to use a star shape in a galaxy-themed mural on her wall. The pentagon in the center is regular, and the triangles forming the points are isosceles.

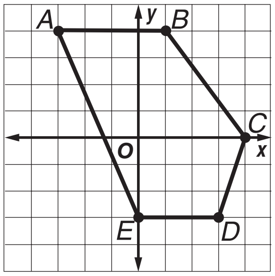



**a.** Find the measures of $\angle 1$, $\angle 2$, and $\angle 3$. Explain your method.

**b.** The approximate dimensions of the design are given. The segment of length 1.3 feet is the apothem of the pentagon. Find the approximate area of the design.

**c.** If Alejandra circumscribes a circle about the star, what is the area of the circle?

**6.** Kareem is using polygon *ABCDE*, shown on a coordinate plane, as a basis for a computer graphics design. He plans to perform various transformations on the polygon to produce a variety of interesting designs.




**a.** First, Kareem creates polygon *A′B′C′D′E′* by rotating *ABCDE* counterclockwise about the origin 270°. Graph polygon *A′B′C′D′E′* and describe the relationship between the coordinates of *ABCDE* and *A′B′C′D′E′*.

**b.** Next, Kareem reflects polygon *A′B′C′D′E′* in the line $y = x$ to produce polygon *A″B″C″D″E″*. Graph *A″B″C″D″E″* and describe the relationship between the coordinates of *A′B′C′D′E′* and *A″B″C″D″E″*.

**c.** Describe how Kareem could transform polygon *ABCDE* to polygon *A″B″C″D″E″* with only one transformation.

## Measurement

**7.** The speed of a satellite orbiting Earth can be found using the equation $v = \sqrt{\frac{\frac{GmE}{r}}{mE}}$. $G$ is the gravitational constant for Earth, $mE$ is the mass of Earth, and $r$ is the radius of the orbit which includes the radius of Earth and the height of the satellite.

**a.** The radius of Earth is $6.38 \times 10^6$ meters. The distance of a particular satellite above Earth is 350 kilometers. What is the value of $r$? (*Hint:* The center of the orbit is the center of Earth.)

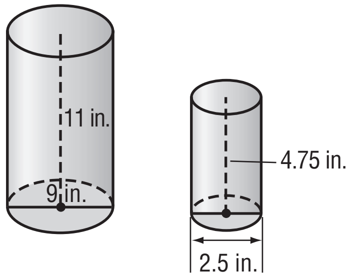
**b.** The gravitational constant for Earth is $6.67 \times 10^{-11}$ N • m$^2$/kg$^2$. The mass of Earth is $5.97 \times 10^{24}$ kg. Find the speed of the satellite in part a.

**c.** As a satellite increases in distance from Earth, what is the effect on the speed of the orbit? Explain your reasoning.

**TEST-TAKING TIP**

**Question 6** When questions require graphing, make sure your graph is accurate to receive full credit for your correct solution.

**8.** A cylindrical cooler has a diameter of 9 inches and a height of 11 inches. Scott plans to use it for soda cans that have a diameter of 2.5 inches and a height of 4.75 inches.




**a.** Scott plans to place two layers consisting of 9 cans each into the cooler. What is the volume of the space that will not be filled with cans?

**b.** Find the ratio of the volume of the cooler to the volume of the cans in part a.

## Data Analysis and Probability

**9.** The table shows the total world population from 1950 through 2000.

| Year | Population | Year | Population |
|---|---|---|---|
| 1950 | 2,566,000,053 | 1980 | 4,453,831,714 |
| 1960 | 3,039,451,023 | 1990 | 5,278,639,789 |
| 1970 | 3,706,618,163 | 2000 | 6,082,966,429 |

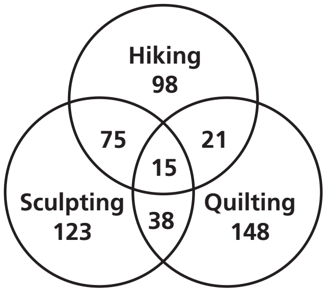
**a.** Between which two decades was the percent increase in population the greatest?

**b.** Make a scatter plot of the data.

**c.** Find a function that models the data.

**d.** Predict the world population for 2030.

**10.** Each year, a university sponsors a conference for women. The Venn diagram shows the number of participants in three activities for the 680 women that attended. Suppose women who attended are selected at randomfor a survey.




**a.** What is the probability that a woman selected participated in hiking or sculpting?

**b.** Describe a set of women such that the probability of their being selected is about 0.39.

# Glossary/Glosario

Math Online A mathematics multilingual glossary is available at www.math.glencoe.com/multilingual_glossary. The glossary includes the following languages.

| | | | |
|---|---|---|---|
| Arabic | Haitian Creole | Portuguese | Tagalog |
| Bengali | Hmong | Russian | Urdu |
| Cantonese | Korean | Spanish | Vietnamese |
| English | | | |

**Cómo usar el glosario en español:**

1. Busca el término en inglés que desees encontrar.
2. El término en español, junto con la definición, se encuentran en la columna de la derecha.

## English / Español

## A

**absolute value** (p. 27) A number's distance from zero on the number line, represented by $|x|$.

**valor absoluto** Distancia entre un número y cero en una recta numérica; se denota con $|x|$.

**absolute value function** (p. 96) A function written as $f(x) = |x|$, where $f(x) \geq 0$ for all values of $x$.

**función del valor absoluto** Una función que se escribe $f(x) = |x|$, donde $f(x) \geq 0$, para todos los valores de $x$.

**algebraic expression** (p. 6) An expression that contains at least one variable.

**expresión algebraica** Expresión que contiene al menos una variable.

**amplitude** (p. 823) For functions in the form $y = a \sin b\theta$ or $y = a \cos b\theta$, the amplitude is $|a|$.

**amplitud** Para funciones de la forma $y = a$ sen $b\theta$ o $y = a \cos b\theta$, la amplitud es $|a|$.

**angle of depression** (p. 764) The angle between a horizontal line and the line of sight from the observer to an object at a lower level.

**ángulo de depresión** Ángulo entre una recta horizontal y la línea visual de un observador a una figura en un nivel inferior.

**angle of elevation** (p. 764) The angle between a horizontal line and the line of sight from the observer to an object at a higher level.

**ángulo de elevación** Ángulo entre una recta horizontal y la línea visual de un observador a una figura en un nivel superior.

**arccosine** (p. 807) The inverse of $y = \cos x$, written as $x = \arccos y$.

**arcocoseno** La inversa de $y = \cos x$, que se escribe como $x = \arccos y$.

**arcsine** (p. 807) The inverse of $y = \sin x$, written as $x = \arcsin y$.

**arcoseno** La inversa de $y =$ sen $x$, que se escribe como $x =$ arcsen $y$.

**arctangent** (p. 807) The inverse of $y = \tan x$ written as $x = \arctan y$.

**arcotangente** La inversa de $y = \tan x$ que se escribe como $x = \arctan y$.

**area diagram** (p. 703) A model of the probability of two events occurring.

**diagrama de área** Modelo de la probabilidad de que ocurran dos eventos.

**arithmetic mean** (p. 624) The terms between any two nonconsecutive terms of an arithmetic sequence.

**media aritmética** Cualquier término entre dos términos no consecutivos de una sucesión aritmética.

**arithmetic sequence** (p. 622) A sequence in which each term after the first is found by adding a constant, the common difference $d$, to the previous term.

**sucesión aritmética** Sucesión en que cualquier término después del primero puede hallarse sumando una constante, la diferencia común $d$, al término anterior.

**arithmetic series** (p. 629) The indicated sum of the terms of an arithmetic sequence.

**serie aritmética** Suma específica de los términos de una sucesión aritmética.

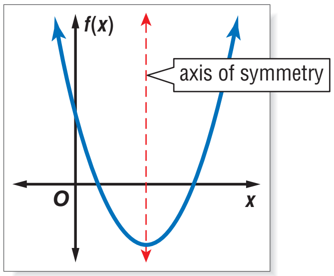
**asymptote** (p. 457, 591) A line that a graph approaches but never crosses.

**asíntota** Recta a la que se aproxima una gráfica, sin jamás cruzarla.

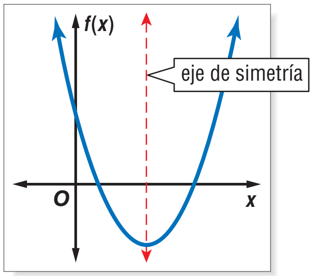
**augmented matrix** (p. 223) A coefficient matrix with an extra column containing the constant terms.

**matriz ampliada** Matriz coeficiente con una columna extra que contiene los términos constantes.

**axis of symmetry** (p. 237) A line about which a figure is symmetric.




**eje de simetría** Recta respecto a la cual una figura es simétrica.




## B

**$b^{\frac{1}{n}}$** (p. 415) For any real number $b$ and for any positive integer $n$, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and $n$ is even .

**$b^{\frac{1}{n}}$** Para cualquier número real $b$ y para cualquier entero positivo $n$, $b^{\frac{1}{n}} = \sqrt[n]{b}$, excepto cuando $b < 0$ y $n$ es par.

**binomial** (p. 7) A polynomial that has two unlike terms.

**binomio** Polinomio con dos términos diferentes.

**binomial experiment** (p. 730) An experiment in which there are exactly two possible outcomes for each trial, a fixed number of independent trials, and the probabilities for each trial are the same.

**experimento binomial** Experimento con exactamente dos resultados posibles para cada prueba, un número fijo de pruebas independientes y en el cual cada prueba tiene igual probabilidad.

**Binomial Theorem** (p. 665) If $n$ is a nonnegative integer, then $(a + b)^n = 1a^nb^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n+1)}{1 \cdot 2}a^{n-2}b^2 + \ldots + 1a^0b^n$.

**teorema del binomio** Si n es un entero no negativo, entonces $(a + b)^n = 1a^nb^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n+1)}{1 \cdot 2}a^{n-2}b^2 + \ldots + 1a^0b^n$.

**bivariate data** (p. 86) Data with two variables.

**datos bivariados** Datos con dos variables.

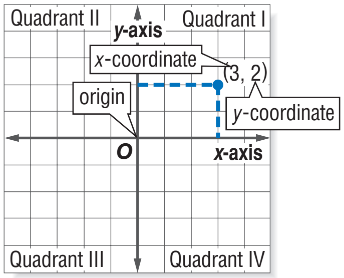
**boundary** (p. 102) A line or curve that separates the coordinate plane into two regions.

**frontera** Recta o curva que divide un plano de coordenadas en dos regiones.

**bounded** (p. 138) A region is bounded when the graph of a system of constraints is a polygonal region.

**acotada** Una región está acotada cuando la gráfica de un sistema de restricciones es una región poligonal.

**Cartesian coordinate plane** (p. 58) A plane divided into four quadrants by the intersection of the $x$-axis and the $y$-axis at the origin.



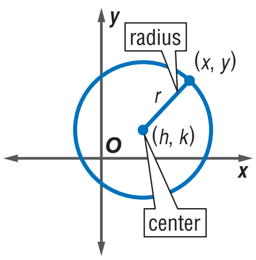

**center of a circle** (p. 574) The point from which all points on a circle are equidistant.

**center of a hyperbola** (p. 591) The midpoint of the segment whose endpoints are the foci.

**center of an ellipse** (p. 582) The point at which the major axis and minor axis of an ellipse intersect.

**Change of Base Formula** (p. 530) For all positive numbers $a$, $b$, and $n$, where $a \neq 1$ and $b \neq 1$, $\log_a n = \dfrac{\log_b n}{\log_b a}$.

**circle** (p. 574) The set of all points in a plane that are equidistant from a given point in the plane, called the center.



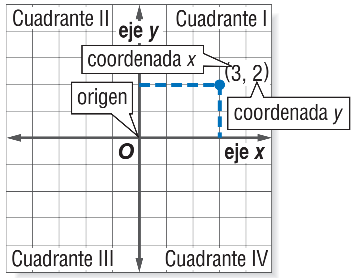

**circular functions** (p. 800) Functions defined using a unit circle.

**coefficient** (p. 7) The numerical factor of a monomial.

**column matrix** (p. 163) A matrix that has only one column.

**combination** (p. 692) An arrangement of objects in which order is not important.

**plano de coordenadas cartesiano** Plano dividido en cuatro cuadrantes mediante la intersección en el origen de los ejes $x$ y $y$.



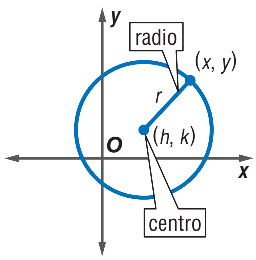

**centro de un círculo** El punto desde el cual todos los puntos de un círculo están equidistantes.

**centro de una hipérbola** Punto medio del segmento cuyos extremos son los focos.

**centro de una elipse** Punto de intersección de los ejes mayor y menor de una elipse.

**fórmula del cambio de base** Para todo número positivo $a$, $b$ y $n$, donde $a \neq 1$ y $b \neq 1$, $\log_a n = \dfrac{\log_b n}{\log_b a}$.

**círculo** Conjunto de todos los puntos en un plano que equidistan de un punto dado del plano llamado centro.




**funciones circulares** Funciones definidas en un círculo unitario.

**coeficiente** Factor numérico de u monomio.

**matriz columna** Matriz que sólo tiene una columna.

**combinación** Arreglo de elementos en que el orden no es importante.

**common difference** (p. 622) The difference between the successive terms of an arithmetic sequence.

**diferencia común** Diferencia entre términos consecutivos de una sucesión aritmética.

**common logarithms** (p. 528) Logarithms that use 10 as the base.

**logaritmos comunes** El logaritmo de base 10.

**common ratio** (p. 636) The ratio of successive terms of a geometric sequence.

**razón común** Razón entre términos consecutivos de una sucesión geométrica.

**completing the square** (p. 269) A process used to make a quadratic expression into a perfect square trinomial.

**completar el cuadrado** Proceso mediante el cual una expresión cuadrática se transforma en un trinomio cuadrado perfecto.

**complex conjugates** (p. 263) Two complex numbers of the form $a + bi$ and $a - bi$.

**conjugados complejos** Dos números complejos de la forma $a + bi$ y $a - bi$.

**complex fraction** (p. 445) A rational expression whose numerator and/or denominator contains a rational expression.

**fracción compleja** Expresión racional cuyo numerador o denominador contiene una expresión racional.

**complex number** (p. 261) Any number that can be written in the form $a + bi$, where $a$ and $b$ are real numbers and $i$ is the imaginary unit.

**número complejo** Cualquier número que puede escribirse de la forma $a + bi$, donde $a$ y $b$ son números reales e $i$ es la unidad imaginaria.

**composition of functions** (p. 385) A function is performed, and then a second function is performed on the result of the first function. The composition of $f$ and $g$ is denoted by $f \circ g$, and $[f \circ g](x) = f[g(x)]$.

**composición de funciones** Se evalúa una función y luego se evalúa una segunda función en el resultado de la primera función. La composición de f y g se define con $f \circ g$, y $[f \circ g](x) = f[g(x)]$.

**compound event** (p. 710) Two or more simple events.

**evento compuesto** Dos o más eventos simples.

**compound inequality** (p. 41) Two inequalities joined by the word *and* or *or*.

**desigualdad compuesta** Dos desigualdades unidas por las palabras *y* u *o*.

**conic section** (p. 567) Any figure that can be obtained by slicing a double cone.

**sección cónica** Cualquier figura obtenida mediante el corte de un cono doble.

**conjugate axis** (p. 591) The segment of length $2b$ units that is perpendicular to the transverse axis at the center.

**eje conjugado** El segmento de $2b$ unidades de longitud que es perpendicular al eje transversal en el centro.

**conjugates** (p. 411) Binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where $a$, $b$, $c$, and $d$ are rational numbers.

**conjugados** Binomios de la forma $a\sqrt{b} + c\sqrt{d}$ y $a\sqrt{b} - c\sqrt{d}$, donde $a$, $b$, $c$, y $d$ son números racionales.

**consistent** (p. 118) A system of equations that has at least one solution.

**consistente** Sistema de ecuaciones que posee por lo menos una solución.

**constant** (p. 7) Monomials that contain no variables.

**constante** Monomios que carecen de variables.

**constant function** (p. 96) A linear function of the form $f(x) = b$.

**función constante** Función lineal de la forma $f(x) = b$.

Glossary/Glosario

**constant of variation** (p. 465) The constant $k$ used with direct or inverse variation.

**constante de variación** La constante $k$ que se usa en variación directa o inversa.

**constant term** (p. 236) In $f(x) = ax^2 + bx + c$, $c$ is the constant term.

**término constante** En $f(x) = ax^2 + bx + c$, $c$ es el término constante.

**constraints** (p. 138) Conditions given to variables, often expressed as linear inequalities.

**restricciones** Condiciones a que están sujetas las variables, a menudo escritas como desigualdades lineales.

**continuity** (p. 457) A graph of a function that can be traced with a pencil that never leaves the paper.

**continuidad** La gráfica de una función que se puede calcar sin levantar nunca el lápiz del papel.

**continuous probability distribution** (p. 724) The outcome can be any value in an interval of real numbers, represented by curves.

**distribución de probabilidad continua** El resultado puede ser cualquier valor de un intervalo de números reales, representados por curvas.

**continuous relation** (p. 59) A relation that can be graphed with a line or smooth curve.

**relación continua** Relación cuya gráfica puede ser una recta o una curva suave.

**cosecant** (p. 759) For any angle, with measure $\alpha$, a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, $\csc \alpha = \frac{r}{y}$.

**cosecante** Para cualquier ángulo de medida $\alpha$, un punto P$(x, y)$ en su lado terminal, $r = \sqrt{x^2} + y^2$, $\csc \alpha = \frac{r}{y}$.

**cosine** (p. 759) For any angle, with measure $\alpha$ , a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, $\cos \alpha = \frac{x}{r}$.

**coseno** Para cualquier ángulo de medida $\alpha$, un punto $P(x, y)$ en su lado terminal, $r = \sqrt{x^2 + y^2}$, $\cos \alpha = \frac{x}{r}$.

**cotangent** (p. 759) For any angle, with measure $\alpha$, a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, $\cot \alpha = \frac{x}{y}$.

**cotangente** Para cualquier ángulo de medida $\alpha$, un punto $P(x, y)$ en su lado terminal, $r = \sqrt{x^2 + y^2}$, $\cot \alpha = \frac{x}{y}$.

**coterminal angles** (p. 771) Two angles in standard position that have the same terminal side.

**ángulos coterminales** Dos ángulos en posición estándar que tienen el mismo lado terminal.

**convergent series** (p. 651) An infinite series with a sum.

**serie convergente** Serie infinita con una suma.

**counterexample** (p. 17) A specific case that shows that a statement is false.

**contraejemplo** Caso específico que demuestra la falsedad de un enunciado.

**Cramer's Rule** (p. 201) A method that uses determinants to solve a system of linear equations.

**regla de Crámer** Método que usa determinantes para resolver un sistema de ecuaciones lineales.

## D

**degree** (p. 7) The sum of the exponents of the variables of a monomial.

**grado** Suma de los exponentes de las variables de un monomio.

**degree of a polynomial** (p. 320) The greatest degree of any term in the polynomial.

**grado de un polinomio** Grado máximo de cualquier término del polinomio.

**dependent events** (p. 686) The outcome of one event does affect the outcome of another event.

**eventos dependientes** El resultado de un evento afecta el resultado de otro evento.

**dependent system** (p. 118) A consistent system of equations that has an infinite number of solutions.

**sistema dependiente** Sistema de ecuaciones que posee un número infinito de soluciones.

**dependent variable** (p. 61) The other variable in a function, usually $y$, whose values depend on $x$.

**variable dependiente** La otra variable de una función, por lo general $y$, cuyo valor depende de $x$.

**depressed polynomial** (p. 357) The quotient when a polynomial is divided by one of its binomial factors.

**polinomio reducido** El cociente cuando se divide un polinomio entre uno de sus factores binomiales.

**determinant** (p. 194) A square array of numbers or variables enclosed between two parallel lines.

**determinante** Arreglo cuadrado de números o variábles encerrados entre dos rectas paralelas

**dilation** (p. 187) A transformation in which a geometric figure is enlarged or reduced.

**homotecia** Transformación en que se amplía o se reduce un figura geométrica.

**dimensional analysis** (p. 315) Performing operations with units.

**anállisis dimensional** Realizar operaciones con unidades.

**dimensions of a matrix** (p. 163) The number of rows, $m$, and the number of columns, $n$, of the matrix written as $m \times n$.

**tamaño de una matriz** El número de filas, $m$, y columnas, $n$, de una matriz, lo que se escribe $m \times n$.

**directrix** (p. 567) See parabola.

**directriz** Véase parábola.

**direct variation** (p. 465) $y$ varies directly as $x$ if there is some nonzero constant $k$ such that $y = kx$. $k$ is called the constant of variation.

**variación directa** $y$ varía directamente con $x$ si hay una constante no nula $k$ tal que $y = kx$. $k$ se llama la constante de variación.

**discrete probability distributions** (p. 724) Probabilities that have a finite number of possible values.

**distribución de probabilidad discreta** Probabilidades que tienen un número finito de valores posibles.

**discrete relation** (p. 59) A relation in which the domain is a set of individual points.

**relación discreta** Relación en la cual el dominio es un conjunto de puntos individuales.

**discriminant** (p. 279) In the Quadratic Formula, the expression $b^2 - 4ac$.

**discriminante** En la fórmula cuadrática, la expresión $b^2 - 4ac$.

**dispersion** (p. 718) Measures of variation of data.

**dispersión** Medidas de variación de los datos.

**Distance Formula** (p. 563) The distance between two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**fórmula de la distancia** La distancia entre dos puntos $(x_1, y_1)$ and $(x_2, y_2)$ viene dada por $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**domain** (p. 58) The set of all $x$-coordinates of the ordered pairs of a relation.

**dominio** El conjunto de todas las coordenadas $x$ de los pares ordenados de una relación.

## E

***e*** (p. 536) The irrational number 2.71828…. $e$ is the base of the natural logarithms.

***e*** El número irracional 2.71828…. $e$ es la base de los logaritmos naturales.

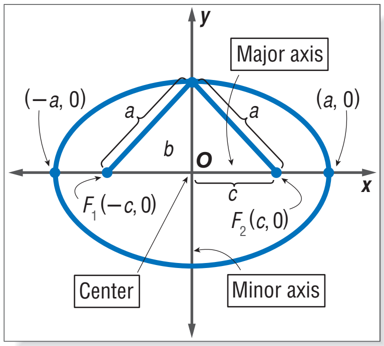
**element** (p. 163) Each value in a matrix.

**elemento** Cada valor de una matriz.

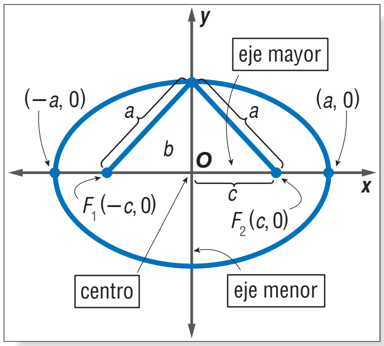
**elimination method** (p. 125) Eliminate one of the variables in a system of equations by adding or subtracting the equations.

**método de eliminación** Eliminar una de las variables de un sistema de ecuaciones sumando o restando las ecuaciones.

**ellipse** (p. 581) The set of all points in a plane such that the sum of the distances from two given points in the plane, called foci, is constant.




**elipse** Conjunto de todos los puntos de un plano en los que la suma de sus distancias a dos puntos dados del plano, llamados focos, es constante.




**empty set** (p. 28) The solution set for an equation that has no solution, symbolized by { } or ø.

**conjunto vacío** Conjunto solución de una ecuación que no tiene solución, denotado por { } o ø.

**end behavior** (p. 334) The behavior of the graph as $x$ approaches positive infinity $(+\infty)$ or negative infinity $(-\infty)$.

**comportamiento final** El comportamiento de una gráfica a medida que $x$ tiende a más infinito $(+\infty)$ o menos infinito $(-\infty)$.

**equal matrices** (p. 164) Two matrices that have the same dimensions and each element of one matrix is equal to the corresponding element of the other matrix.

**matrices iguales** Dos matrices que tienen las mismas dimensiones y en las que cada elemento de una de ellas es igual al elemento correspondiente en la otra matriz.

**equation** (p. 18) A mathematical sentence stating that two mathematical expressions are equal.

**ecuación** Enunciado matemático que afirma la igualdad de dos expresiones matemáticas.

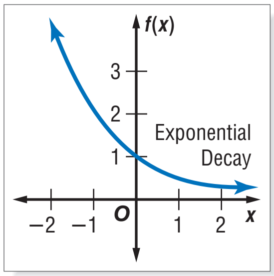
**event** (p. 684) One or more outcomes of a trial.

**evento** Uno o más resultados de una prueba.

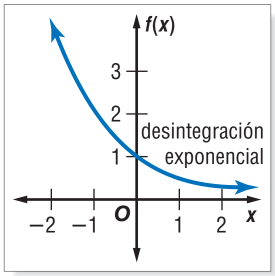
**expansion by minors** (p. 195) A method of evaluating a third or high order determinant by using determinants of lower order.

**expansión por determinantes menores** Un método de calcular el determinante de tercer orden o mayor mediante el uso de determinantes de orden más bajo.

**exponential decay** (p. 500) Exponential decay occurs when a quantity decreases exponentially over time.



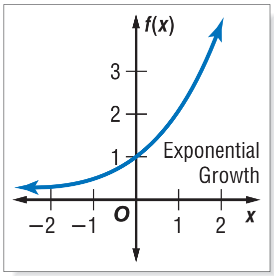

**desintegración exponencial** Ocurre cuando una cantidad disminuye exponencialmente con el tiempo.



**exponential equation** (p. 501) An equation in which the variables occur as exponents.

**exponential function** (p. 499) A function of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

**exponential growth** (p. 500) Exponential growth occurs when a quantity increases exponentially over time.




**exponential inequality** (p. 502) An inequality involving exponential functions.

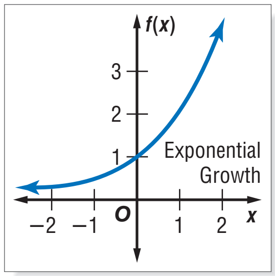
**extraneous solution** (p. 422) A number that does not satisfy the original equation.

**extrapolation** (p. 87) Predicting for an $x$-value greater than any in the data set.

**ecuación exponencial** Ecuación en que las variables aparecen en los exponentes.

**función exponencial** Una función de la forma $y = ab^x$, donde $a \neq 0$, $b > 0$, y $b \neq 1$.

**crecimiento exponencial** El que ocurre cuando una cantidad aumenta exponencialmente con el tiempo.




**desigualdad exponencial** Desigualdad que contiene funciones exponenciales.

**solución extraña** Número que no satisface la ecuación original.

**extrapolación** Predicción para un valor de $x$ mayor que cualquiera de los de un conjunto de datos.

## F

**factorial** (p. 666) If n is a positive integer, then $n! = n(n - 1)(n - 2) \ldots 2 \cdot 1$.

**failure** (p. 697) Any outcome other than the desired outcome.

**family of graphs** (p. 73) A group of graphs that displays one or more similar characteristics.

**feasible region** (p. 138) The intersection of the graphs in a system of constraints.

**Fibonacci sequence** (p. 658) A sequence in which the first two terms are 1 and each of the additional terms is the sum of the two previous terms.

**focus** (pp. 567, 581, 590) See parabola, ellipse, hyperbola.

**FOIL method** (p. 253) The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

**formula** (p. 7) A mathematical sentence that expresses the relationship between certain quantities.

**factorial** Si n es un entero positivo, entonces $n! = n(n - 1)(n - 2) \ldots 2 \cdot 1$.

**fracaso** Cualquier resultado distinto del deseado.

**familia de gráficas** Grupo de gráficas que presentan una o más características similares.

**región viable** Intersección de las gráficas de un sistema de restricciones.

**sucesión de Fibonacci** Sucesión en que los dos primeros términos son iguales a 1 y cada término que sigue es igual a la suma de los dos anteriores.

**foco** Véase parábola, elipse, hipérbola.

**método FOIL** El producto de dos binomios es la suma de los productos de los primeros (*First*) términos, los términos exteriores (*Outer*), los términos interiores (*Inner*) y los últimos (*Last*) términos.

**fórmula** Enunciado matemático que describe la relación entre ciertas cantidades.

**function** (p. 59) A relation in which each element of the domain is paired with exactly one element in the range.

**función** Relación en que a cada elemento del dominio le corresponde un solo elemento del rango.

**function notation** (p. 61) An equation of $y$ in terms of $x$ can be rewritten so that $y = f(x)$. For example, $y = 2x + 1$ can be written as $f(x) = 2x + 1$.

**notación funcional** Una ecuación de y en términos de x puede escribirse en la forma $y = f(x)$. Por ejemplo, $y = 2x + 1$ puede escribirse como $f(x) = 2x + 1$.

**Fundamental Counting Principle** (p. 685) If event $M$ can occur in $m$ ways and is followed by event $N$ that can occur in $n$ ways, then event $M$ followed by event $N$ can occur in $m \cdot n$ ways.

**principio fundamental de conteo** Si el evento $M$ puede ocurrir de $m$ maneras y es seguido por el evento $N$ que puede ocurrir de $n$ maneras, entonces el evento $M$ seguido por el evento $N$ pueden ocurrir de $m \cdot n$ maneras.

## G

**geometric mean** (p. 638) The terms between any two nonsuccessive terms of a geometric sequence.

**media geométrica** Cualquier término entre dos términos no consecutivos de una sucesión geométrica.

**geometric sequence** (p. 636) A sequence in which each term after the first is found by multiplying the previous term by a constant $r$, called the common ratio.

**sucesión geométrica** Sucesión en que cualquier término después del primero puede hallarse multiplicando el término anterior por una constante $r$, llamada razón común .

**geometric series** (p. 643) The sum of the terms of a geometric sequence.

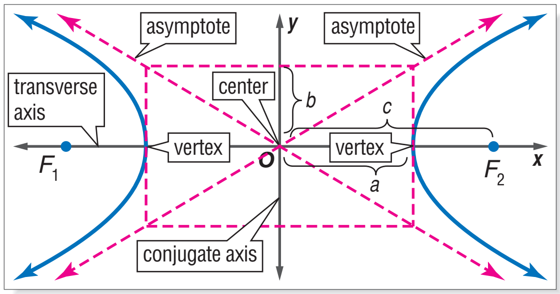
**serie geométrica** La suma de los términos de una sucesión geométrica.

**greatest integer function** (p. 95) A step function, written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer less than or equal to $x$.

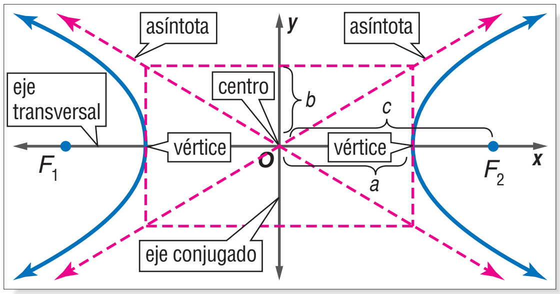
**función del máximo entero** Una función etapa que se escribe $f(x) = [\![x]\!]$, donde $f(x)$ es el meaximo entero que es menor que o igual a $x$.

## H

**hyperbola** (p. 590) The set of all points in the plane such that the absolute value of the difference of the distances from two given points in the plane, called foci, is constant.




**hipérbola** Conjunto de todos los puntos de un plano en los que el valor absoluto de la diferencia de sus distancias a dos puntos dados del plano, llamados focos, es constante.



# I

**identity function** (p. 96, 393) The function $I(x) = x$.

**identity matrix** (p. 208) A square matrix that, when multiplied by another matrix, equals that same matrix. If $A$ is any $n \times n$ matrix and $I$ is the $n \times n$ identity matrix, then $A \cdot I = A$ and $I \cdot A = A$.

**image** (p. 185) The graph of an object after a transformation.

**imaginary unit** (p. 260) $\boldsymbol{i}$, or the principal square root of $-1$.

**inclusive** (p. 712) Two events whose outcomes may be the same.

**inconsistent** (p. 118) A system of equations that has no solutions.

**independent events** (p. 684) Events that do not affect each other.

**independent system** (p. 118) A system of equations that has exactly one solution.

**independent variable** (p. 61) In a function, the variable, usually $x$, whose values make up the domain.

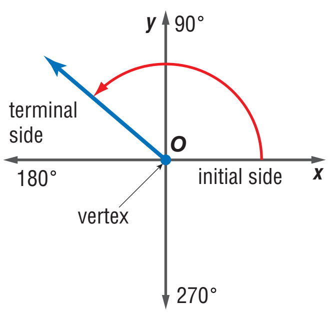
**index of summation** (p. 631) The variable used with the summation symbol. In the expression below, the index of summation is $n$.

$$\sum_{n=1}^{3} 4n$$

**inductive hypothesis** (p. 670) The assumption that a statement is true for some positive integer $k$, where $k \geq n$.

**infinite geometric series** (p. 650) A geometric series with an infinite number of terms.

**initial side of an angle** (p. 768) The fixed ray of an angle.




**unción identidad** La función $I(x) = x$.

**matriz identidad** Matriz cuadrada que al multiplicarse por otra matriz, es igual a la misma matriz. Si $A$ es una matriz de $n \times n$ e $I$ es la matriz identidad de $n \times n$, entonces $A \cdot I = A$ y $I \cdot A = A$.

**imagen** Gráfica de una figura después de una transformación.

**unidad imaginaria** $\boldsymbol{i}$, o la raíz cuadrada principal de $-1$.

**inclusivo** Dos eventos que pueden tener los mismos resultados.

**inconsistente** Sistema de ecuaciones que no tiene solución alguna.

**eventos independientes** Eventos que no se afectan mutuamente.

**sistema independiente** Sistema de ecuaciones que sólo tiene una solución.

**variable independiente** En una función, la variable, por lo general $x$, cuyos valores forman el dominio.

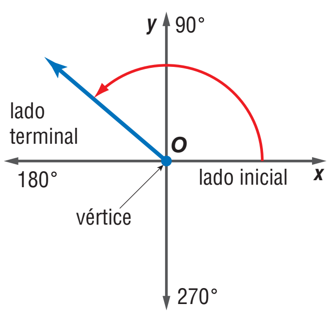
**índice de suma** Variable que se usa con el símbolo de suma. En la siguiente expresión, el índice de suma es $n$.

$$\sum_{n=1}^{3} 4n$$

**hipótesis inductiva** El suponer que un enunciado es verdadero para algún entero positivo $k$, donde $k \geq n$.

**serie geométrica infinita** Serie geométrica con un número infinito de términos.

**lado inicial de un ángulo** El rayo fijo de un ángulo.



**intercept form** (p. 253) A quadratic equation in the form $y = a(x - p)(x - q)$ where $p$ and $q$ represent the $x$-intercept of the graph.

**forma intercepción** Ecuación cuadrática de la forma $y = a(x - p)(x - q)$ donde $p$ y $q$ representan la intersección $x$ de la gráfica.

**interpolation** (p. 87) Predicting for an $x$-value between the least and greatest values of the set.

**interpolación** Predecir un valor de $x$ entre los valores máximo y mínimo del conjunto de datos.

**intersection** (p. 41) The graph of a compound inequality containing *and*.

**intersección** Gráfica de una desigualdad compuesta que contiene la palabra *y*.

**inverse** (p. 209) Two $n \times n$ matrices are inverses of each other if their product is the identity matrix.

**inversa** Dos matrices de $n \times n$ son inversas mutuas si su producto es la matriz identidad.

**inverse function** (p. 392) Two functions $f$ and $g$ are inverse functions if and only if both of their compositions are the identity function.

**función inversa** Dos funciones $f$ y $g$ son inversas mutuas si y sólo si las composiciones de ambas son la función identidad.

**inverse of a trigonometric function** (p. 806) The arccosine, arcsine, and arctangent relations.

**inversa de una función trigonométrica** Las relaciones arcocoseno, arcoseno y arcotangente.

**inverse relations** (p. 391) Two relations are inverse relations if and only if whenever one relation contains the element $(a, b)$ the other relation contains the element $(b, a)$.

**relaciones inversas** Dos relaciones son relaciones inversas mutuas si y sólo si cada vez que una de las relaciones contiene el elemento $(a, b)$, la otra contiene el elemento $(b, a)$.

**inverse variation** (p. 467) $y$ varies inversely as $x$ if there is some nonzero constant $k$ such that $xy = k$ or $y = \frac{k}{x}$, where $x \neq 0$ and $y \neq 0$.

**variación inversa** $y$ varía inversamente con $x$ si hay una constante no nula $k$ tal que $xy = k$ o $y = \frac{k}{x}$, donde $x \neq 0$ y $y \neq 0$.

**irrational number** (p. 11) A real number that is not rational. The decimal form neither terminates nor repeats.

**número irracional** Número que no es racional. Su expansión decimal no es ni terminal ni periódica.

**iteration** (p. 660) The process of composing a function with itself repeatedly.

**iteración** Proceso de componer una función consigo misma repetidamente.

## J

**joint variation** (p. 466) $y$ varies jointly as $x$ and $z$ if there is some nonzero constant $k$ such that $y = kxz$.

**variación conjunta** $y$ varía conjuntamente con $x$ y $z$ si hay una constante no nula $k$ tal que $y = kxz$.

## L

**latus rectum** (p. 569) The line segment through the focus of a parabola and perpendicular to the axis of symmetry.

**latus rectum** El segmento de recta que pasa por el foco de una parábola y que es perpendicular a su eje de simetría.

**Law of Cosines** (pp. 793–794) Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of sides, and opposite angles with measures $A$, $B$, and $C$, respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Ley de los cosenos** Sea $\triangle ABC$ un triángulo cualquiera, con $a$, $b$ y $c$ las longitudes de los lados y con ángulos opuestos de medidas $A$, $B$ y $C$, respectivamente. Entonces se cumplen las siguientes ecuaciones.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Law of Sines** (p. 786) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measurements $A$, $B$, and $C$, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

**Ley de los senos** Sea $\triangle ABC$ cualquier triángulo con $a$, $b$ y $c$ las longitudes de los lados y con ángulos opuestos de medidas $A$, $B$ y $C$, respectivamente.
Entonces $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

**leading coefficient** (p. 331) The coefficient of the term with the highest degree.

**coeficiente líder** Coeficiente del término de mayor grado.

**like radical expressions** (p. 411) Two radical expressions in which both the radicands and indices are alike.

**expresiones radicales semejantes** Dos expresiones radicales en que tanto los radicandos como los índices son semejantes.

**like terms** (p. 7) Monomials that can be combined.

**términos semejantes** Monomios que pueden combinarse.

**limit** (p. 642) The value that the terms of a sequence approach.

**límite** El valor al que tienden los términos de una sucesión.

**linear correlation coefficient** (p. 92) A value that shows how close data points are to a line.

**coeficiente de correlación lineal** Valor que muestra la cercanía de los datos a una recta.

**linear equation** (p. 66) An equation that has no operations other than addition, subtraction, and multiplication of a variable by a constant.

**ecuación lineal** Ecuación sin otras operaciones que las de adición, sustracción y multiplicación de una variable por una constante.

**linear function** (p. 66) A function whose ordered pairs satisfy a linear equation.

**función lineal** Función cuyos pares ordenados satisfacen una ecuación lineal.

**linear permutation** (p. 690) The arrangement of objects or people in a line.

**permutación lineal** Arreglo de personas o figuras en una línea.

**linear programming** (p. 140) The process of finding the maximum or minimum values of a function for a region defined by inequalities.

**programación lineal** Proceso de hallar los valores máximo o mínimo de una función lineal en una región definida por las desigualdades.

**linear term** (p. 236) In the equation $f(x) = ax^2 + bx + c$, $bx$ is the linear term.

**término lineal** En la ecuación $f(x) = ax^2 + bx + c$, el término lineal es $bx$.

**line of best fit** (p. 92) A line that best matches a set of data.

**recta de óptimo ajuste** Recta que mejor encaja un conjunto de datos.

**line of fit** (p. 86) A line that closely approximates a set of data.

**recta de ajuste** Recta que se aproxima estrechamente a un conjunto de datos.

**Location Principle** (p. 340) Suppose $y = f(x)$ represents a polynomial function and $a$ and $b$ are two numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between $a$ and $b$.

**principio de ubicación** Sea $y = f(x)$ una función polinómica con $a$ y $b$ dos números tales que $f(a) < 0$ y $f(b) > 0$. Entonces la función tiene por lo menos un resultado real entre $a$ y $b$.

**logarithm** (p. 510) In the function $x = b^y$, $y$ is called the logarithm, base $b$, of $x$. Usually written as $y = \log_b x$ and is read "$y$ equals log base $b$ of $x$."

**logaritmo** En la función $x = b^y$, $y$ es el logaritmo en base $b$, de $x$. Generalmente escrito como $y = \log_b x$ y se lee "$y$ es igual al logaritmo en base $b$ de $x$."

**logarithmic equation** (p. 512) An equation that contains one or more logarithms.

**ecuación logarítmica** Ecuación que contiene uno o más logaritmos.

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**logarithmic function** (p. 511) The function $y = \log_b x$, where $b > 0$ and $b \neq 1$, which is the inverse of the exponential function $y = bx$.

**función logarítmica** La función $y = \log_b x$, donde $b > 0$ y $b \neq 1$, inversa de la función exponencial $y = bx$.

**logarithmic inequality** (p. 512) An inequality that contains one or more logarithms.

**desigualdad logarítmica** Desigualdad que contiene uno o más logaritmos.

## M

**major axis** (p. 582) The longer of the two line segments that form the axes of symmetry of an ellipse.

**eje mayor** El más largo de dos segmentos de recta que forman los ejes de simetría de una elipse.

**mapping** (p. 58) How each member of the domain is paired with each member of the range.

**transformaciones** La correspondencia entre cada miembro del dominio con cada miembro del rango.

**margin of sampling error (ME)** (p. 735) The limit on the difference between how a sample responds and how the total population would respond.

**margen de error muestral (EM)** Límite en la diferencia entre las respuestas obtenidas con una muestra y cómo pudiera responder la población entera.

**mathematical induction** (p. 670) A method of proof used to prove statements about positive integers.

**inducción matemática** Método de demostrar enunciados sobre los enteros positivos.

**matrix** (p. 162) Any rectangular array of variables or constants in horizontal rows and vertical columns.

**matriz** Arreglo rectangular de variables o constantes en filas horizontales y columnas verticales.

**matrix equation** (p. 216) A matrix form used to represent a system of equations.

**ecuación matriz** Forma de matriz que se usa para representar un sistema de ecuaciones.

**maximum value** (p. 238) The y-coordinate of the vertex of the quadratic function $f(x) = ax^2 + bx + c$, where $a < 0$.

**valor máximo** La coordenada y del vértice de la función cuadrática $f(x) = ax^2 + bx + c$, where $a < 0$.

**measure of central tendency** (p. 717) A number that represents the center or middle of a set of data.

**medida de tendencia central** Número que representa el centro o medio de un conjunto de datos.

**measure of variation** (p. 718) A representation of how spread out or scattered a set of data is.

**medida de variación** Número que representa la dispersión de un conjunto de datos.

**midline** (p. 831) A horizontal axis used as the reference line about which the graph of a periodic function oscillates.

**recta central** Eje horizontal que se usa como recta de referencia alrededor de la cual oscila la gráfica de una función periódica.

**minimum value** (p. 238) The y-coordinate of the vertex of the quadratic function $f(x) = ax^2 + bx + c$, where $a > 0$.

**valor mínimo** La coordenada y del vértice de la función cuadrática $f(x) = ax^2 + bx + c$, donde $a > 0$.

**minor** (p. 195) The determinant formed when the row and column containing that element are deleted.

**determinante menor** El que se forma cuando se descartan la fila y columna que contienen dicho elemento.

**minor axis** (p. 582) The shorter of the two line segments that form the axes of symmetry of an ellipse.

**eje menor** El más corto de los dos segmentos de recta de los ejes de simetría de una elipse.

**monomial** (p. 6) An expression that is a number, a variable, or the product of a number and one or more variables.

**monomio** Expresión que es un número, una variable o el producto de un número por una o más variables.

**mutually exclusive** (p. 710) Two events that cannot occur at the same time.

**mutuamente exclusivos** Dos eventos que no pueden ocurrir simultáneamente.

## N

***nth* root** (p. 402) For any real numbers $a$ and $b$, and any positive integer $n$, if $a^n = b$, then a is an *nth* root of $b$.

**raíz *enésima*** Para cualquier número real $a$ y $b$ y cualquier entero positivo $n$, si $a^n = b$, entonces $a$ se llama una raíz *enésima* de $b$.

**natural base, *e*** (p. 536) An irrational number approximately equal to 2.71828… .

**base natural, *e*** Número irracional aproximadamente igual a 2.71828…

**natural base exponential function** (p. 536) An exponential function with base $e$, $y = e^x$.

**función exponencial natural** La función exponencial de base $e$, $y = e^x$.

**natural logarithm** (p. 537) Logarithms with base $e$, written $\ln x$.

**logaritmo natural** Logaritmo de base $e$, el que se escribe $\ln x$.

**natural logarithmic function** (p. 537) $y = \ln x$, the inverse of the natural base exponential function $y = e^x$.

**función logarítmica natural** $y = \ln x$, la inversa de la función exponencial natural $y = e^x$.

**negative exponent** (p. 312) For any real number $a \neq 0$ and any integer $n$, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

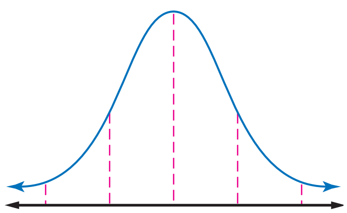
**exponente negativo** Para cualquier número real $a \neq 0$ cualquier entero positivo $n$, $a^{-n} = \frac{1}{a^n}$ y $\frac{1}{a^{-n}} = a^n$.

**nonrectangular hyperbola** (p. 596) A hyperbola with asymptotes that are not perpendicular.

**hipérbola no rectangular** Hipérbola con asíntotas que no son perpendiculares.

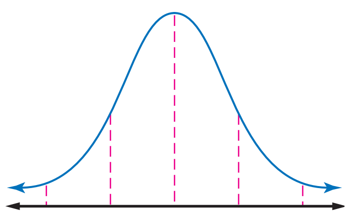
**normal distribution** (p. 724) A frequency distribution that often occurs when there is a large number of values in a set of data: about 68% of the values are within one standard deviation of the mean, 95% of the values are within two standard deviations from the mean, and 99% of the values are within three standard deviations.

Normal Distribution



**distribución normal** Distribución de frecuencia que aparece a menudo cuando hay un número grande de datos: cerca del 68% de los datos están dentro de una desviación estándar de la media, 95% están dentro de dos desviaciones estándar de la media y 99% están dentro de tres desviaciones estándar de la media.

Distribución normal



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## O

**one-to-one function** (p. 394) **1.** A function where each element of the range is paired with exactly one element of the domain **2.** A function whose inverse is a function.

**open sentence** (p. 18) A mathematical sentence containing one or more variables.

**ordered pair** (p. 58) A pair of coordinates, written in the form $(x, y)$, used to locate any point on a coordinate plane.

**ordered triple** (p. 146) **1.** The coordinates of a point in space **2.** The solution of a system of equations in three variables $x$, $y$, and $z$.

**Order of Operations** (p. 6)
**Step 1** Evaluate expressions inside grouping symbols.
**Step 2** Evaluate all powers.
**Step 3** Do all multiplications and/or divisions from left to right.
**Step 4** Do all additions and subtractions from left to right.

**outcomes** (p. 684) The results of a probability experiment or an event.

**outlier** (p. 87) A data point that does not appear to belong to the rest of the set.

**función biunívoca** **1.** Función en la que a cada elemento del rango le corresponde sólo un elemento del dominio. **2.** Función cuya inversa es una función.

**enunciado abierto** Enunciado matemático que contiene una o más variables.

**par ordenado** Un par de números, escrito en la forma $(x, y)$, que se usa para ubicar cualquier punto en un plano de coordenadas.

**triple ordenado** **1.** Las coordenadas de un punto en el espacio **2.** Solución de un sistema de ecuaciones en tres variables $x$, $y$ y $z$.

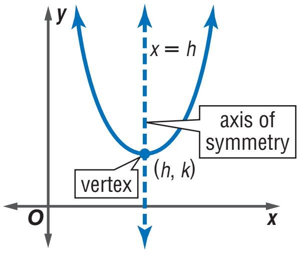
**orden de las operaciones**
**Paso 1** Evalúa las expresiones dentro de símbolos de agrupamiento.
**Paso 2** Evalúa todas las potencias.
**Paso 3** Ejecuta todas las multiplicaciones y divisiones de izquierda a derecha.
**Paso 4** Ejecuta todas las adiciones y sustracciones de izquierda a derecha.

**resultados** Lo que produce un experimento o evento probabilístico.

**valor atípico** Dato que no parece pertenecer al resto el conjunto.

## P

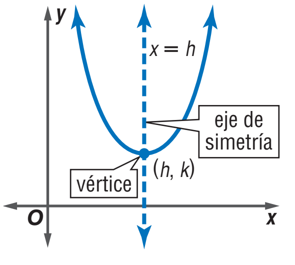
**parabola** (p. 236, 567) The set of all points in a plane that are the same distance from a given point, called the focus, and a given line, called the directrix.




**parallel lines** (p. 73) Nonvertical coplanar lines with the same slope.

**parent graph** (p. 73) The simplest of graphs in a family.

**parábola** Conjunto de todos los puntos de un plano que están a la misma distancia de un punto dado, llamado foco, y de una recta dada, llamada directriz.




**rectas paralelas** Rectas coplanares no verticales con la misma pendiente.

**gráfica madre** La gráfica más sencilla en una familia de gráficas.

Glossary/Glosario

**partial sum** (p. 650) The sum of the first $n$ terms of a series.

**Pascal's triangle** (p. 664) A triangular array of numbers such that the $(n + 1)^{th}$ row is the coefficient of the terms of the expansion $(x + y)^n$ for $n = 0, 1, 2 \ldots$

**period** (p. 801) The least possible value of $a$ for which $f(x) = f(x + a)$.

**periodic function** (p. 801) A function is called periodic if there is a number $a$ such that $f(x) = f(x + a)$ for all $x$ in the domain of the function.

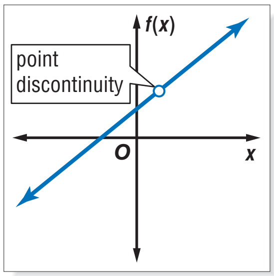
**permutation** (p. 690) An arrangement of objects in which order is important.

**perpendicular lines** (p. 74) In a plane, any two oblique lines the product of whose slopes is 21.

**phase shift** (p. 829) A horizontal translation of a trigonometric function.

**piecewise function** (p. 97) A function that is written using two or more expressions.

**point discontinuity** (p. 457) If the original function is undefined for $x = a$ but the related rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.




**point-slope form** (p. 80) An equation in the form $y - y_1 = m(x - x_1)$ where $(x_1, y_1)$ are the coordinates of a point on the line and $m$ is the slope of the line.

**polynomial** (p. 7) A monomial or a sum of monomials.

**polynomial function** (p. 332) A function that is represented by a polynomial equation.

**suma parcial** La suma de los primeros $n$ términos de una serie.

**triángulo de Pascal** Arreglo triangular de números en el que la fila $(n + 1)^n$ proporciona los coeficientes de los términos de la expansión de $(x + y)^n$ para $n = 0, 1, 2 \ldots$

**período** El menor valor positivo posible para $a$, para el cual $f(x) = f(x + a)$.

**función periódica** Función para la cual hay un número $a$ tal que $f(x) = f(x + a)$ para todo $x$ en el dominio de la función .

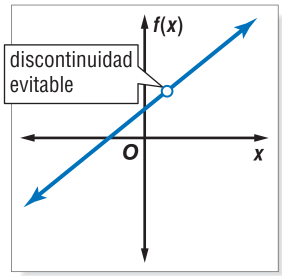
**permutación** Arreglo de elementos en que el orden es importante.

**rectas perpendiculares** En un plano, dos rectas oblicuas cualesquiera cuyas pendientes tienen un producto igual a 21.

**desvío de fase** Traslación horizontal de una función trigonométrica.

**función a intervalos** Función que se escribe usando dos o más expresiones.

**discontinuidad evitable** Si la función original no está definida en $x = a$ pero la expresión racional reducida correspondiente de la función está definida en $x = a$, entonces la gráfica tiene una ruptura o corte en $x = a$.




**forma punto-pendiente** Ecuación de la forma $y - y_1 = m(x - x_1)$ donde $(x_1, y_1)$ es un punto en la recta y $m$ es la pendiente de la recta.

**polinomio** Monomio o suma de monomios.

**función polinomial** Función representada por una ecuación polinomial.

**polynomial in one variable** (p. 331) $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, where the coefficients $a_n, a_{n-1}, \ldots, a_0$ represent real numbers, and $a_n$ is not zero and $n$ is a nonnegative integer.

**polinomio de una variable** $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, donde los coeficientes $a_n, a_{n-1}, \ldots, a_0$ son números reales, $a_n$ no es nulo y $n$ es un entero no negativo.

**power** (p. 7) An expression of the form $x^n$.

**potencia** Expresión de la forma $x^n$.

**power function** (p. 762) An equation in the form $f(x) = ax^b$, where $a$ and $b$ are real numbers.

**función potencia** Ecuación de la forma $f(x) = ax^b$, donde $a$ y $b$ son números reales.

**prediction equation** (p. 86) An equation suggested by the points of a scatter plot that is used to predict other points.

**ecuación de predicción** Ecuación sugerida por los puntos de una gráfica de dispersión y que se usa para predecir otros puntos.

**preimage** (p. 185) The graph of an object before a transformation.

**preimagen** Gráfica de una figura antes de una transformación.

**principal root** (p. 402) The nonnegative root.

**raíz principal** La raíz no negativa.

**principal values** (p. 806) The values in the restricted domains of trigonometric functions.

**valores principales** Valores en los dominios restringidos de las funciones trigonométricas.

**probability** (p. 697) A ratio that measures the chances of an event occurring.

**probabilidad** Razón que mide la posibilidad de que ocurra un evento.

**probability distribution** (p. 699) A function that maps the sample space to the probabilities of the outcomes in the sample space for a particular random variable.

**distribución de probabilidad** Función que aplica el espacio muestral a las probabilidades de los resultados en el espacio muestral obtenidos para una variable aleatoria particular.

**pure imaginary number** (p. 260) The square roots of negative real numbers. For any positive (real number $b$, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$, or $b\boldsymbol{i}$.

**número imaginario puro** Raíz cuadrada de un número real negativo. Para cualquier número (real positivo $b$, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$, ó $b\boldsymbol{i}$.

## Q

**quadrantal angle** (p. 778) An angle in standard position whose terminal side coincides with one of the axes.

**ángulo de cuadrante** Ángulo en posición estándar cuyo lado terminal coincide con uno de los ejes.

**quadrants** (p. 58) The four areas of a Cartesian coordinate plane.

**cuadrantes** Las cuatro regiones de un plano de coordenadas Cartesiano.

**quadratic equation** (p. 246) A quadratic function set equal to a value, in the form $ax^2 + bx + c$, where $a \neq 0$.

**ecuación cuadrática** Función cuadrática igual a un valor, de la forma $ax^2 + bx + c$, donde $a \neq 0$.

**quadratic form** (p. 351) For any numbers $a$, $b$, and $c$, except for $a = 0$, an equation that can be written in the form a$[f(x)^2] + b[f(x)] + c = 0$, where $f(x)$ is some expression in $x$.

**forma de ecuación cuadrática** Para cualquier número $a$, $b$, y $c$, excepto $a = 0$, una ecuación que puede escribirse de la forma $[f(x)^2] + b[f(x)] + c = 0$, donde $f(x)$ es una expresión en $x$.

**Quadratic Formula** (p. 276) The solutions of a quadratic equation of the form $ax^2 + bx + c$, where $a \neq 0$, are given by the Quadratic Formula, which is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**fórmula cuadrática** Las soluciones de una ecuación cuadrática de la forma $ax^2 + bx + c$, donde $a \neq 0$, se dan por la fórmula cuadrática, que es $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**quadratic function** (p. 236) A function described by the equation $f(x) = ax^2 + bx + c$, where $a \neq 0$.

**quadratic inequality** (p. 294) A quadratic equation in the form $y > ax^2 + bx + c$, $y \geq ax^2 + bx + c$, $y < ax^2 + bx + c$, or $y \leq ax^2 + bx + c$.

**quadratic term** (p. 236) In the equation $f(x) = ax^2 + bx + c$, $ax^2$ is the quadratic term.

**función cuadrática** Función descrita por la ecuación $f(x) = ax^2 + bx + c$, donde $a \neq 0$.

**desigualdad cuadrática** Ecuación cuadrática de la forma $y > ax^2 + bx + c$, $y \geq ax^2 + bx + c$, $y < ax^2 + bx + c$, $y \leq ax^2 + bx + c$.

**término cuadrático** En la ecuación $f(x) = ax^2 + bx + c$, el término cuadrático es $ax^2$.

## R

**radian** (p. 770) The measure of an angle $\theta$ in standard position whose rays intercept an arc of length 1 unit on the unit circle.

**radical equation** (p. 422) An equation with radicals that have variables in the radicands.

**radical inequality** (p. 424) An inequality that has a variable in the radicand.

**random** (p. 697) All outcomes have an equally likely chance of happening.

**random variable** (p. 699) The outcome of a random process that has a numerical value.

**range** (p. 58) The set of all $y$-coordinates of a relation.

**rate of change** (p. 71) How much a quantity changes on average, relative to the change in another quantity, often time.

**rate of decay** (p. 544) The percent decrease $r$ in the equation $y = a(1 - r)^t$.

**rate of growth** (p. 546) The percent increase $r$ in the equation $y = a(1 + r)^t$.

**rational equation** (p. 479) Any equation that contains one or more rational expressions.

**rational exponent** (p. 416) For any nonzero real number $b$, and any integers $m$ and $n$, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, except when $b < 0$ and $n$ is even.

**rational expression** (p. 457) A ratio of two polynomial expressions.

**rational function** (p. 472) An equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions, and $q(x) \neq 0$.

**radián** Medida de un ángulo $\theta$ en posición normal cuyos rayos intersecan un arco de 1 unidad de longitud en el círculo unitario.

**ecuación radical** Ecuación con radicales que tienen variables en el radicando.

**desigualdad radical** Desigualdad que tiene una variable en el radicando.

**aleatorio** Todos los resultados son equiprobables.

**variable aleatoria** El resultado de un proceso aleatorio que tiene un valor numérico.

**rango** Conjunto de todas las coordenadas y de una relación.

**tasa de cambio** Lo que cambia una cantidad en promedio, respecto al cambio en otra cantidad, por lo general el tiempo.

**tasa de desintegración** Disminución porcentual $r$ en la ecuación $y = a(1 - r)^t$.

**tasa de crecimiento** Aumento porcentual $r$ en la ecuación $y = a(1 + r)^t$.

**ecuación racional** Cualquier ecuación que contiene una o más expresiones racionales.

**exponent racional** Para cualquier número real no nulo $b$ y cualquier entero $m$ y $n$, con $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, excepto cuando $b < 0$ y $n$ es par.

**expresión racional** Razón de dos expresiones polinomiales.

**función racional** Ecuación de la forma $f(x) = \frac{p(x)}{q(x)}$, donde $p(x)$ y $q(x)$ son funciones polinomiales y $q(x) \neq 0$.

**rational inequality** (p. 483) Any inequality that contains one or more rational expressions.

**rationalizing the denominator** (p. 409) To eliminate radicals from a denominator or fractions from a radicand.

**rational number** (p. 11) Any number $\frac{m}{n}$, where $m$ and $n$ are integers and $n$ is not zero. The decimal form is either a terminating or repeating decimal.

**real numbers** (p. 11) All numbers used in everyday life; the set of all rational and irrational numbers.

**rectangular hyperbola** (p. 596) A hyperbola with perpendicular asymptotes.

**recursive formula** (p. 658) Each term is formulated from one or more previous terms.

**reference angle** (p. 778) The acute angle formed by the terminal side of an angle in standard position and the $x$-axis.

**reflection** (p. 188) A transformation in which every point of a figure is mapped to a corresponding image across a line of symmetry.

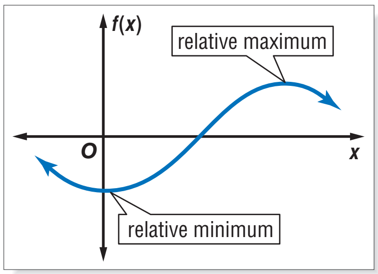
**reflection matrix** (p. 188) A matrix used to reflect an object over a line or plane.

**regression line** (p. 92) A line of best fit.

**relation** (p. 58) A set of ordered pairs.

**relative frequency histogram** (p. 699) A table of probabilities or a graph to help visualize a probability distribution.

**relative maximum** (p. 340) A point on the graph of a function where no other nearby points have a greater y-coordinate.




**relative minimum** (p. 340) A point on the graph of a function where no other nearby points have a lesser y-coordinate.

**desigualdad racional** Cualquier desigualdad que contiene una o más expresiones racionales.

**racionalizar el denominador** La eliminación de radicales de un denominador o de fracciones de un radicando.

**número racional** Cualquier número $\frac{m}{n}$, donde $m$ y $n$ son enteros y $n$ no es cero. Su expansión decimal es o terminal o periódica.

**números reales** Todos los números que se usan en la vida cotidiana; el conjunto de los todos los números racionales e irracionales.

**hipérbola rectangular** Hipérbola con asíntotas perpendiculares.

**fórmula recursiva** Cada término proviene de uno o más términos anteriores.

**ángulo de referencia** El ángulo agudo formado por el lado terminal de un ángulo en posición estándar y el eje $x$.

**reflexión** Transformación en que cada punto de una figura se aplica a través de una recta de simetría a su imagen correspondiente.

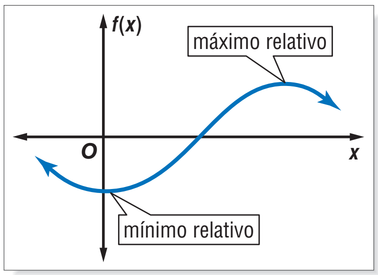
**matriz de reflexión** Matriz que se usa para reflejar una figura sobre una recta o plano.

**reca de regresión** Una recta de óptimo ajuste.

**relación** Conjunto de pares ordenados.

**histograma de frecuencia relativa** Tabla de probabilidades o gráfica para asistir en la visualización de una distribución de probabilidad.

**máximo relativo** Punto en la gráfica de una función en donde ningún otro punto cercano tiene una coordenada y mayor.




**mínimo relativo** Punto en la gráfica de una función en donde ningún otro punto cercano tiene una coordenada y menor.

**root** (p. 246) The solutions of a quadratic equation.

**raíz** Las soluciones de una ecuación cuadrática.

**rotation** (p. 188) A transformation in which an object is moved around a center point, usually the origin.

**rotación** Transformación en que una figura se hace girar alrededor de un punto central, generalmente el origen.

**rotation matrix** (p. 188) A matrix used to rotate an object.

**matriz de rotación** Matriz que se usa para hacer girar un objeto.

**row matrix** (p. 163) A matrix that has only one row.

**matriz fila** Matriz que sólo tiene una fila.

## S

**sample space** (p. 684) The set of all possible outcomes of an experiment.

**espacio muestral** Conjunto de todos los resultados posibles de un experimento probabilístico.

**scalar** (p. 171) A constant.

**escalar** Una constante.

**scalar multiplication** (p. 171) Multiplying any matrix by a constant called a scalar; the product of a scalar $k$ and an $m \times n$ matrix.

**multiplicación por escalares** Multiplicación de una matriz por una constante llamada escalar; producto de un escalar $k$ y una matriz de $m \times n$.

**scatter plot** (p. 86) A set of data graphed as ordered pairs in a coordinate plane.

**gráfica de dispersión** Conjuntos de datos graficados como pares ordenados en un plano de coordenadas.

**scientific notation** (p. 315) The expression of a number in the form $a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer.

**notación científica** Escritura de un número en la forma $a \times 10^n$, donde $1 \leq a < 10$ y $n$ es un entero.

**secant** (p. 759) For any angle, with measure $\alpha$, a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, $\sec \alpha = \frac{r}{x}$.

**secante** Para cualquier ángulo de medida $\alpha$, un punto $P(x, y)$ en su lado terminal, $r = \sqrt{x^2 + y^2}$, $\sec \alpha = \frac{r}{x}$.

**second-order determinant** (p. 194) The determinant of a $2 \times 2$ matrix.

**determinante de segundo orden** El determinante de una matriz de $2 \times 2$.

**sequence** (p. 622) A list of numbers in a particular order.

**sucesión** Lista de números en un orden particular.

**series** (p. 629) The sum of the terms of a sequence.

**serie** Suma específica de los términos de una sucesión.

**set-builder notation** (p. 35) The expression of the solution set of an inequality, for example $\{x|x > 9\}$.

**notación de construcción de conjuntos** Escritura del conjunto solucion de una desigualdad, por ejemplo, $\{x|x > 9\}$.

**sigma notation** (p. 631) For any sequence $a_1, a_2, a_3, \ldots$, the sum of the first $k$ terms may be written $\sum_{n=1}^{k} a_n$, which is read "the summation from $n = 1$ to $k$ of $a_n$." Thus, $\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \ldots + a_k$, where $k$ is an integer value.

**notación de suma** Para cualquier sucesión $a_1, a_2, a_3, \ldots$, la suma de los $k$ primeros términos puede escribirse $\sum_{n=1}^{k} a_n$, lo que se lee "la suma de $n = 1$ a $k$ de los $a_n$." Así, $\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \ldots + a_k$, donde $k$ es un valor entero.

Glossary/Glosario

**simple event** (p. 710) One event.

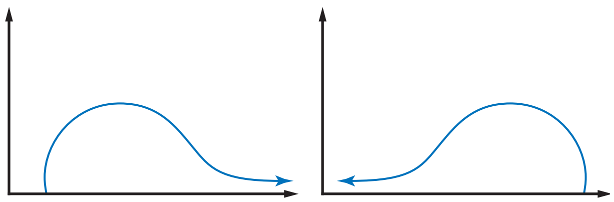
**simplify** (p. 312) To rewrite an expression without parentheses or negative exponents.

**simulation** (p. 734) The use of a probability experiment to mimic a real-life situation.

**sine** (p. 759) For any angle, with measure $\alpha$, a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, $\sin \alpha = \frac{y}{r}$.

**skewed distribution** (p. 724) A curve or histogram that is not symmetric.

Positively Skewed    Negatively Skewed



**slope** (p. 71) The ratio of the change in $y$-coordinates to the change in $x$-coordinates.

**slope-intercept form** (p. 79) The equation of a line in the form $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept.

**solution** (p. 19) A replacement for the variable in an open sentence that results in a true sentence.

**solving a right triangle** (p. 762) The process of finding the measures of all of the sides and angles of a right triangle.

**square matrix** (p. 163) A matrix with the same number of rows and columns.

**square root** (p. 259) For any real numbers $a$ and $b$, if $a^2 = b$, then a is a square root of $b$.

**square root function** (p. 397) A function that contains a square root of a variable.

**square root inequality** (p. 399) An inequality involving square roots.

**Square Root Property** (p. 260) For any real number $n$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

**standard deviation** (p. 718) The square root of the variance, represented by $a$.

**evento simple** Un solo evento.

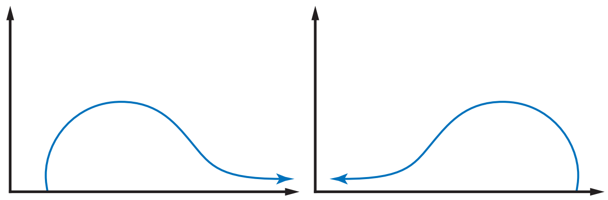
**reducir** Escribir una expresión sin paréntesis o exponentes negativos.

**simulación** Uso de un experimento probabilístico para imitar una situación de la vida real.

**seno** Para cualquier ángulo de medida $\alpha$, un punto $P(x, y)$ en su lado terminal, $r = \sqrt{x^2 + y^2}$, $\sin \alpha = \frac{y}{r}$.

**distribución asimétrica** Curva o histograma que no es simétrico.

Positivamente Alabeada    Negativamente Alabeada



**pendiente** La razón del cambio en coordenadas $y$ al cambio en coordenadas $x$.

**forma pendiente-intersección** Ecuación de una recta de la forma $y = mx + b$, donde $m$ es la pendiente $y$ b la intersección.

**solución** Sustitución de la variable de un enunciado abierto que resulta en un enunciado verdadero.

**resolver un triángulo rectángulo** Proceso de hallar las medidas de todos los lados y ángulos de un triángulo rectángulo.

**matriz cuadrada** Matriz con el mismo número de filas y columnas.

**raíz cuadrada** Para cualquier número real $a$ y $b$, si $a^2 = b$, entonces a es una raíz cuadrada de $b$.

**función radical** Función que contiene la raíz cuadrada de una variable.

**desigualdad radical** Desigualdad que presenta raíces cuadradas.

**Propiedad de la raíz cuadrada** Para cualquier número real $n$, si $x^2 = n$, entonces $x = \pm\sqrt{n}$.

**desviación estándar** La raíz cuadrada de la varianza, la que se escribe a.

**standard form** (p. 67, 246) **1.** A linear equation written in the form $Ax + By = C$, where $A$, $B$, and $C$ are integers whose greatest common factor is 1, $A \geq 0$, and $A$ and $B$ are not both zero. **2.** A quadratic equation written in the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are integers, and $a \neq 0$.

**standard notation** (p. 315) Typical form for written numbers.

**standard position** (p. 767) An angle positioned so that its vertex is at the origin and its initial side is along the positive $x$-axis.

**step function** (p. 95) A function whose graph is a series of line segments.

**substitution method** (p. 123) A method of solving a system of equations in which one equation is solved for one variable in terms of the other.

**success** (p. 697) The desired outcome of an event.

**synthetic division** (p. 327) A method used to divide a polynomial by a binomial.

**synthetic substitution** (p. 356) The use of synthetic division to evaluate a function.

**system of equations** (p. 116) A set of equations with the same variables.

**system of inequalities** (p. 130) A set of inequalities with the same variables.

**forma estándar** **1.** Ecuación lineal escrita de la forma $Ax + By = C$, donde $A$, $B$, y $C$ son enteros cuyo máximo común divisores 1, $A \geq 0$, y $A$ y $B$ no son cero simultáneamente. **2.** Una ecuación cuadrática escrita en la forma $ax^2 + bx + c = 0$, donde $a$, $b$, and $c$ are integers, and $a \neq 0$.

**notación estándar** Forma típica de escribir números.

**posición estándar** Ángulo en posición tal que su vértice está en el origen y su lado inicial está a lo largo del eje $x$ positivo.

**fución etapa** Función cuya gráfica es una serie de segmentos de recta.

**método de sustitución** Método para resolver un sistema de ecuaciones en que una de las ecuaciones se resuelve en una de las variables en términos de la otra.

**éxito** El resultado deseado de un evento.

**división sintética** Método que se usa para dividir un polinomio entre un binomio.

**sustitución sintética** Uso de la división sintética para evaluar una función polinomial.

**sistema de ecuaciones** Conjunto de ecuaciones con las mismas variables.

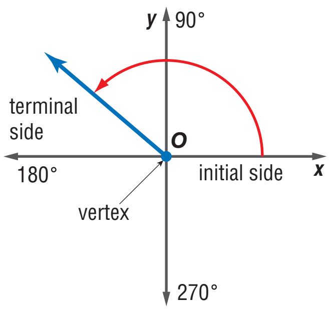
**sistema de desigualdades** Conjunto de desigualdades con las mismas variables.

## T

**tangent** (pp. 427, 759) **1.** A line that intersects a circle at exactly one point. **2.** For any angle, with measure $\alpha$, a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, $\tan \alpha = \frac{y}{x}$.

**term** (p. 7, 622) **1.** The monomials that make up a polynomial. **2.** Each number in a sequence or series.

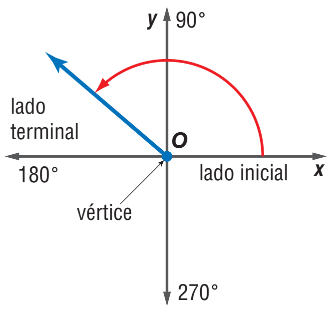
**terminal side of an angle** (p. 767) A ray of an angle that rotates about the center.




**tangente** **1.** Recta que interseca un círculo en un solo punto. **2.** Para cualquier ángulo, de medida $\alpha$, un punto $P(x, y)$ en su lado terminal, $r = \sqrt{x^2 + y^2}$, $\tan \alpha = \frac{y}{x}$.

**término** **1.** Los monomios que constituyen un polinomio. **2.** Cada número de una sucesión o serie.

**lado terminal de un ángulo** Rayo de un ángulo que gira alrededor de un centro.




**determinante de tercer orden** Determinante de

**third-order determinant** (p. 195) Determinant of a $3 \times 3$ matrix.

**transformation** (p. 185) Functions that map points of a pre-image onto its image.

**translation** (p. 185) A figure is moved from one location to another on the coordinate plane without changing its size, shape, or orientation.

**translation matrix** (p. 185) A matrix that represents a translated figure.

**transverse axis** (p. 591) The segment of length $2a$ whose endpoints are the vertices of a hyperbola.

**trigonometric equation** (p. 861) An equation containing at least one trigonometric function that is true for some but not all values of the variable.

**trigonometric functions** (pp. 759, 775) For any angle, with measure $\alpha$, a point $P(x, y)$ on its terminal side, $r = \sqrt{x^2 + y^2}$, the trigonometric functions of a are as follows.

$\sin \alpha = \frac{y}{r}$ $\quad \cos \alpha = \frac{x}{r}$ $\quad \tan \alpha = \frac{y}{x}$

$\csc \alpha = \frac{r}{y}$ $\quad \sec \alpha = \frac{r}{x}$ $\quad \cot \alpha = \frac{x}{y}$

**trigonometric identity** (p. 837) An equation involving a trigonometric function that is true for all values of the variable.

**trigonometry** (p. 759) The study of the relationships between the angles and sides of a right triangle.

**trinomial** (p. 7) A polynomial with three unlike terms.

**third-order determinant** → **determinante de tercer orden** una matriz de $3 \times 3$.

**transformación** Funciones que aplican puntos de una preimagen en su imagen.

**traslación** Se mueve una figura de un lugar a otro en un plano de coordenadas sin cambiar su tamaño, forma u orientación.

**matriz de traslación** Matriz que representa una figura trasladada.

**eje transversal** El segmento de longitud $2a$ cuyos extremos son los vértices de una hipérbola.

**ecuación trigonométrica** Ecuación que contiene por lo menos una función trigonométrica y que sólo se cumple para algunos valores de la variable.

**funciones trigonométricas** Para cualquier ángulo, de medida $\alpha$, un punto $P(x, y)$ en su lado terminal, $r = \sqrt{x^2 + y^2}$, las funciones trigonométricas de a son las siguientes.

$\operatorname{sen} \alpha = \frac{y}{r}$ $\quad \cos \alpha = \frac{x}{r}$ $\quad \tan \alpha = \frac{y}{x}$

$\csc \alpha = \frac{r}{y}$ $\quad \sec \alpha = \frac{r}{x}$ $\quad \cot \alpha = \frac{x}{y}$

**identidad trigonométrica** Ecuación que involucra una o más funciones trigonométricas y que se cumple para todos los valores de la variable.

**trigonometría** Estudio de las relaciones entre los lados y ángulos de un triángulo rectángulo.

**trinomio** Polinomio con tres términos diferentes.

## U

**unbiased sample** (p. 735) A sample in which every possible sample has an equal chance of being selected.

**unbounded** (p. 139) A system of inequalities that forms a region that is open.

**uniform distribution** (p. 699) A distribution where all of the probabilities are the same.

**union** (p. 42) The graph of a compound inequality containing *or*.

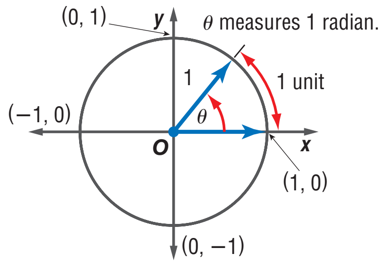
**muestra no sesgada** Muestra en que cualquier muestra posible tiene la misma posibilidad de seleccionarse.

**no acotado** Sistema de desigualdades que forma una región abierta.

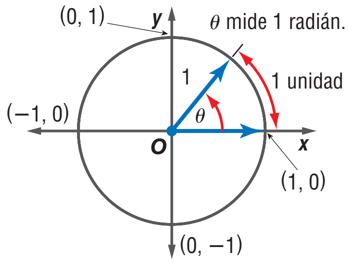
**distribución uniforme** Distribución donde todas las probabilidades son equiprobables.

**unión** Gráfica de una desigualdad compuesta que contiene la palabra o.

**unit circle** (p. 768) A circle of radius 1 unit whose center is at the origin of a coordinate system.




**círculo unitario** Círculo de radio 1 cuyo centro es el origen de un sistema de coordenadas.




**univariate date** (p. 717) Data with one variable.

**datos univariados** Datos con una variable.

## V

**variable** (p. 6) Symbols, usually letters, used to represent unknown quantities.

**variables** Símbolos, por lo general letras, que se usan para representar cantidades desconocidas.

**variance** (p. 718) The mean of the squares of the deviations from the arithmetic mean.

**varianza** Media de los cuadrados de las desviaciones de la media aritmética.

**vertex** (p. 138, 237, 591) **1.** Any of the points of intersection of the graphs of the constraints that determine a feasible region. **2.** The point at which the axis of symmetry intersects a parabola. **3.** The point on each branch nearest the center of a hyperbola.

**vértice** **1.** Cualqeiera de los puntos de intersección de las gráficas que los contienen y que determinan una región viable. **2.** Punto en el que el eje de simetría interseca una parábola. **3.** El punto en cada rama más cercano al centro de una hipérbola.

**vertex form** (p. 286) A quadratic function in the form $y = a(x - h)^2 + k$, where $(h, k)$ is the vertex of the parabola and $x = h$ is its axis of symmetry.

**forma de vértice** Función cuadrática de la forma $y = a(x - h)^2 + k$, donde $(h, k)$ es el vértice de la parábola y $x = h$ es su eje de simetría.

**vertex matrix** (p. 185) A matrix used to represent the coordinates of the vertices of a polygon.

**matriz de vértice** Matriz que se usa para escribir las coordenadas de los vértices de un polígono.

**vertical asymptote** (p. 457) If the related rational expression of a function is written in simplest form and is undefined for $x = a$, then $x = a$ is a vertical asymptote.

**asíntota vertical** Si la expresión racional que corresponde a una función racional se reduce y está no definida en $x = a$, entonces $x = a$ es una asíntota vertical.

**vertical line test** (p. 59) If no vertical line intersects a graph in more than one point, then the graph represents a function.

**prudba de la recta vertical** Si ninguna recta vertical interseca una gráfica en más de un punto, entonces la gráfica representa una función.

## X

***x*-intercept** (p. 68) The $x$-coordinate of the point at which a graph crosses the $x$-axis.

**intersección $x$** La coordenada $x$ del punto o puntos en que una gráfica interseca o cruza el eje $x$.

## Y

**$y$-intercept** (p. 68) The $y$-coordinate of the point at which a graph crosses the $y$-axis.

**intersección $y$** La coordenada $y$ del punto o puntos en que una gráfica interseca o cruza el eje $y$.

## Z

**zeros** (p. 246) The $x$-intercepts of the graph of a quadratic equation; the points for which $f(x) = 0$.

**ceros** Las intersecciones $x$ de la gráfica de una ecuación cuadrática; los puntos $x$ para los que $f(x) = 0$.

**zero matrix** (p. 163) A matrix in which every element is zero.

**matriz nula** matriz cuyos elementos son todos igual a cero.

# Selected Answers

## Chapter 1 Equations and Inequalities

***Page 5*** ***Chapter 1*** ***Get Ready***

**1.** 19.84 **3.** $-\frac{5}{12}$ **5.** $-2\frac{1}{6}$ **7.** 0.48 **9.** $-2\frac{2}{3}$ **11.** $8\frac{4}{5}$
**13.** \$7.31 **15.** 125 **17.** $-1$ **19.** $-1.44$ **21.** $\frac{25}{81}$
**23.** $2^5$ or 32 **25.** true **27.** true **29.** false **31.** false

***Pages 8–10*** ***Lesson 1-1***

**1.** $-2.5$ **3.** 10.5 **5.** 24 **7.** \$432 **9.** 3.4 **11.** 45
**13.** 5.3 **15.** 40 **17.** $-1$ **19.** $\frac{1}{4}$ **21.** 31.25 drops per min **23.** $\pi\left(\frac{y+5}{2}\right)^2$ **25.** 75 **27.** $-4$ **29.** 36.01
**31.** $-16$ **33.** \$15,954.39 **35.** 98.6 **37.** Sample answer: $4 - 4 + 4 \div 4 = 1$; $4 \div 4 + 4 \div 4 = 2$; $(4 + 4 + 4) \div 4 = 3$; $4 \times (4 - 4) + 4 = 4$; $(4 \times 4 + 4) \div 4 = 5$; $(4 + 4) \div 4 + 4 = 6$; $44 \div 4 - 4 = 7$; $(4 + 4) \times (4 \div 4) = 8$; $4 + 4 + 4 \div 4 = 9$; $(44 - 4) \div 4 = 10$ **39.** A table of IV flow rates is limited to those situations listed, while a formula can be used to find any IV flow rate. If a formula used in a nursing setting is applied incorrectly, a patient could die. **41.** H **43.** 4 **45.** 13
**47.** $-5$ **49.** $\frac{6}{7}$

***Pages 15–17*** ***Lesson 1-2***

**1.** Z, Q, R **3.** Q, R **5.** Assoc. (+) **7.** 8, $-\frac{1}{8}$ **9.** $-1.5$, $\frac{2}{3}$
**11.** \$175.50 **13.** $-17a - 1$ **15.** Q, R **17.** I, R
**19.** N, W, Z, Q, R **21.** Z, Q, R **23.** Add. Iden.
**25.** Comm. (+) **27.** Distributive **29.** $-2.5$; 0.4
**31.** $\frac{5}{8}$; $-\frac{8}{5}$ **33.** $4\frac{3}{5}$; $-\frac{5}{23}$
**35.** $3\left(2\frac{1}{4}\right) + 2\left(1\frac{1}{8}\right)$

| | |
|---|---|
| $= 3\left(2 + \frac{1}{4}\right) + 2\left(1 + \frac{1}{8}\right)$ | Def. of a mixed number |
| $= 3(2) + 3\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{1}{8}\right)$ | Distributive Prop. |
| $= 6 + \frac{3}{4} + 2 + \frac{1}{4}$ | Multiply. |
| $= 6 + 2 + \frac{3}{4} + \frac{1}{4}$ | Commutative Property |
| $= 8 + \frac{3}{4} + \frac{1}{4}$ | Add. |
| $= 8 + \left(\frac{3}{4} + \frac{1}{4}\right)$ | Associative Property |
| $= 8 + 1$ or 9 | Add. |

**37.** $10x + 2y$ **39.** $11m + 10a$ **41.** $32c - 46d$
**43.** $4.4p - 2.9q$ **45.** 3.6; \$327.60 **47.** $-m$; Add. Inv.
**49.** 1 **51.** $\sqrt{2}$ units **53.** W, Z, Q, R **55.** I, R
**57.** Sample answer: $-2$ **59.** true **61.** false; 6
**63.** Yes; $\frac{6+8}{2} = \frac{6}{2} + \frac{8}{2} = 7$; dividing by a number is the same as multiplying by its reciprocal. **65.** B
**67.** 9 **69.** $-5$ **71.** 358 in$^2$ **73.** $\frac{7}{10}$ **75.** 36

***Pages 22–26*** ***Lesson 1-3***

**1.** $5 + 4n$ **3.** 9 times a number decreased by 3 is 6.
**5.** Reflexive (=) **7.** $-21$ **9.** $-4$ **11.** 1.5
**13.** $y = \frac{9 + 2n}{4}$ **15.** D **17.** $5 + 3n$ **19.** $n^2 - 4$
**21.** $5(9 + n)$ **23.** Sample answer: 5 less than a number is 12. **25.** Sample answer: A number squared is equal to 4 times the number. **27.** Substitution (=) **29.** Trans. (=)
**31.** 7 **33.** 3.2 **35.** $-8$ **37.** $\frac{d}{t} = r$ **39.** $\frac{3V}{\pi r^2} = h$
**41.** $\frac{1}{3}$ **43.** $s$ = length of a side; $8s = 124$; 15.5 in.
**45.** $(n - 7)^3$ **47.** $2\pi r(h + r)$ **49.** Sample answer: 7 minus half a number is equal to 3 divided by the square of the number. **51.** $\frac{4x}{1 - x} = y$ **53.** $-7$ **55.** 1
**57.** $\frac{10}{17}$ **59.** $n$ = number of students that can attend each meeting; $2n + 3 = 83$; 40 students **61.** $c$ = cost per student; $50(30 - c) + \frac{50}{5}(45) = 1800$; \$3 **63.** $h$ = height of can A; $\pi(1.2^2)h = \pi(2^2)3$; $8\frac{1}{3}$ units
**65.** Central: 690 mi; Union: 1085 mi **67.** \$295
**69.** Sample answer: $2x - 5 = -19$ **71.** The Symmetric Property of Equality allows the two sides of an equation to be switched; the order is changed. The Commutative Property of Addition allows the order of terms in an expression on one side of an equation to be changed; the order of terms is changed, but not necessarily on both sides of an equation. **73.** D
**75.** $-6x + 8y + 4z$ **77.** 6.6 **79.** 105 cm$^2$ **81.** $-\frac{1}{4}$
**83.** $-5 + 6y$

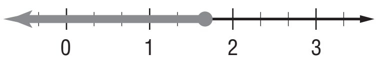
***Pages 29–31*** ***Lesson 1-4***

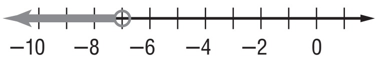
**1.** 8 **3.** $-10.8$ **5.** least: 158°F; greatest: 162°F
**7.** $\{-21, 13\}$ **9.** $\{-11, 29\}$ **11.** $\varnothing$ **13.** $\{8\}$ **15.** 15
**17.** 0 **19.** 3 **21.** $-4$ **23.** $\{8, 42\}$ **25.** $\{-45, 21\}$
**27.** $\{-2, 16\}$ **29.** $\left\{\frac{3}{2}\right\}$ **31.** $\varnothing$ **33.** $|x - 200| = 5$; maximum: 205°F; minimum: 195°F **35.** $\left\{2, \frac{9}{2}\right\}$
**37.** $\{-5, 11\}$ **39.** $\left\{-\frac{11}{3}, -3\right\}$ **41.** $\{8\}$ **43.** 5 **45.** $-22$
**47.** $|x - 13| = 5$; maximum: 18 km, minimum: 8 km
**49.** Sometimes; it is true only if $a \geq 0$ and $b \geq 0$ or if $a \leq 0$ and $b \leq 0$. **51.** Always; since the opposite of 0 is still 0, this equation has only one case, $ax + b = 0$. The solution is $-\frac{b}{a}$. **53.** B **55.** $\frac{16}{3}$ **57.** 14
**59.** Distributive **61.** 364 ft$^2$ **63.** 8 **65.** $\frac{2}{3}$

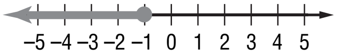
***Pages 37–39*** ***Lesson 1-5***

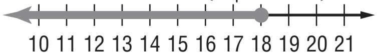
**1.** $\{a \mid a < 1.5\}$

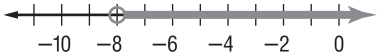


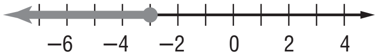
**3.** $\left\{x \,\middle|\, x \leq \frac{5}{3}\right\}$



**5.** $\{w \mid w < -7\}$


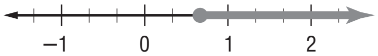
**7.** $\{n \mid n \le -1\}$ 

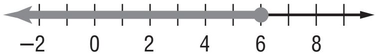

**9.** at least 92 **11.** $\{b \mid b \le 18\}$


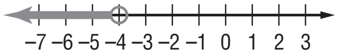

**13.** $\{d \mid d > -8\}$ 


**15.** $\{p \mid p \le -3\}$ 

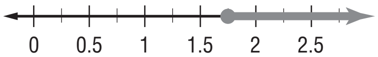

**17.** $\{y \mid y < 5\}$ 

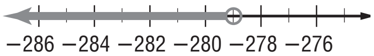

**19.** $\left\{b \,\middle|\, b \ge \frac{2}{3}\right\}$ 

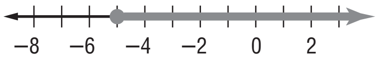

**21.** $\{r \mid r \le 6\}$ 

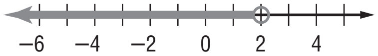

**23.** $\{k \mid k < -4\}$ 

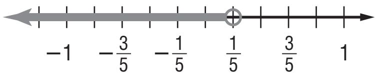

**25.** at least 25 h **27.** $12n > 36$; $n > 3$ **29.** $n + 8 > 2$; $n > -6$ **31.** $\frac{1}{2}n - 7 \ge 5$; $n \ge 24$

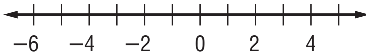
**33.** $\{n \mid n \ge 1.75\}$ 


**35.** $\{x \mid x < -279\}$ 


**37.** $\{d \mid d \ge -5\}$ 


**39.** $\{g \mid g < 2\}$ 

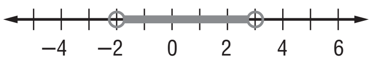

**41.** $\left\{y \,\middle|\, y < \frac{1}{5}\right\}$ 

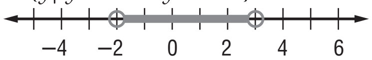

**43.** $\varnothing$ 

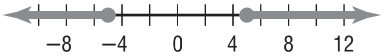

**45.** $34{,}000 + 0.015(30{,}500n) \ge 50{,}000$ **47.** $2(n + 5) \le 3n + 11$; $n \ge -1$ **49.** $2(7m) \ge 17$; $m \ge \frac{17}{14}$; at least 2 child-care staff members **51.** $s \ge 91$; Flavio must score at least 91 on his next test to have an A test average. **53.** $x \ge -2$ **55.** Sample answer: $x + 2 < x + 1$ **57a.** It holds only for $\le$ or $\ge$; $2 < 2$. **57b.** $1 < 2$ but $2 \not< 1$ **57c.** For all real numbers $a$, $b$, and $c$, if $a < b$ and $b < c$ then $a < c$. **59.** Let $n$ equal the number of minutes used. Write an expression representing the cost of Plan 1 and for Plan 2 for $n$ minutes. The cost for Plan 1 would include a monthly access fee of $35 plus 40¢ for each minute over 400 minutes or $35 + 0.4(n - 400)$. The cost for Plan 2 for 650 minutes or less would be $55. To find where Plan 2 would cost less than Plan 1, solve $55 < 35 + 0.4(n - 400)$ for $n$. The solution set is $\{n \mid n > 450\}$, which means that for more than 450 minutes of calls, Plan 2 is cheaper. **61.** J

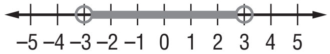
**63.** $\left\{-\frac{5}{4}, \frac{11}{4}\right\}$ **65.** $b$ = billions of dollars spent online each year; $4b + 28.3 = 69.2$; about $10.2 billion each year **67.** Q, R **69.** $4.25(5.5 + 8)$; $4.25(5.5) + 4.25(8)$ **71.** $\{13, -23\}$ **73.** $\{11, 25\}$ **75.** $\{-18, 10\}$

***Pages 45–48*** ***Lesson 1-6***

**1.** $\{d \mid -2 < d < 3\}$ 


**3.** $\{y \mid y > 4 \text{ or } y < -1\}$


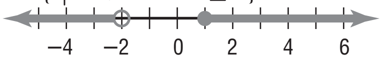

**5.** $\{a \mid a \ge 5 \text{ or } a \le -5\}$


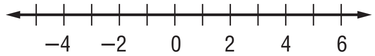

**7.** $\{h \mid -3 < h < 3\}$ 

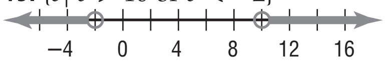

**9.** $\{k \mid -3 < k < 7\}$ 


**11.** $54.45 \le c \le 358.8$ between $54.45 and $358.80

**13.** $\{x \mid -2 < x < 4\}$ 

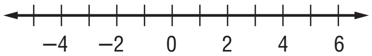

**15.** $\{c \mid c < -2 \text{ or } c \ge 1\}$


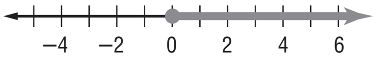

**17.** $\varnothing$ 


**19.** $\{b \mid b > 10 \text{ or } b < -2\}$



**21.** $\{r \mid -3 < r < 4\}$ 


**23.** $45 \le s \le 55$
**25.** all real numbers 


**27.** $\varnothing$ 


**29.** $\{n \mid -9\ n- \ge 0\}$ 

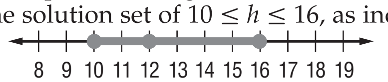

**31.** $\{n \mid -3 < n < 1\}$ 


**33.** $|n| > 1$ **35.** $|n| \ge 1.5$ **37.** $|n + 1| > 1$

**39.** $|b - 98.6| \ge 8$; $\{b \mid b > 106.6 \text{ or } b < 90.6\}$

**41.** 84 in. $< L \le$ 106 in. **43.** red: $24.35 \le x \le 24.49$; blue: $24.17 \le x \le 24.67$; green: $23.92 \le x \le 24.92$

**45.** $a - b < c < a + b$ **47.** $2 < x < 3$ **49.** abs$(2x - 6) > 10$; $\{x \mid x < -2 \text{ or } x > 8\}$ **51.** Sabrina; an absolute value inequality of the form $|a| > b$ should be rewritten as an *or* compound inequality, $a > b$ or $a < -b$.

**53.** Compound inequalities can be used to describe the acceptable time frame for the fasting state before a glucose tolerance test is administered to a patient suspected of having diabetes. $10 \le h \le 16$; 12 hours would be an acceptable fasting time for this test since it is part of the solution set of $10 \le h \le 16$, as indicated on the graph. 

**55.** G

**57.** $\{x \mid x < 4\}$ 


**59.** $|x - 587| = 5$; highest: 592 keys, lowest: 582 keys **61.** $\{-11, 4\}$ **63.** Addition (=) **65.** Transitive (=) **67.** 616.69 **69.** $-2m - 7n - 18$ **71.** 92

*Pages 49–52* *Chapter 1* *Study Guide and Review*

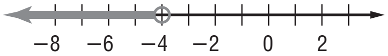
**1.** empty set **3.** rational numbers **5.** absolute value **7.** coefficient **9.** equation **11.** 22 **13.** 14 **15.** 18 **17.** 7 **19.** 260 mi **21.** Q, R **23.** $-4m + 2n$ **25.** $7x - 16y$ **27.** \$75 **29** $-21$ **31.** 3 **33.** $-4$

**35.** $x = \frac{C - By}{A}$ **37.** $p = \frac{A}{1 + rt}$ **39.** about 1.5 in.

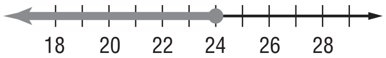
**41.** $\{6, -18\}$

**43.** $\{6\}$ **45.** $\left\{-\frac{3}{2}, -1\right\}$

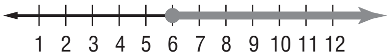
**47.** $\{w \mid w < -4\}$




**49.** $\{n \mid n \leq 24\}$

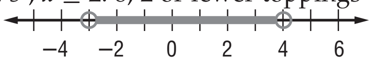



**51.** $\{z \mid z \geq 6\}$

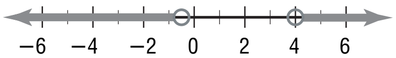



**53.** $6(9 + 1.25x) \leq 75$ , $x \leq 2.8$; 2 or fewer toppings

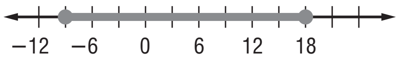
**55.** $\{a \mid -1 < a < 4\}$



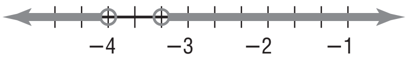

**57.** $\left\{y \,\middle|\, y > 4 \text{ or } y < -\frac{1}{3}\right\}$




**59.** $\{y \mid -9 \leq y \leq 18\}$




**61.** $\left\{b \,\middle|\, b > \frac{-10}{3} \text{ or } b < -4\right\}$




# Chapter 2 Linear Relations and Functions

*Page 57* *Chapter 2* *Get Ready*

**1.** $(-3, 3)$ **3.** $(-3, -1)$ **5.** $(0, -4)$

**7.**

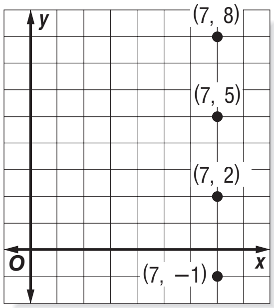
| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| 1 | 9 | (1, 9) |
| 2 | 18 | (2, 18) |
| 3 | 27 | (3, 27) |
| 4 | 36 | (4, 36) |

**9.** $-2$ **11.** 9 **13.** 2 **15.** $x + 1$ **17.** $2x + 6$
**19.** $(120x + 165)$ mi

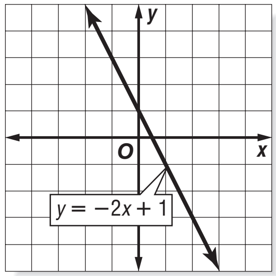
*Pages 62–64* *Lesson 2-1*

**1.** D = {−6, 2, 3}, R = {1, 5}; yes **3.** D = {−1, 2, 3}, R = {1, 2, 3, 4}; no **5.** {(97, 134), (78, 117), (86, 109), (98, 119)}

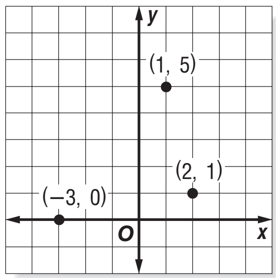
**7.** D = {7}, R = {−1, 2, 5, 8}; no; discrete



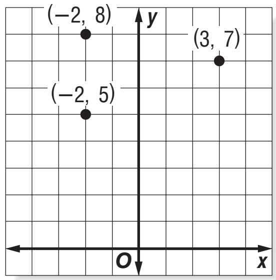

**9.** D = all reals, R = all reals; yes; continuous



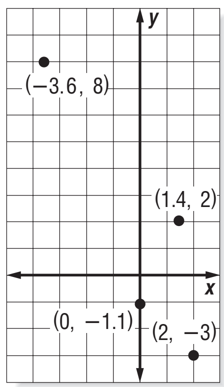

**11.** 10 **13.** D = {10, 20, 30}, R = {1, 2, 3}; yes
**15.** D = {0.5, 2}, R = {−3, 0.8, 8}; no
**17.** D = all reals, R = all reals; no **19.** discrete
**21.** discrete **23.** D = {−3, 1, 2}, R = {0, 1, 5}; yes; discrete



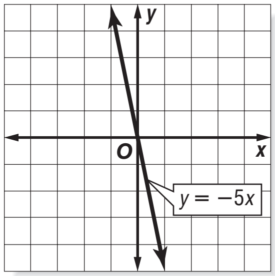

**25.** D = {−2, 3}, R = {5, 7, 8}; no; discrete



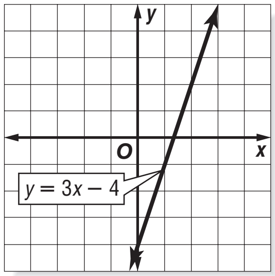

**27.** D = {−3.6, 0, 1.4, 2}, R = {−3, −1.1, 2, 8}; yes; discrete



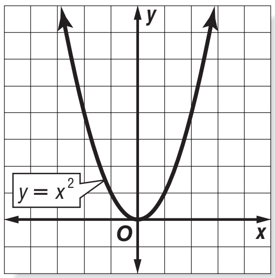
**29.** D = all reals, R = all reals; yes; continuous




**31.** D = all reals, R = all reals; yes; continuous

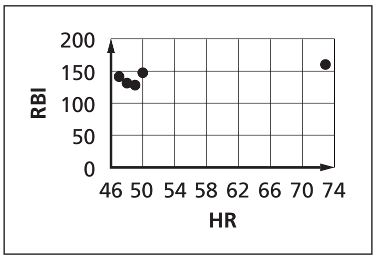



**33.** D = all reals, R = $\{y|\ y \geq 0\}$; yes; continuous



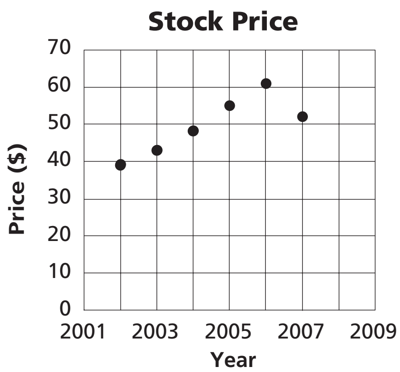

**35.** −14 **37.** $-\frac{2}{9}$ **39.** $3a - 5$ **41.** −4

**43.**



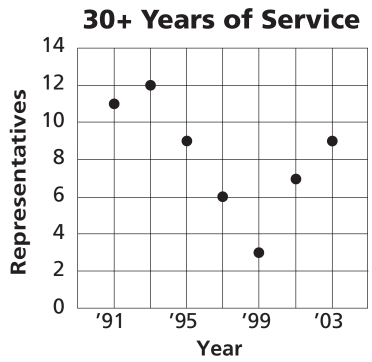

**45.** Yes; each domain value is paired with only one range value so the relation is a function.

**47.**




**49.** Yes; each domain value is paired with only one range value.

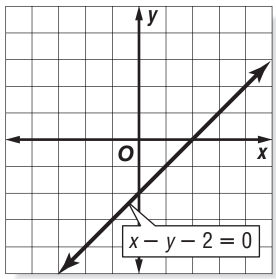
**51.**




**53.** Yes; each domain value is paired with only one range value so the relation is a function. **55.** Sample answer: {(−4, 3), (−2, 3), (1, 5), (−2, 1)}. For $x = -2$, there are two different $y$-values. **57.** Sample answer: $f(x) = 4x - 3$ **59.** C **61.** $\{y|\ -8 < y < 6\}$ **63.** $\{x|\ x < 5.1\}$ **65.** 362 **67.** −1 **69.** 6

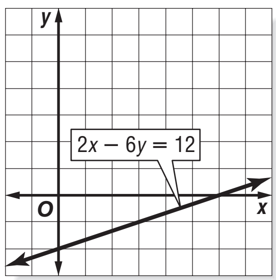
***Pages 68–70*** ***Lesson 2-2***

**1.** No; the variables have an exponent other than 1. **3.** $177.62 **5.** $3x - y = 5$; 3, −1, 5 **7.** $2x - 3y = -3$; 2, −3, −3 **9.** 2, −2

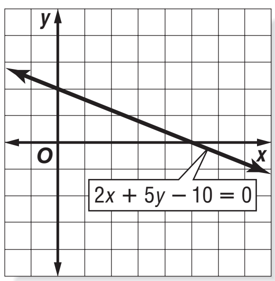



**11.** yes **13.** No; $x$ has exponents other than 1. **15.** No; $x$ appears in a denominator. **17.** No; $x$ is inside a square root. **19.** Sound travels only 1715 m in 5 seconds in air, so it travels faster in water. **21.** 35,000 ft **23.** $12x - y = 0$; 12, −1, 0 **25.** $x - 7y = 2$; 1, −7, 2 **27.** $x - 2y = -3$; 1, −2, −3

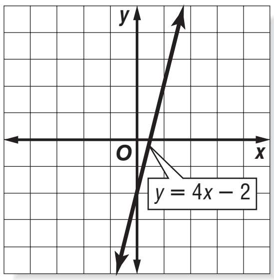
**29.** 6, −2



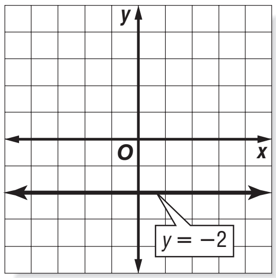

**31.** 5, 2



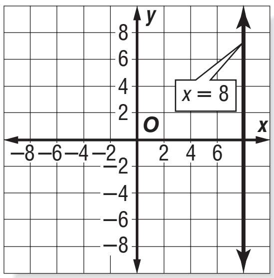
**33.** $\frac{1}{2}$, −2



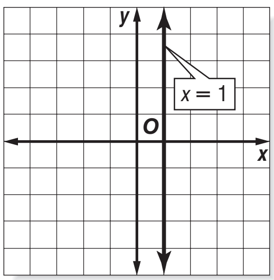

**35.** $x + y = 12$; 1, 1, 12 **37.** $x = 6$; 1, 0, 6
**39.** $25x + 2y = 9$; 25, 2, 9
**41.** none, −2



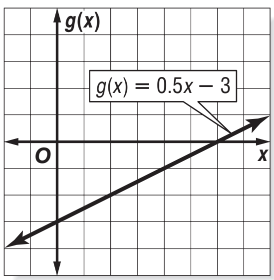

**43.** 8, none



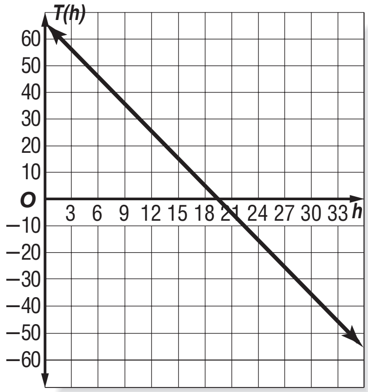

**45.** 1, none



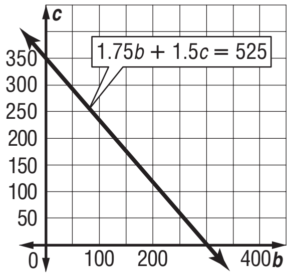

**47.** 6, –3




**49.**



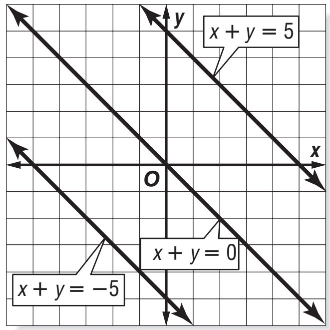

**51.**




Yes; the graph passes the vertical line test.

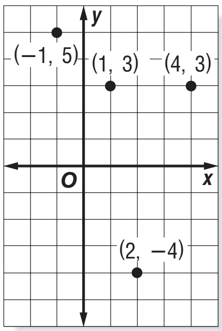
**53.** Sample answer: $x + y = 2$

**55.**




The lines are parallel but have different $y$-intercepts.

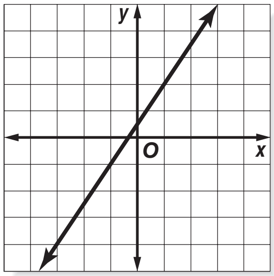
**57.** Regardless of whether 0 is substituted in for $x$ or $y$, the value of the other variable is also 0. So the only intercept is (0, 0). **59.** B **61.** D = {−1, 1, 2, 4}, R = {−4, 3, 5}; yes



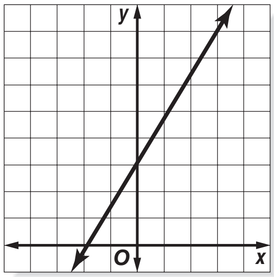

**63.** $\{x \mid -1 < x < 2\}$ **65.** \$7.95 **67.** 2 **69.** −0.8

***Pages 74–77*** ***Lesson 2-3***

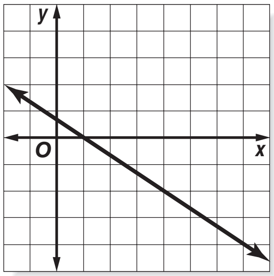
**1.** $-\frac{1}{2}$ **3.** 1 **5.**



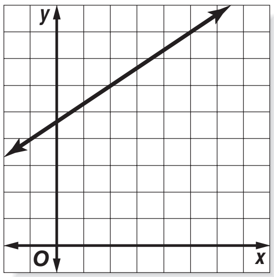

**7.** 1.25°/h **9.**



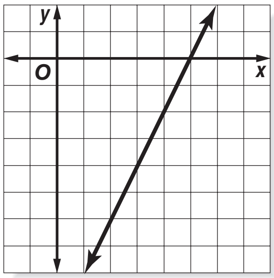

**11.**



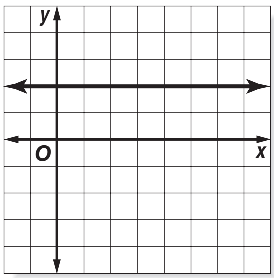

**13.** $-\frac{5}{2}$ **15.** 13 **17.** 0 **19.**




**21.**

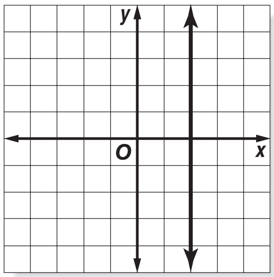



**23.**

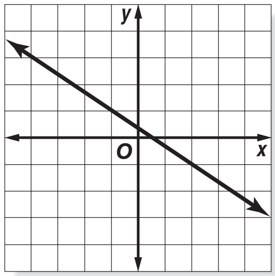



**25.** about 11 million per year **27.** 55 mph **29.** speed or velocity

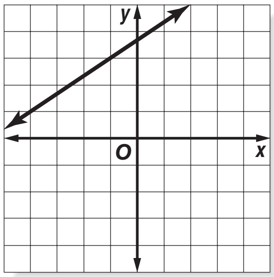
**31.**



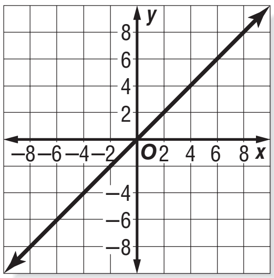

**33.**




**35.**

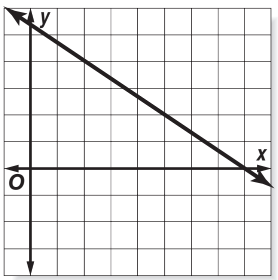



**37.**



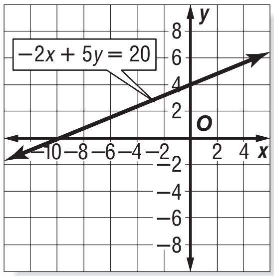

**39.** $-\frac{5}{4}$ **41.** 0 **43.** 9

**45.**

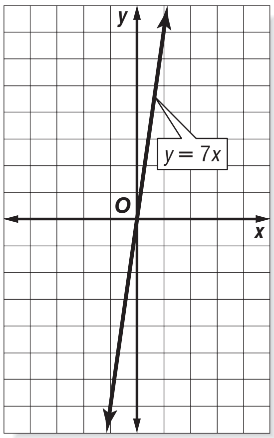



**47.** Yes; slopes show that adjacent sides are perpendicular. **49.** The graphs have the same *y*-intercept. As the slopes become more negative, the lines get steeper. **51.** −1 **53.** Sometimes; the slope of a vertical line is undefined. **55.** D

**57.** $-10, 4$




**59.** $0, 0$



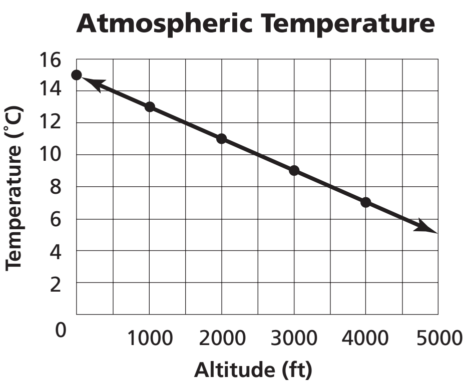

**61.** 5 **63.** $3a - 4$ **65.** $\{z \mid z \geq 735\}$ **67.** $17a - b$
**69.** $y = 9 - x$ **71.** $y = -3x + 7$ **73.** $y = \frac{3}{5}x + \frac{4}{5}$

***Pages 82–84*** ***Lesson 2-4***

**1.** $y = 0.5x + 1$ **3.** $y = 3x - 6$ **5.** $y = -\frac{5}{2}x + 16$
**7.** $y = 0.8x$ **9.** B **11.** $y = -\frac{4}{3}x + \frac{8}{3}$ **13.** $y = 3x - 6$
**15.** $y = -\frac{1}{2}x + \frac{7}{2}$ **17.** $y = -\frac{4}{5}x + \frac{17}{5}$ **19.** $y = \frac{2}{3}x + \frac{10}{3}$
**21.** $y = -4$ **23.** $y = 75x + 6000$ **25.** $d = 180c - 360$
**27.** 540° **29.** 68°F **31.** $y = -0.5x - 2$ **33.** $y = x + 4$
**35.** $y = -\frac{1}{15}x - \frac{23}{5}$ **37.** Sample answer: $y = 3x + 2$
**39.** $y = 2x + 4$ **41.** A **43.** $-2$ **45.** 0 **47.** $\varnothing$
**49.** $\{r \mid r \geq 6\}$ **51.** 6.5 **53.** 5.85

***Pages 88–91*** ***Lesson 2-5***

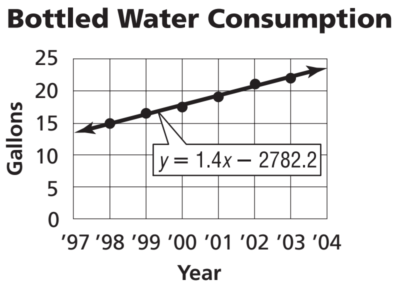
**1a.**




strong negative correlation

**1b.** Sample answer using (2000, 11.0) and (3000, 9.1): $y = -0.0019x + 14.8$ **1c.** Sample answer: 5.3°C
**3a.** strong positive correlation

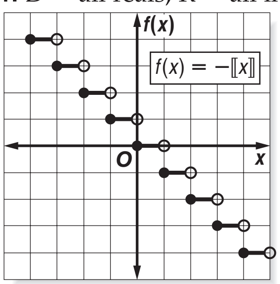

**3b.** Sample answer using (1999, 19.1) and (2003, 22.8): $y = 0.93x - 1830$ **3c.** Sample answer: 33,900
**5a.** strong positive correlation



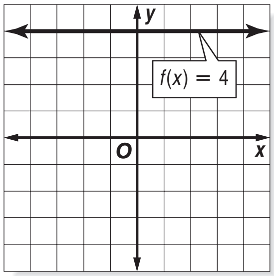

**5b.** Sample answer using (1998, 15) and (2003, 22): $y = 1.4x - 2782.2$ **5c.** Sample answer: 38.8 gal
**7.** Sample answer using (2000, 1309.9) and (2003, 1678.9): $y = 123x - 244{,}690.1$ **9.** The value predicted by the equation is significantly lower than the one given in the graph. **11.** No. Past performance is no guarantee of the future performance of a stock. Other factors that should be considered include the companies' earnings data and how much debt they have. **13.** Sample answer using (483.8, $-166$) and (3647.2, $-375$): $y = -0.07x - 134.04$ **15.** Sample answer: The predicted value differs from the actual value by only 2°F, less than 1%. **17.** Sample answer using (1980, 66.5) and (1995, 81.7): 102% **19.** Sample answer using (4, 152.5) and (8, 187.6): $y = 8.78x + 117.4$ **21.** D **23.** $y = 4x + 6$ **25.** $y = 0.35x + 1.25$
**27.** $-\frac{1}{2}$ **29.** undefined **31.** 3 **33.** 0 **35.** 1.5

***Pages 99–101*** ***Lesson 2-6***

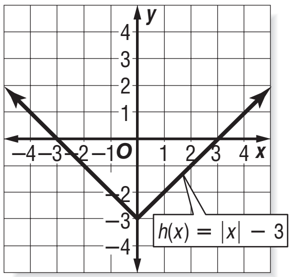
**1.** D = all reals, R = all integers



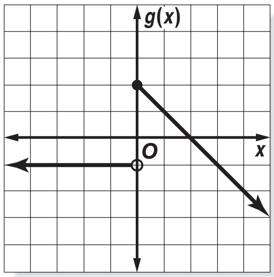
**3.** D = all reals, R = {4}



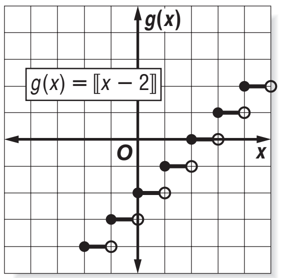

**5.** D = all reals, R = $\{y \mid y \geq -3\}$



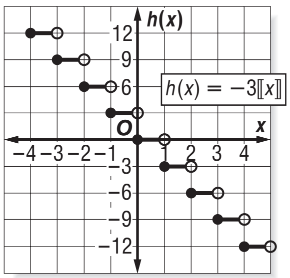

**7.** D = all reals, R = $\{y \mid y \leq 2\}$



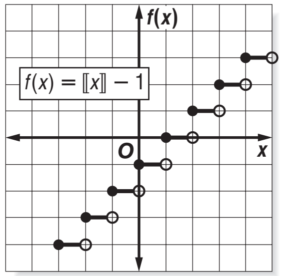

**9.** A **11.** step function **13.** $6 **15.** D = all reals, R = all integers



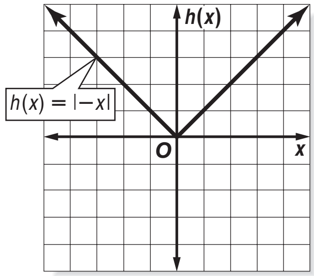

**17.** D = all reals, R = $\{3a \mid a$ is an integer$\}$



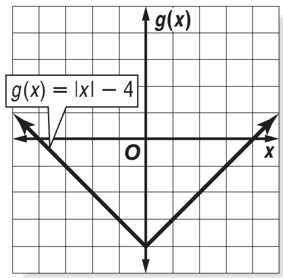

**19.** D = all reals, R = all integers



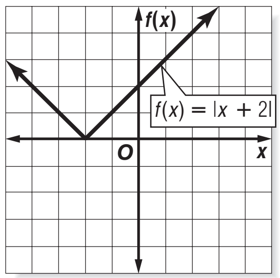

**21.** D = all reals, R = all nonnegative reals



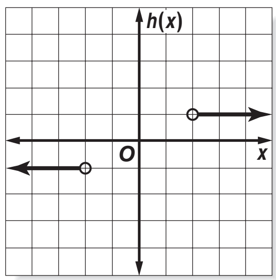

**23.** D = all reals, R = $\{y \mid y \geq -4\}$



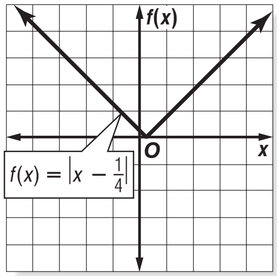

**25.** D = all reals, R = all nonnegative reals



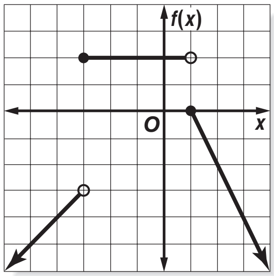

**27.** D = $\{x \mid x < -2$ or $x > 2\}$, R = $\{-1, 1\}$



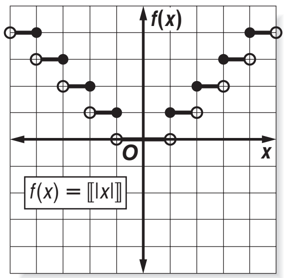
**29.** A **31.** S **33.** P **35.** D = all reals, R = all nonnegative reals



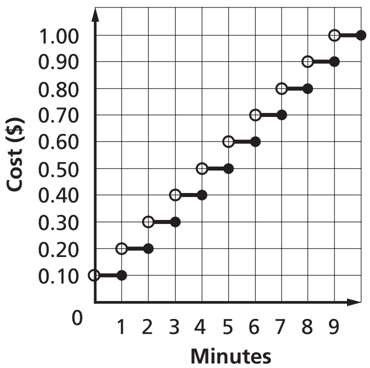

**37.** D = all reals, R = $\{y \mid y \leq 0 \text{ or } y = 2\}$




**39.** D = all reals, R = all whole numbers

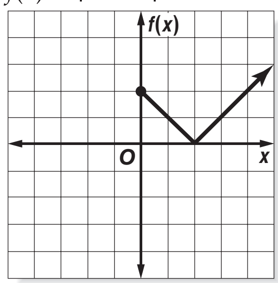



**41.**



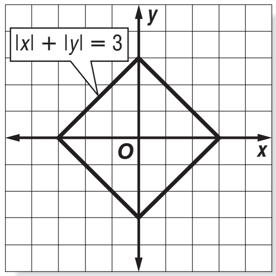

**43.** $f(x) = |x - 2|$

**45.**



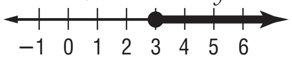

**47.** Sample answer: $f(x) = |x - 1|$

**49.**




**51.** B **53.** B **55.** Sample answer using (10, 69.7) and (47, 76.5): $y = 0.18x + 66.1$ **57.** $y = 3x + 10$

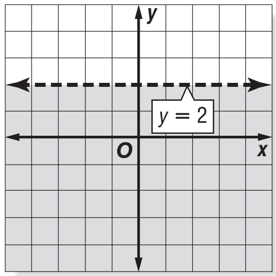
**59.** $\{x \mid x \geq 3\}$



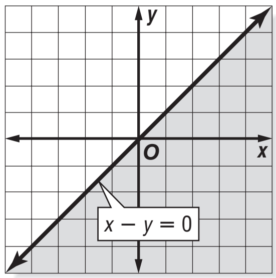

**61.** yes **63.** no **65.** no

***Pages 104–105*** ***Lesson 2-7***

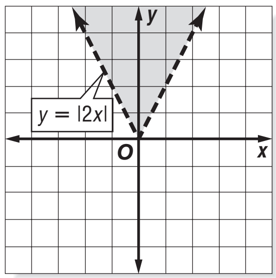
**1.**




**3.**

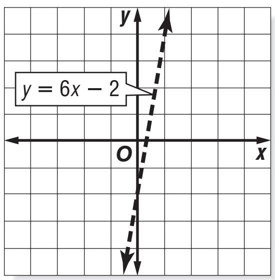



**5.**

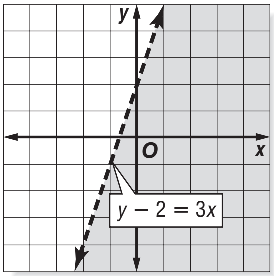



**7.** $10c + 13d \leq 40$ **9.** No; (2, 3) is not in the shaded region.

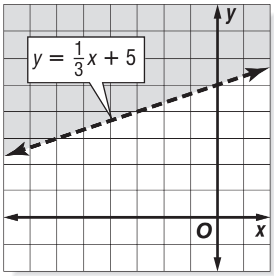
**11.**



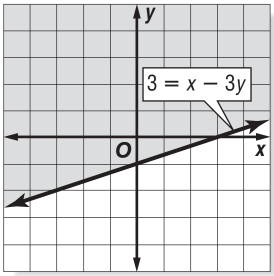
**13.**



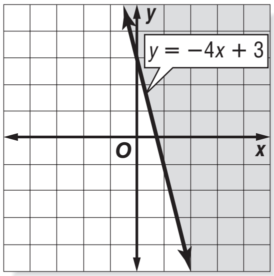

**15.**



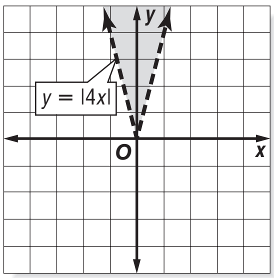

**17.**




**19.**

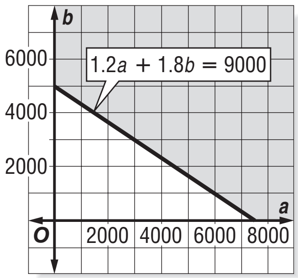



**21.**

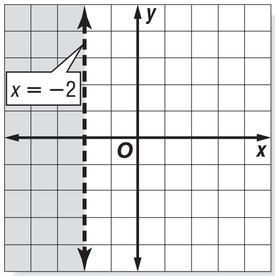



**23.** yes

**25.**

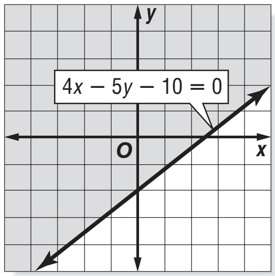



**27.**

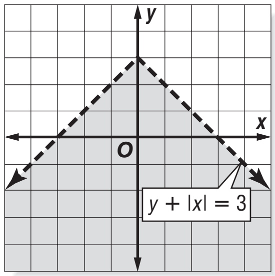



$x < -2$

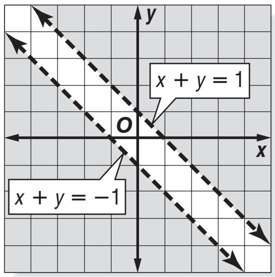
**29.**



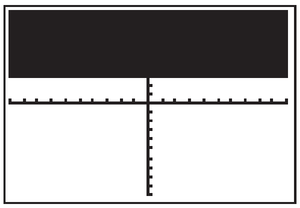

**31.**




**33.**

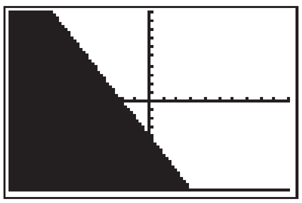



**35.**



[−10, 10] scl: 1 by [−10, 10] scl: 1

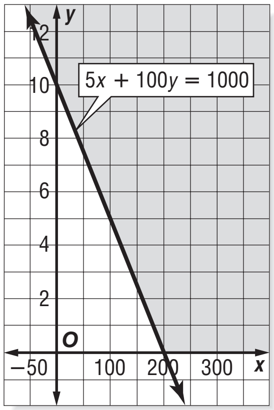
**37.**



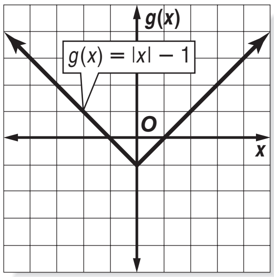
[−10, 10] scl: 1 by [−10, 10] scl: 1

**39.** Substitute the coordinates of a point not on the boundary into the inequality. If the inequality is satisfied, shade the region containing the point. If the inequality is not satisfied, shade the region that does not contain the point.

**41.** Linear inequalities can be used to track the performance of players in fantasy football leagues. Let $x$ be the number of passing yards and let $y$ be the number of touchdowns. The number of points Dana gets from passing yards is $5x$ and the number of points he gets from touchdowns is $100y$. His total number of points is $5x + 100y$. He wants at least 1000 points, so the inequality $5x + 100y \geq 1000$ represents the situation.




**43.** J **45.**




D = all reals, R = $\{y \mid y \geq -1\}$

**47.**

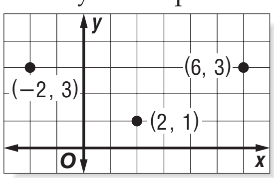

There is a strong positive correlation between salary and experience.

**49.** Sample answer: \$64,000 **51.** 3

***Pages 106–110*** ***Chapter 2*** ***Study Guide and Review***

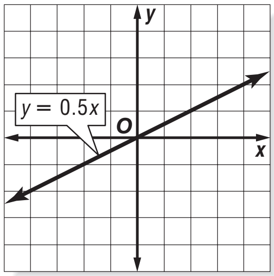
**1.** identity **3.** slope-intercept **5.** vertical line test

**7.**




D = {−2, 2, 6}, R = {1, 3}; function; discrete

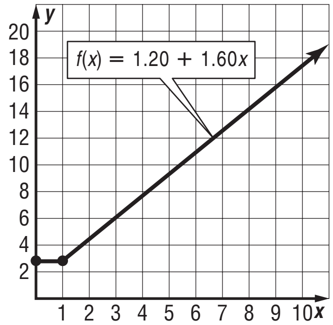
**9.**




D = all reals, R = all reals; function; continuous

**11.** 21 **13.** $5y - 9$

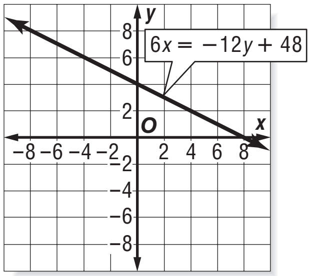
**15.**




D = all reals, $\{x \mid x \geq 0\}$
R = $\{y \mid y \geq 2.80\}$
function; continuous

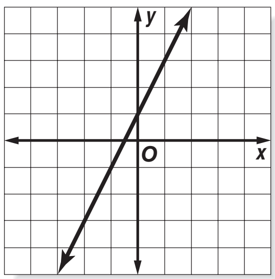
**17.** No; this function is not linear because the $x$ is under a square root. **19.** $5x + 2y = -4$; 5, 2, −4

**21.** 8, 4

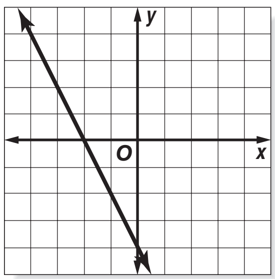



**23.** $\frac{5}{6}$

**25.**

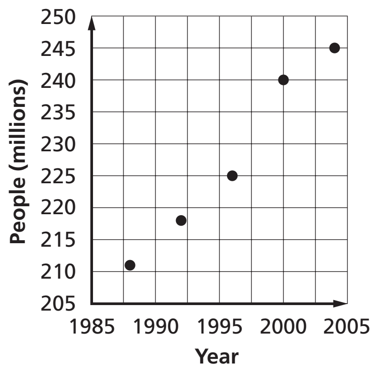



**27.**



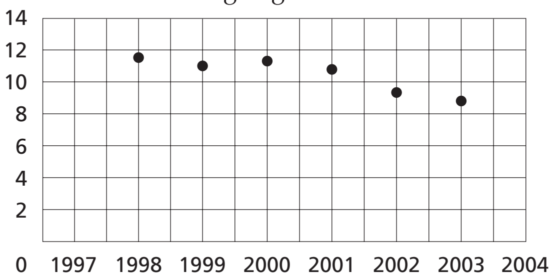
**29.** $\frac{3}{5}$ **31.** $y = \frac{1}{3}x + \frac{7}{3}$ **33.** $y = -\frac{3}{4}x + \frac{17}{4}$

**35.**

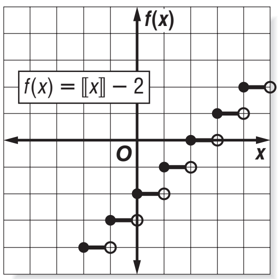



There is a strong positive correlation.

**37.** There is a strong negative correlation.

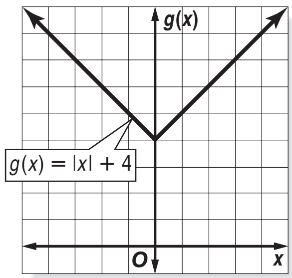



**39.**



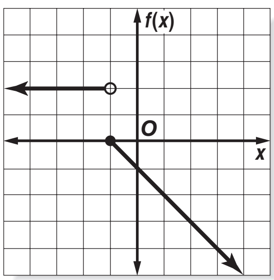

D = all reals,
R = all integers

**41.**



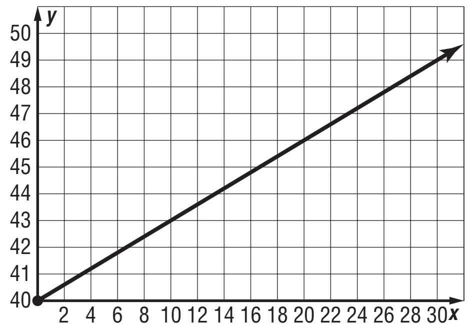

D = all reals,
R = $\{y|\ y \geq 4\}$

**43.**

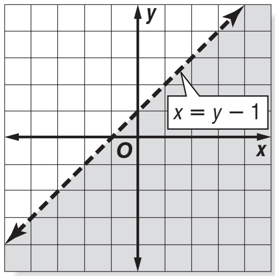



D = all reals,
R = $\{y|\ y \leq 0 \text{ or } y = 2\}$

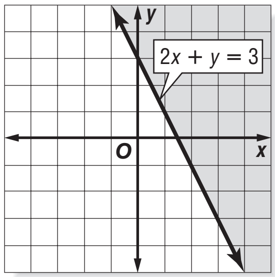
**45.**



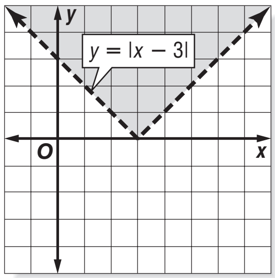

**47.**




**49.**



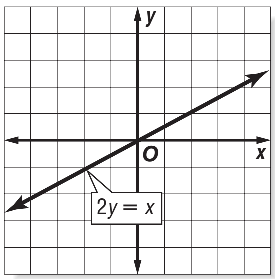

**51.**



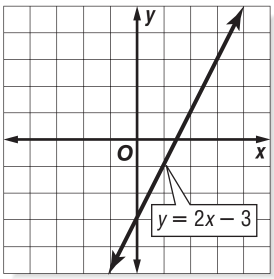

## Chapter 3 Systems of Equations and Inequalities

***Page 115*** ***Chapter 3*** ***Get Ready***

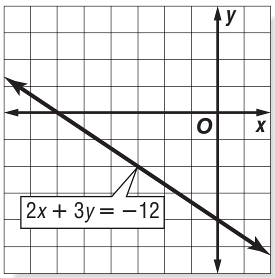
**1.**



**3.**

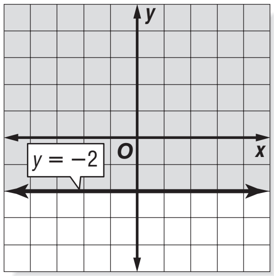



**5.**

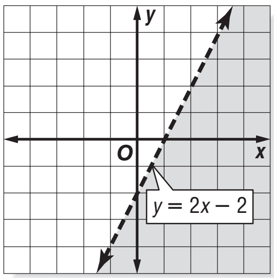



**7.** $1.75b + 1.5c = 525$ **9.** Yes; the graph passes the vertical line test.

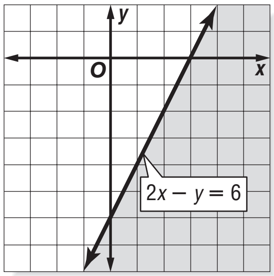
**11.**



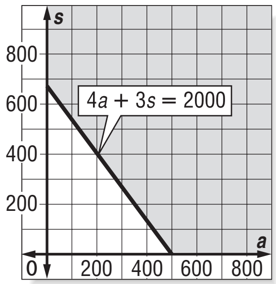

**13.**




**15.**




**17.**

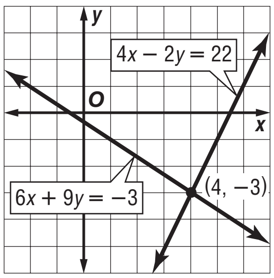



***Pages 120–122*** ***Lesson 3-1***

**1.** $(-2, 5)$

| $x$ | $y_1$ | $y_2$ |
|---|---|---|
| 0 | 9 | 3 |
| −1 | 7 | 4 |
| −2 | 5 | 5 |
| −3 | 3 | 6 |

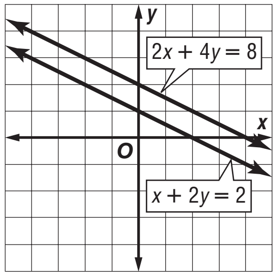
**3.**




$(4, -3)$

**5.** $y = 0.15x + 2.70$, $y = 0.25x$ **7.** You should use Ez Online photos if you are printing more than 27 digital photos and the local pharmacy if you are printing fewer than 27 digital photos.

**9.**

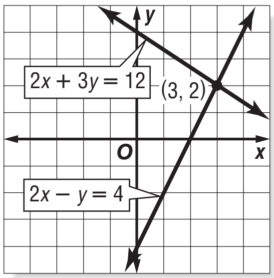



inconsistent

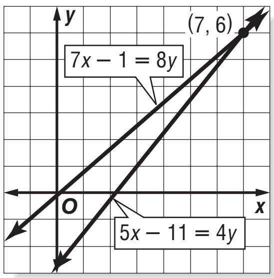
**11.** $(0, -8)$

| $x$ | $y_1$ | $y_2$ |
|---|---|---|
| −1 | −11 | −9 |
| 0 | −8 | −8 |
| 1 | −5 | −7 |
| 2 | −2 | −6 |

**13.**

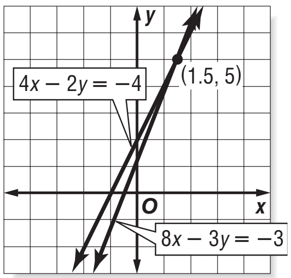



$(3, 2)$



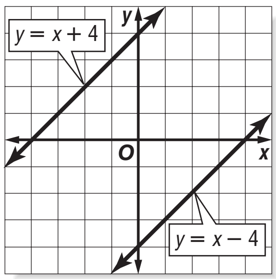

$(7, 6)$

**17.**

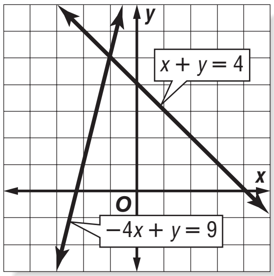



$(1.5, 5)$

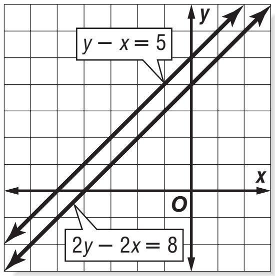
**19.**




inconsistent

**21.**


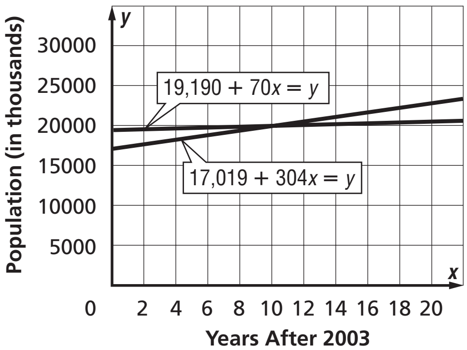

consistent and independent

**23.**


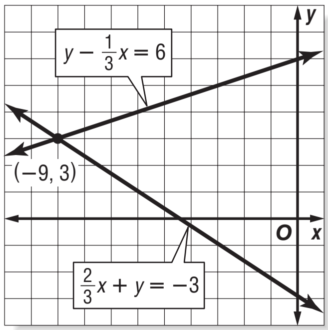

inconsistent

**25.** $(-3, 1)$

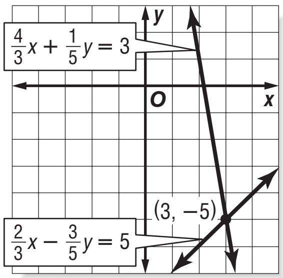
**27.** Supply, 200,000; demand, 300,000; prices will tend to rise. **29.** 250,000; \$10

**31.**


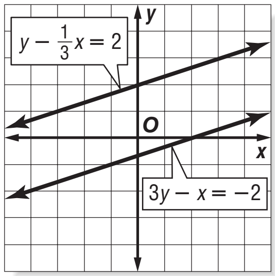

2012

**33.**



$(-9, 3)$

**35.**



$(3, -5)$

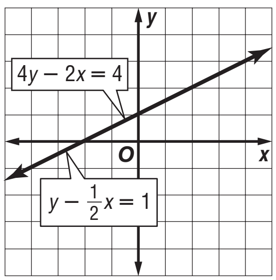
**37.**



inconsistent

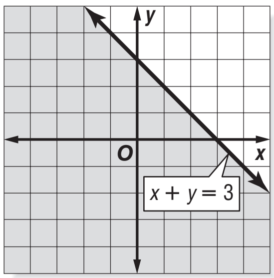
**39.** $(3.40, -2.58)$ **41.** $(4, 3.42)$ **43.** Two lines cannot intersect in exactly two points. **45.** You can use a system of equations to track sales and make predictions about future growth based on past performance and trends in the graphs. The coordinates (6, 54.2) represent that 6 years after 1999, both the in-store sales and online sales will be \$54,200. It would not be very reasonable. The unpredictability of the market, companies, and consumers makes models such as this one accurate for only a short period of time. **47.** H **49.** A **51.** P **53.** $9y + 1$
**55.** $12x + 18y - 6$ **57.** $x + 4y$

***Pages 127–129*** ***Lesson 3-2***

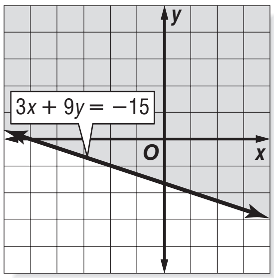
**1.** $(4, 8)$ **3.** $(9, 7)$ **5.** C **7.** $(6, -20)$ **9.** $(3, 2)$
**11.** infinitely many solutions **13.** $(2, 7)$ **15.** $(-6, 8)$
**17.** $(1, 1)$ **19.** $(2, -7)$ **21.** $(3, -1)$ **23.** no solution
**25.** 18 members rented skis and 10 members rented snowboards. **27.** 18 printers, 12 monitors **29.** $(6, 5)$
**31.** $(7, -1)$ **33.** $(-5, 8)$ **35.** $\left(\frac{1}{3}, 2\right)$ **37.** $(2, 4)$
**39.** $(12, -3)$ **41.** 10 true/false, 20 multiple-choice
**43.** $a + s = 40$, $11a + 4s = 335$ **45.** one equation should have a variable with a coefficient of 1.
**47.** Jamal; Juanita subtracted the two equations incorrectly; $-y - y = -2y$, not 0.
**49.** You can use a system of equations to find the monthly fee and rate per minute charged during the months of January and February. The coordinates of the point of intersection are (0.08, 3.5). Currently, Yolanda is paying a monthly fee of \$3.50 and an additional 8¢ per minute. **51.** J
**53.** consistent and dependent



55.



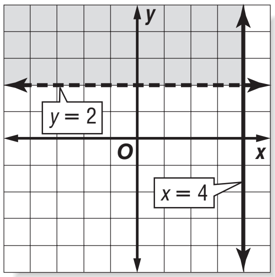

57.



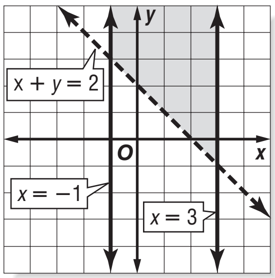

**59.** $x - y = 0$; 1, −1, 0 **61.** $2x - y = -3$; 2,−1,−3
**63.** $3x + 2y = 21$; 3, 2, 21 **65.** yes **67.** no

***Pages 132–135*** ***Lesson 3-3***

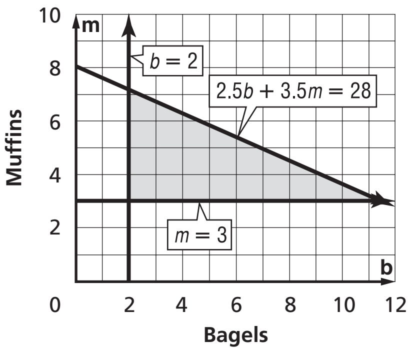
**1.**




**3.**

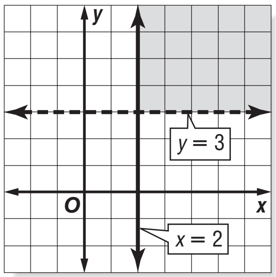



**5.**

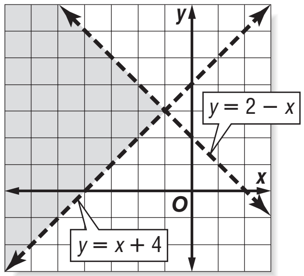



**7.** (−3, −3), (2, 2), (5, −3)

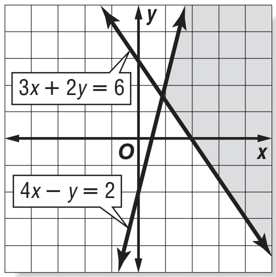
**9.**



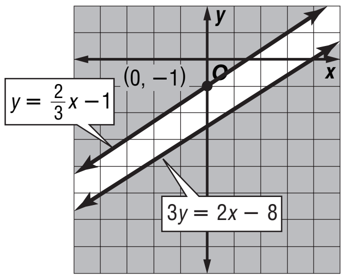

**11.**




**13.**

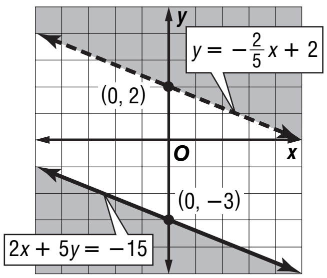



**15.**



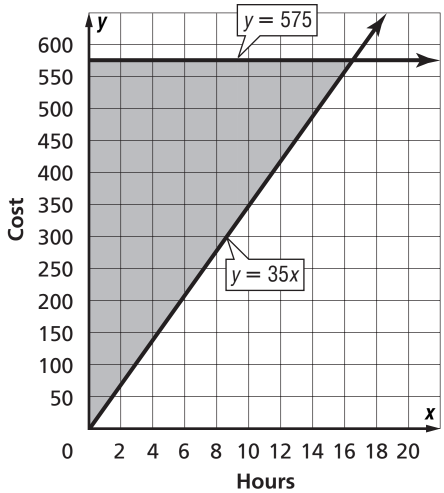

no solution

**17.**



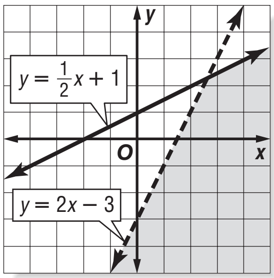

no solution

**19.**

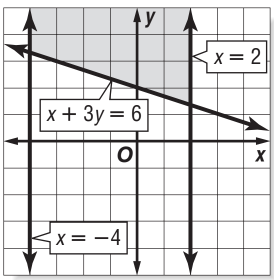



**21.** (−3, −4), (5, −4), (1, 4) **23.** (−6, −9), (2, 7), (10, −1) **25.** 64 units$^2$ **27.** category 4; 13–18 ft
**29.** Sample answer: 2 pumpkin, 8 soda; 4 pumpkin, 6 soda; 8 pumpkin, 4 soda

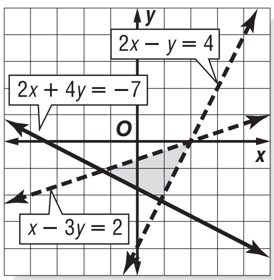
**31.**




**33.**

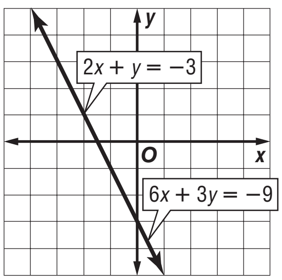



**35.**




**37.** Sample answer: $y > x + 3$, $y < x - 2$ **39.** 42 units$^2$
**41.** B **43.** (−3, 8) **45.** (8, −5)

**47.**

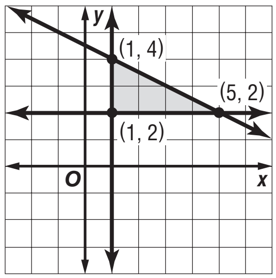



infinitely many

**49.** $75x + 200$ **51.** −12 **53.** −8.25

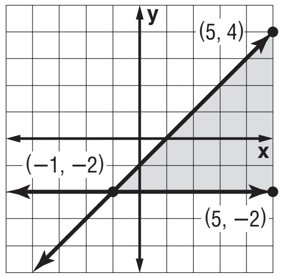
**Pages 141–144** **Lesson 3-4**

**1.**



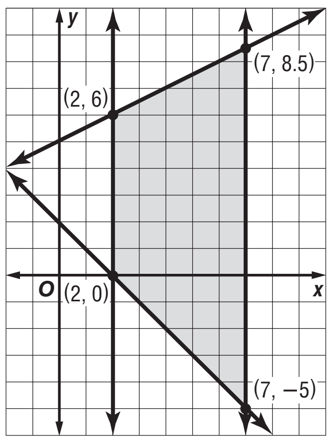

vertices: (1, 2), (1, 4), (5, 2); max: $f(5, 2) = 4$; min: $f(1, 4) = -10$

**3.**



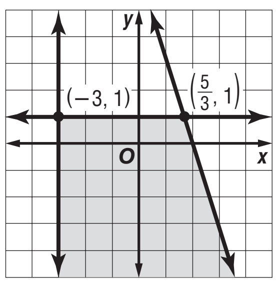

vertices: (−1, −2), (5, −2), (5, 4); max: $f(5, -2) = 9$; min: $f(5, 4) = -3$

**5.**




vertices: (2, 0), (2, 6), (7, 8.5), (7, −5); max: $f(7, 8.5) = 81.5$; min: $f(2, 0) = 16$

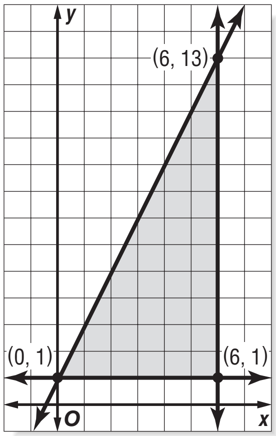
**7.**




vertices: (−3, 1), $\left(\frac{5}{3}, 1\right)$; no maximum; min: $f(-3, 1) = -17$

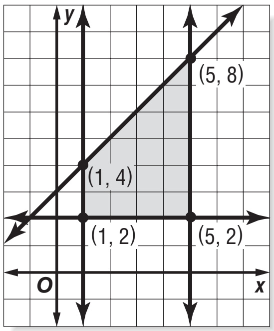
**9.** $c \geq 0$, $\ell \geq 0$, $c + 3\ell \leq 56$, $4c + 2\ell \leq 104$ **11.** (0, 0), (26, 0), (20, 12), $\left(0, 18\frac{2}{3}\right)$ **13.** Make 20 canvas tote bags and 12 leather tote bags.

**15.**



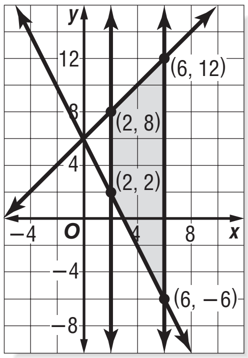

vertices: (0, 1), (6, 1), (6, 13); max $f(6, 13) = 19$; min; $f(0, 1) = 1$

**17.**



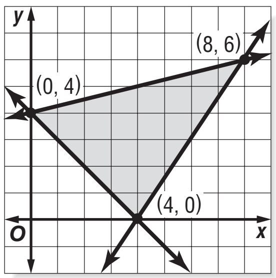

vertices: (1, 4), (5, 8), (5, 2), (1, 2); max $f(5, 2) = 11$; min; $f(1, 4) = -5$

**19.**



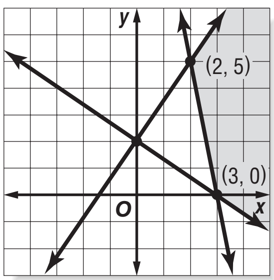

vertices: (2, 2), (2, 8), (6, 12), (6, −6); max $f(6, 12) = 30$; min; $f(6, -6) = -24$

**21.**



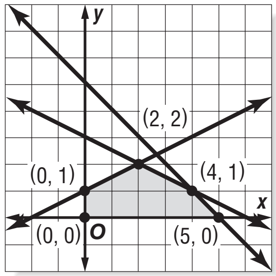

vertices: (0, 4), (4, 0), (8, 6); max $f(4, 0) = 4$; min; $f(0, 4) = -8$

**23.**



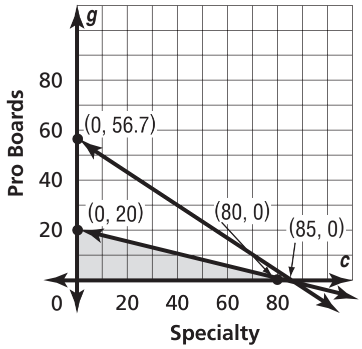

vertices: (2, 5), (3, 0); no maximum; no minimum

**25.**



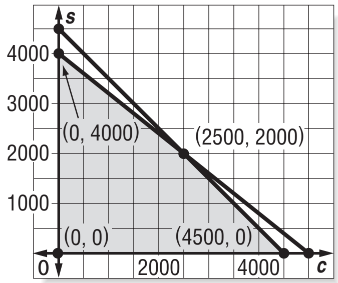

vertices: (0, 0), (0, 1), (2, 2), (4, 1), (5, 0); max $f(5, 0) = 19$; min; $f(0, 1) = -5$

**29.**




**31.** $f(c, g) = 65c + 50g$ **33.** \$5200

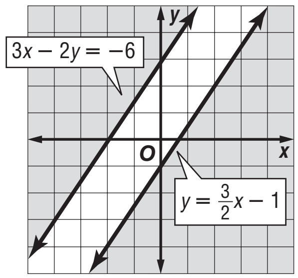
**35.**




(0, 0), (0, 4000), (2500, 2000), (4500, 0)

**37.** 4500 acres corn, 0 acres soybeans; \$130,500
**39.** Sample answer: $y \geq -x$, $y \geq x - 5$, $y \leq 0$
**41.** (−2,6); the other coordinates are solutions of the system of inequalities. **43.** There are many variables in scheduling tasks. Linear programming can help make sure that all the requirements are met. Let $x =$ the number of buoy replacements and let $y =$ the number of buoy repairs. Then, $x \geq 0$, $y \geq 0$, $x \leq 8$ and $2.5x + y \leq 24$. The captain would want to maximize the number of buoys that a crew could repair and replace so $f(x, y) = x + y$. **45.** J

**47.**




no solution

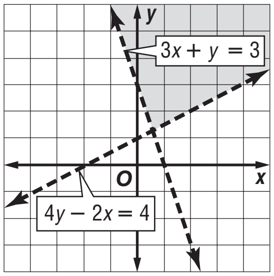
**49.** (2, 3) **51.** $y = 2x + 50$ **53.** 5 **55.** −3 **57.** −4

***Pages 149–152*** ***Lesson 3-5***

**1.** (6, 3, −4) **3.** infinitely many **5.** no solution
**7.** $6c + 3s + r = 42$, $c + s + r = 13\frac{1}{2}$, $r = 2s$

**9.** (3, 4, 7) **11.** (2, −3, 6) **13.** no solution
**15.** (1, 2, −1) **17.** infinitely many **19.** 8, 1, 3
**21.** 52 3-point goals, 168 2-point goals, 142 1-point free throws **23.** $7.80 **25.** $\left(\frac{1}{3}, -\frac{1}{2}, \frac{1}{4}\right)$ **27.** (−5, 9, 4)
**29.** You can use elimination or substitution to eliminate one of the variables. Then you can solve two equations in two variables. **31.** $a = \frac{4}{3}$, $b = \frac{1}{3}$, $c = 3$; $y = \frac{4}{3}x^2 + \frac{1}{3}x + 3$ **33.** A **35.** 179 gallons of skim and 21 of whole milk

**37.**

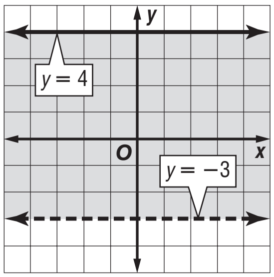



**39.** Sample answer using (0, 8) and (35, 39): $y = 0.89x + 8$
**41.** 3830 feet; the $y$-intercept
**43.** 5 **45.** 30

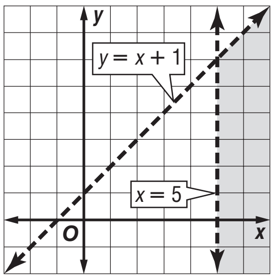
***Pages 153–156*** ***Chapter 3*** ***Study Guide and Review***

**1.** constraints **3.** feasible region **5.** consistent system
**7.** elimination method **9.** system of equations
**11.** (4, 0) **13.** (−8, −8) **15.** 1 hr **17.** (−3, −5)
**19.** (6.25, −2.25) **21.** (−1, 2)

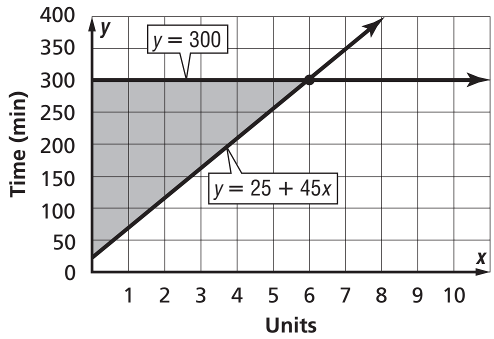
**23.**




**25.**




**27.**




**29.** (1, 2, 3) **31.** (3, −1, 5)

## Chapter 4 Matrices

***Page 161*** ***Chapter 4*** ***Get Ready***

**1.** $-3; \frac{1}{3}$ **3.** $-8; \frac{1}{8}$ **5.** −1.25; 0.8 **7.** $\frac{8}{3}; -\frac{3}{8}$ **9.** 4
**11.** (3, 4) **13.** (9, 2)

***Pages 165–167*** ***Lesson 4-1***

**1.**
$$\begin{array}{l} \\ \text{High} \\ \text{Low} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Fri} & \text{Sat} & \text{Sun} & \text{Mon} & \text{Tue} \end{array} \\ \begin{bmatrix} 88 & 88 & 90 & 86 & 85 \\ 54 & 54 & 56 & 53 & 52 \end{bmatrix} \end{array}$$

**3.** 1 × 5 **5.** (5, 6)

**7.** $\begin{bmatrix} 60{,}060{,}700 & 13{,}215 \\ 29{,}637{,}900 & 12{,}880 \\ 26{,}469{,}500 & 13{,}002 \\ 7{,}848{,}300 & 16{,}400 \\ 5{,}427{,}000 & 3953 \end{bmatrix}$

**9.** 2 × 3 **11.** 4 × 3 **13.** 2 × 5 **15.** $\left(3, -\frac{1}{3}\right)$

**17.** (5, 3) **19.** (4, −3) **21.** $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 3 & 3 \\ 2 & 4 & 4 & 3 \end{bmatrix}$

**23.**
$$\begin{array}{l} \\ \text{Adult} \\ \text{Child} \\ \text{Senior} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Evening} & \text{Matinee} & \text{Twilight} \end{array} \\ \begin{bmatrix} 7.50 & 5.50 & 3.75 \\ 4.50 & 4.50 & 3.75 \\ 5.50 & 5.50 & 3.75 \end{bmatrix} \end{array}$$

**25.**
$$\begin{array}{l} \\ \text{Single} \\ \text{Double} \\ \text{Suite} \end{array} \begin{array}{c} \begin{array}{cc} \text{Weekday} & \text{Weekend} \end{array} \\ \begin{bmatrix} 60 & 79 \\ 70 & 89 \\ 75 & 95 \end{bmatrix} \end{array}$$

**29.** $\begin{bmatrix} 1 & 3 & 6 & 10 & 15 & 21 \\ 2 & 5 & 9 & 14 & 20 & 27 \\ 4 & 8 & 13 & 19 & 26 & 34 \\ 7 & 12 & 18 & 25 & 33 & 42 \\ 11 & 17 & 24 & 32 & 41 & 51 \\ 16 & 23 & 31 & 40 & 50 & 61 \\ 22 & 30 & 39 & 49 & 60 & 72 \end{bmatrix}$

**31.** Matrices are used to organize information so it can be read and compared more easily. For example, Sabrina can see that the hybrid SUV has the best price and fuel economy; the standard SUV has the most horsepower, exterior length, and cargo space; the mid-size SUV has a lower price than the standard but high horsepower and cargo space; and the compact SUV has the a low price and good fuel economy. **33.** J **35.** (7, 3, −9) **37.** 0 dresses, 120 skirts **39.** $2x - 3y = -3$ **41.** 2 **43.** 20 **45.** −18 **47.** 75

### *Pages 173–176* *Lesson 4-2*

**1.** impossible **3.** $\begin{bmatrix} 1 & 10 \\ -7 & 5 \end{bmatrix}$

**5.** Males $\begin{bmatrix} 17{,}389 & 544{,}811 \\ 15{,}221 & 504{,}801 \\ 14{,}984 & 457{,}146 \\ 10{,}219 & 349{,}785 \\ 5758 & 96{,}562 \end{bmatrix}$ Females $\begin{bmatrix} 17{,}061 & 457{,}986 \\ 15{,}089 & 418{,}322 \\ 14{,}181 & 362{,}468 \\ 9490 & 309{,}032 \\ 6176 & 144{,}565 \end{bmatrix}$

**7.** No; many schools offer the same sport for males and females, so those schools would be counted twice.

**9.** $\begin{bmatrix} -10 & 20 \\ 30 & -15 \\ 45 & 5 \end{bmatrix}$ **11.** $\begin{bmatrix} -21 & 29 \\ 12 & -22 \end{bmatrix}$ **13.** impossible

**15.** $\begin{bmatrix} -13 & -1 \\ 2 & 3 \end{bmatrix}$ **17.** $\begin{bmatrix} -7 & 7 & -2 \\ -6 & 5 & 8 \\ -5 & -16 & 8 \end{bmatrix}$ **19.** $\begin{bmatrix} 15 & 0 & 4 \\ 0 & 13 & -5 \end{bmatrix}$

**21.** $\begin{bmatrix} -8 & -1 & -13 \\ 15 & 13 & -2 \\ -9 & -7 & 19 \end{bmatrix}$ **23.** $\begin{bmatrix} 631 & 377 & 292 & 533 \\ 552 & 431 & 163 & 317 \\ 363 & 367 & 199 & 258 \end{bmatrix}$

**25.** $\begin{bmatrix} -4 & 8 & -2 \\ 6 & -10 & -16 \\ -14 & -12 & 4 \end{bmatrix}$ **27.** $\begin{bmatrix} 15 & -29 & 65 & -2 \end{bmatrix}$

**29.** $\begin{bmatrix} 13 & 10 \\ 4 & 7 \\ 7 & -5 \end{bmatrix}$ **31.** $\begin{bmatrix} 0 & 16 \\ -8 & 20 \\ 28 & -4 \end{bmatrix}$ **33.** $\begin{bmatrix} -12 & -13 \\ 3 & -8 \\ 13 & 37 \end{bmatrix}$

**35.** $\begin{bmatrix} 1.8 & 9.08 \\ 3.18 & 31.04 \\ 10.41 & 56.56 \end{bmatrix}$ **37.** $\begin{bmatrix} -4 & -15 \\ \frac{3}{2} & -2 \end{bmatrix}$ **39.** $\begin{bmatrix} 0.5 \\ 0.47 \\ 0.87 \\ 0 \end{bmatrix}$

**41.** 50m and 100m **43.** $\begin{bmatrix} 1.00 & 1.00 \\ 1.50 & 1.50 \end{bmatrix}$

**45.** Sample answer: [−3, 1], [3, −1] **47.** You can use matrices to track dietary requirements and add them to find the total each day or each week.

Breakfast $\begin{bmatrix} 566 & 18 & 7 \\ 482 & 12 & 17 \\ 530 & 10 & 11 \end{bmatrix}$ Lunch $\begin{bmatrix} 785 & 22 & 19 \\ 622 & 23 & 20 \\ 710 & 26 & 12 \end{bmatrix}$ Dinner $\begin{bmatrix} 1257 & 40 & 26 \\ 987 & 32 & 45 \\ 1380 & 29 & 38 \end{bmatrix}$

add the three matrices: $\begin{bmatrix} 2608 & 80 & 52 \\ 2091 & 67 & 82 \\ 2620 & 65 & 61 \end{bmatrix}$.

**49.** J **51.** $1 \times 4$ **53.** $3 \times 3$ **55.** $4 \times 3$ **57.** (5, 3, 7) **59.** (2, 5) **61.** (6, −1) **63.** No, it would cost $6.30. **65.** Assoc. (+) **67.** Comm. (×)

### *Pages 182–184* *Lesson 4-3*

**1.** $3 \times 2$ **3.** $3 \times 22$ **5.** $\begin{bmatrix} 0 & 44 \\ 8 & -34 \end{bmatrix}$

**7.** $\begin{bmatrix} 15 & -5 & 20 \\ 24 & -8 & 32 \end{bmatrix}$ **9.** $\begin{bmatrix} 24 \\ 41 \end{bmatrix}$ **11.** $74,525

**13.** yes; $A(BC) = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \cdot \left(\begin{bmatrix} -4 & 1 \\ 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}\right)$

$= \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -13 & -6 \\ 24 & 16 \end{bmatrix} = \begin{bmatrix} -50 & -28 \\ 81 & 62 \end{bmatrix}$

$(AB)C = \left(\begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -4 & 1 \\ 8 & 0 \end{bmatrix}\right) \cdot \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -16 & 2 \\ 28 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -50 & -28 \\ 81 & 62 \end{bmatrix}$

**15.** $2 \times 2$ **17.** $1 \times 5$ **19.** $3 \times 5$ **21.** $\begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}$

**23.** $\begin{bmatrix} -6 & 3 \\ 44 & -19 \end{bmatrix}$ **25.** $\begin{bmatrix} -39 \\ 18 \end{bmatrix}$ **27.** $\begin{bmatrix} 12 & 4 \\ -24 & -8 \end{bmatrix}$ **29.** $\begin{bmatrix} 14{,}285 \\ 13{,}270 \\ 4295 \end{bmatrix}$

**31.** $c(AB) = 3\left(\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 4 & 3 \end{bmatrix}\right)$

$= 3\begin{bmatrix} -13 & -4 \\ -8 & 17 \end{bmatrix} = \begin{bmatrix} -39 & -12 \\ -24 & 51 \end{bmatrix}$

$A(cB) = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} \cdot \left(3\begin{bmatrix} -5 & 2 \\ 4 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} -16 & 6 \\ 12 & 9 \end{bmatrix} = \begin{bmatrix} -39 & -12 \\ -24 & 51 \end{bmatrix}$

The equation is true.

**33.** $AC + BC = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 9 \\ 26 & -8 \end{bmatrix} + \begin{bmatrix} -21 & -13 \\ 26 & -8 \end{bmatrix}$

$= \begin{bmatrix} -20 & -4 \\ 52 & -16 \end{bmatrix}$

$(A + B)C = \left(\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ 4 & 3 \end{bmatrix}\right) \cdot \begin{bmatrix} 5 & 1 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 \\ 8 & 8 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 \\ 2 & -4 \end{bmatrix}$

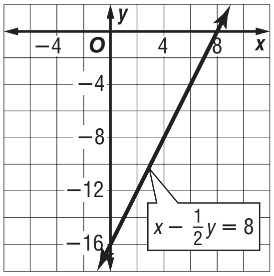
$= \begin{bmatrix} -20 & -4 \\ 52 & -16 \end{bmatrix}$ The equation is true.

**35.** $\begin{bmatrix} 72 & 49 \\ 68 & 63 \\ 90 & 56 \\ 86 & 62 \end{bmatrix} \cdot \begin{bmatrix} 1.00 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 96.50 \\ 99.50 \\ 118 \\ 117 \end{bmatrix}$; juniors

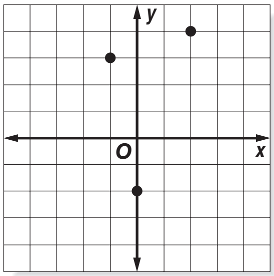
**37.** \$24,900 **39.** \$1460 **41.** Never; the inner dimensions will never be equal. **43.** $a = 1, b = 0, c = 0, d = 1$; the original matrix **45.** C **47.** $\begin{bmatrix} 12 & -6 \\ -3 & 21 \end{bmatrix}$ **49.** $\begin{bmatrix} -20 & 2 \\ -28 & 12 \end{bmatrix}$

**51.** $x = 5, y = -9$ **53.** \$2.50; \$1.50

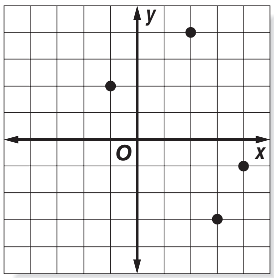
**55.** 8, −16




**57.**

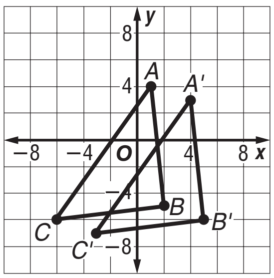



**59.**




***Pages 189–192*** ***Lesson 4-4***

**1.** $\begin{bmatrix} 3 & 3 & 3 \\ -1 & -1 & -1 \end{bmatrix}$ **3.**

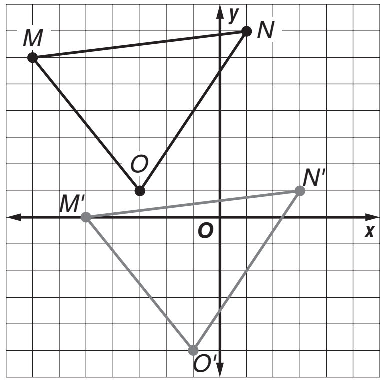



**5.** $\begin{bmatrix} 0 & 5 & 5 & 0 \\ 4 & 4 & 0 & 0 \end{bmatrix}$ **7.** $A'(0, 2), B'\left(\frac{5}{2}, 2\right), C'\left(\frac{5}{2}, 0\right), D'(0, 0)$

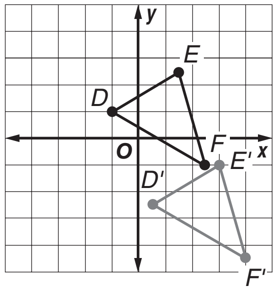
**9.** $A'(0, 4), B'(-5, 4), C'(-5, 0), D'(0, 0)$

**11.** $A'(4, 0), B'(4, -5), C'(0, -5), D'(0, 0)$

**13.** $\begin{bmatrix} 2 & 2 & 2 \\ -6 & -6 & -6 \end{bmatrix}$; $M'(-5, 0), N'(3, 1), O'(-1, -5)$;

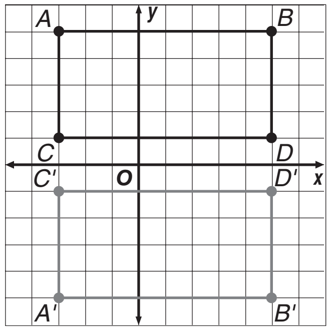



**15.** $\begin{bmatrix} 3 & 3 & 3 \\ -7 & -7 & -7 \end{bmatrix}$, $E'(6, -2), F'(8, -9)$

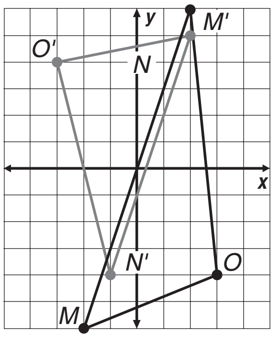



**17.** $\begin{bmatrix} -6 & 4 & 2 \\ 2 & 8 & -6 \end{bmatrix}$, $X'(-3, 1), Y'(2. 4), Z'(1, -3)$;

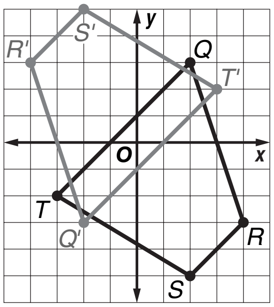
**19.** $\begin{bmatrix} -3 & 5 & 5 & -3 \\ 5 & 5 & -1 & -1 \end{bmatrix}$; $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; $A'(-3, -5), B'(5, -5), D'(5,1), C'(-3,1)$;



**21.** $\begin{bmatrix} -2 & 1 & 3 \\ -6 & 4 & -4 \end{bmatrix}$; $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$; $M'(2, 6)$, $N'(-1, -4)$, $O'(-3, 4)$;




**23.**




**25.** $J(-5, 3)$, $K(7, 2)$, $L(4, -1)$ **27.** $P(2, 2)$, $Q(-4, 1)$, $R(1, -5)$, $S(3, -4)$ **29.** $\begin{bmatrix} 4 & -4 & -4 & 4 \\ -4 & -4 & 4 & 4 \end{bmatrix}$

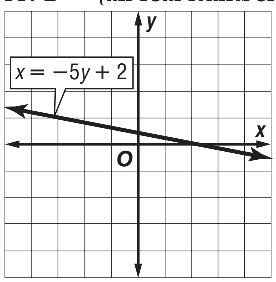
**31.** The figures in Exercise 28 and Exercise 29 have the same coordinates, but the figure in Exercise 30 has different coordinates. **33.** (6.5, 6.25) **35.** (−3.75, −2.625) **37.** The object is reflected over the $x$-axis, then translated 6 units to the right. **39.** No; since the translation does not change the $y$-coordinate, it does not matter whether or not you do the translation or the reflection first. However, if the translation did change the y-coordinate, the order would be important.

**41.** $\begin{bmatrix} -3 & -3 & -3 \\ -2 & -2 & -2 \end{bmatrix}$ **43.** Sample answer: $\begin{bmatrix} -4 & -4 & -4 \\ 1 & 1 & 1 \end{bmatrix}$

**45.** Sometimes; the image of a dilation is congruent to its preimage if and only if the scale factor is 1 or −1.

**47.** A. 3 **49.** $2 \times 2$ **51.** $2 \times 5$ **53.** $\begin{bmatrix} 20 & 10 & -24 \\ 31 & -46 & -9 \\ -10 & 3 & 7 \end{bmatrix}$

**55.** D = {all real numbers}; R = {all real numbers}; yes



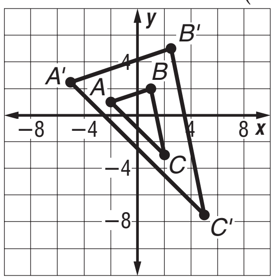

**57.** $|x| \geq 4$ **59.** $513\frac{2}{3}$ mi **61.** 5 **63.** $\frac{9}{4}$ **65.** $\frac{5}{3}$

## Pages 198–200 Lesson 4-5

**1.** −38 **3.** −28 **5.** 0 **7.** 26 units$^2$ **9.** 20 **11.** −22 **13.** −14 **15.** 32 **17.** −58 **19.** 62 **21.** 172 **23.** −22 **25.** −5 **27.** 20 ft$^2$ **29.** 14.5 units$^2$ **31.** $\frac{5}{3}$, −1 **33.** 6 or 4

**35.** 0 **37.** Sample answer: $\begin{bmatrix} 2 & 1 \\ 8 & 4 \end{bmatrix}$

**39.** Sample answer: $\begin{bmatrix} 3 & 1 \\ 6 & 5 \end{bmatrix}$, $\begin{bmatrix} 4 & 3 \\ 1 & 3 \end{bmatrix}$ **41.** If you know the coordinates of the vertices of a triangle, you can use a determinant to find the area. This is convenient since you don't need to know any additional information such as the measure of the angles. You could place a coordinate grid over a map of the Bermuda Triangle with one vertex at the origin. By using the scale of the map, you could determine coordinates to represent the other two vertices and use a determinant to estimate the area. The determinant method is advantageous because you don't need to physically measure the lengths of each side or the measure of the angles at the vertices. **43.** H **45.** $A'(-5, 2.5)$, $B'(2.5, 5)$, $C'(5, -7.5)$;




**47.** undefined **49.** 138,435 ft **51.** $y = -\frac{4}{3}x$ **53.** $y = \frac{1}{2}x + 5$ **55.** (1, 9)

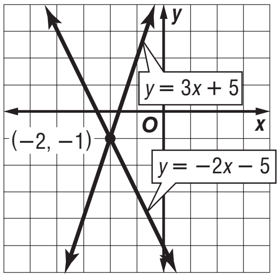
## Pages 205-207 Lesson 4-6

**1.** (5, 1) **3.** $s + d = 5000$, $0.035s + 0.05d = 227.50$ **5.** no solution **7.** (2, −1) **9.** (3, 5) **11.** (−4, −1.75) **13.** (−1.5, 2) **15.** $6g + 15r = 93$; $7g + 12r = 81$ **17.** \$1.99, \$2.49 **19.** (2, −1, 3) **21.** $\left(\frac{141}{29}, -\frac{102}{29}, \frac{244}{29}\right)$

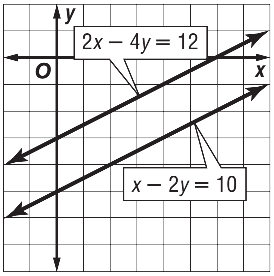
**23.** $\left(-\frac{155}{28}, \frac{143}{70}, \frac{673}{140}\right)$ **25.** (−8.5625, −19.0625)

**27.** $\left(\frac{2}{3}, \frac{5}{6}\right)$ **29.** $p + r + c = 5$, $2r - c = 0$, $3.2p + 2.4r + 4c = 16.8$; peanuts, 2 lb; raisins, 1 lb; pretzels, 2 lb **31.** $3x + 5y = -6$, $4x - 2y = 30$ **33.** Cramer's Rule is a formula for the variables $x$ and $y$ where $(x, y)$ is a solution for a system of equations. Cramer's Rule uses determinants composed of the coefficients and constant terms in a system of linear equations to solve the system. Cramer's rule is convenient when coefficients are large or involve fractions or decimals. Finding the value of the determinant is sometimes easier than trying to find a greatest common factor if you are solving by using elimination or substituting complicated numbers. **35.** J **37.** 40 **39.** $\begin{bmatrix}1 & 1 & 1\\ 3 & 3 & 3\end{bmatrix}$

**41.** (−2, −1)




**43.** no solution




**45.** $[-4\ \ 32]$ **47.** $\begin{bmatrix}21\\ 43\end{bmatrix}$

***Pages 212–215*** ***Lesson 4-7***

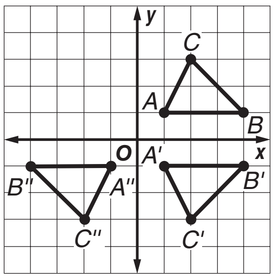
**1.** no **3.** yes **5.** $\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$ **7.** $-\frac{1}{27}\begin{bmatrix}4 & -1\\ -7 & -5\end{bmatrix}$ **9.** yes

**11.** no **13.** $\frac{1}{5}\begin{bmatrix}1 & 0\\ 0 & 5\end{bmatrix}$ **15.** No inverse exists. **17.** $\frac{1}{7}\begin{bmatrix}1 & -1\\ 4 & 3\end{bmatrix}$

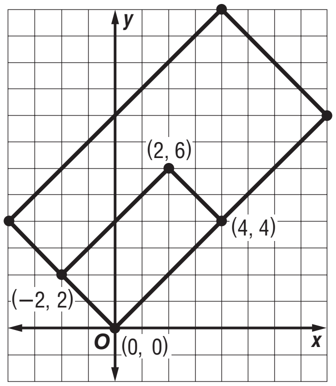
**19.** $\frac{1}{34}\begin{bmatrix}7 & 3\\ -2 & 4\end{bmatrix}$ **21.** No inverse exists.

**23.** AT_SIX_THIRTY **27.** true **29.** false **31.** yes

**33.** $4\begin{bmatrix}\frac{1}{4} & \frac{3}{4}\\ -\frac{1}{6} & \frac{1}{2}\end{bmatrix}$ **35a.** no **35b.** yes




**37.** $\begin{bmatrix}0 & -2 & 2 & 4\\ 0 & 2 & 6 & 4\end{bmatrix}$ **39.** dilation by a scale factor of 2




**41.** $B^{-1} = \begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}$; the graph of the inverse transformation is the original figure. **43.** No inverse exists. **45.** Exchange the values for a and d in the first diagonal in the matrix. Multiply the values for b and c by −1 in the second diagonal in the matrix. Find the determinant of the original matrix. Multiply the negative reciprocal of the determinant by the matrix with the above mentioned changes. **47.** $a = \pm 1$, $d = \pm 1$, $b = c = 0$ **49.** B **51.** (2, −4) **53.** (−5, 4, 1) **55.** −14 **57.** $[-4]$ **59.** $[14\ \ -8]$ **61.** (2, 5) **63.** 1 **65.** −5 **67.** $\frac{5}{2}$ **69.** 7.82 tons/in$^2$ **71.** −2 **73.** 4 **75.** −34

***Pages 219–222*** ***Lesson 4-8***

**1.** $\begin{bmatrix}1 & -1\\ 1 & 3\end{bmatrix} \cdot \begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-3\\ 5\end{bmatrix}$ **3.** $h = 1$, $c = 12$, $o = 16$

**5.** (1, 1.75) **7.** no solution **9.** $\begin{bmatrix}4 & -7\\ 3 & 5\end{bmatrix} \cdot \begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}2\\ 9\end{bmatrix}$

**11.** $\begin{bmatrix}3 & -7\\ 6 & 5\end{bmatrix} \cdot \begin{bmatrix}m\\ n\end{bmatrix} = \begin{bmatrix}-43\\ -10\end{bmatrix}$ **13.** 27 h of flight instruction and 23 h in the simulator **15.** no solution **17.** (2, −3)

**19.** no solution **21.** (3, 1.5) **23.** $\left(\frac{3}{2}, \frac{1}{3}\right)$ **25.** carbon = 12; hydrogen = 1 **27.** 2010 **29.** (−6, 2, 5) **31.** (0, −1, 3) **33.** Sample answer: $x + 3y = 8$ and $2x + 6y = 16$ **35.** The solution set is the empty set or infinitely many solutions. **37.** C **39.** D **41.** $\begin{bmatrix}4 & -5\\ -7 & 9\end{bmatrix}$

**43.** (4, −2) **45.** (−6, −8)

***Pages 224–228*** ***Chapter 4*** ***Study Guide and Review***

**1.** identity matrix **3.** rotation **5.** matrix equation **7.** matrix **9.** inverses **11.** dilation **13.** (−5, −1)

**15.** (−1, 0) **17.** $\begin{bmatrix}17 & 20 & 23\\ 12 & 19 & 22\\ 6 & 7 & 11\end{bmatrix}$ row 3, column 1

**19.** $[-1.8\ \ -0.4\ \ -3]$ **21.** $\begin{bmatrix}5 & -6 & -13\\ 10 & -3 & -2\end{bmatrix}$ **23.** $[-18]$

**25.** No product exists. **27.** $A'(1, 0)$, $B'(8, -2)$, $C'(3, -7)$ **29.** $A'(3, 5)$, $B'(-4, 3)$, $C'(1, -2)$ **31.** $A'(1, 1)$, $B'(3, 1)$, $C'(3, 3)$, $D'(1, 3)$ **33.** 53 **35.** −36 **37.** −35 **39.** $\left(\frac{2}{3}, 5\right)$
**41.** $(2, -2)$ **43.** $(1, 2, -1)$ **45.** ($5.25, $4.75)
**47.** $\frac{1}{2}\begin{bmatrix} 7 & -6 \\ -9 & 8 \end{bmatrix}$ **49.** No inverse exists. **51.** $(4, 2)$
**53.** $(-3, 1)$ **55.** 720 mL of the 50% solution and 780 mL of the 75% solution

# Chapter 5 Quadratic Functions and Inequalities

***Page 235 Chapter 5 Get Ready***

**1.** −4 **3.** −6 **5.** −4 **7.** 0 **9.** $f(x) = 9x$ **11.** $(x + 6)(x + 5)$ **13.** $(x - 8)(x + 7)$ **15.** prime **17.** $(x - 11)^2$ **19.** $(x + 7)$ feet

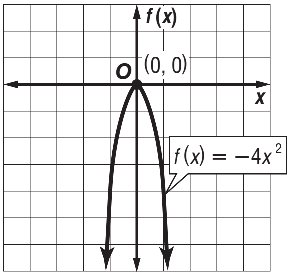
***Pages 241–244 Lesson 5–1***

**1a.** 0; $x = 0$; 0

**1b.**

| x | f(x) |
|---|---|
| −1 | −4 |
| 0 | 0 |
| 1 | −4 |

**1c.**

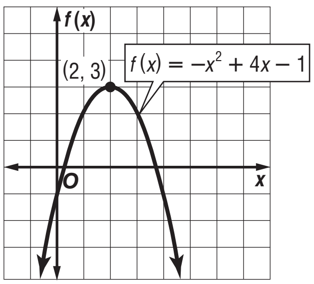



**3a.** −1; $x = 2$; 2

**3b.**

| x | f(x) |
|---|---|
| 0 | −1 |
| 1 | 2 |
| 2 | 3 |
| 3 | 2 |
| 4 | −1 |

**3c.**

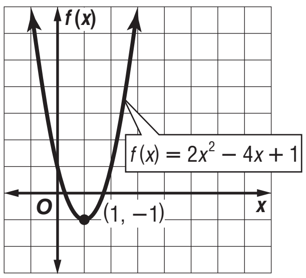



**5a.** 1; $x = 1$; 1

**5b.**

| x | f(x) |
|---|---|
| −1 | 7 |
| 0 | 1 |
| 1 | −1 |
| 2 | 1 |
| 3 | 7 |

**5c.**



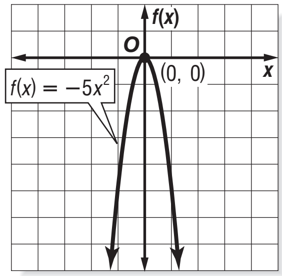

**7.** max.; 7; D = all reals; R = $\{y \mid y \leq 7\}$
**9.** min.; 0; D = all reals; R = $\{y \mid y \geq 0\}$
**11.** $8.75

**13a.** 0; $x = 0$; 0

**13b.**

| x | f(x) |
|---|---|
| −2 | −20 |
| −1 | −5 |
| 0 | 0 |
| 1 | −5 |
| 2 | −20 |

**13c.**

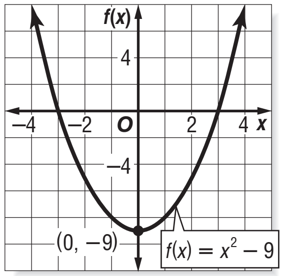



**15a.** −9; $x = 0$; 0

**15b.**

| x | f(x) |
|---|---|
| −2 | −5 |
| −1 | −8 |
| 0 | −9 |
| 1 | −8 |
| 2 | −5 |

**15c.**

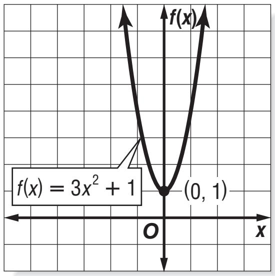



**17a.** 1; $x = 0$; 0

**17b.**

| x | f(x) |
|---|---|
| −2 | 13 |
| −1 | 4 |
| 0 | 1 |
| 1 | 4 |
| 2 | 13 |

**17c.**

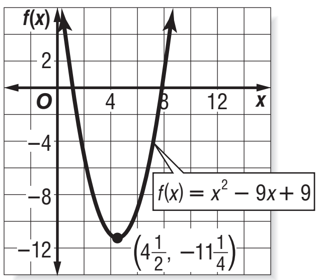



**19a.** 9; $x = 4.5$; 4.5

**19b.**

| x | f(x) |
|---|---|
| 3 | −9 |
| 4 | −11 |
| 4.5 | −11.25 |
| 5 | −11 |
| 6 | −9 |

**19c.**

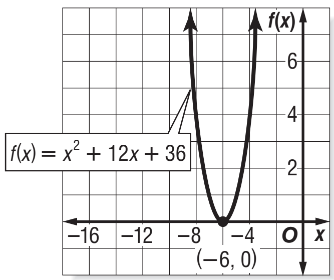



**21a.** 36; $x = -6$; −6

**21b.**

| x | f(x) |
|---|---|
| −8 | 4 |
| −7 | 1 |
| −6 | 0 |
| −5 | 1 |
| −4 | 4 |

**21c.**



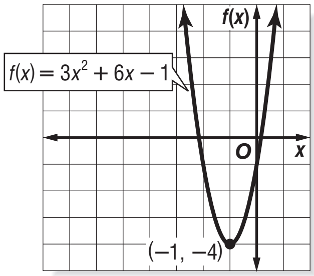

**23.** max.; −9; D = all reals, R = $\{y \mid y \leq -9\}$
**25.** min.; −11; D = all reals, R = $\{y \mid y \geq -11\}$
**27.** max.; 12; D = all reals, R = $\{y \mid y \leq 12\}$
**29.** min.; −1; D = all reals, R = $\{y \mid y \geq -1\}$
**31.** max.; −60; D = all reals, R = $\{y \mid y \leq -60\}$
**33.** 40 m **35.** 300 ft, 2.5 s

**37a.** $-1$; $x = -1$; $-1$

**37b.**

| $x$ | $f(x)$ |
|---|---|
| $-3$ | 8 |
| $-2$ | $-1$ |
| $-1$ | $-4$ |
| 0 | $-1$ |
| 1 | 8 |

**37c.**

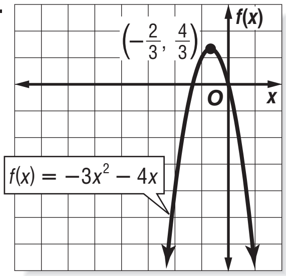



**39a.** 0; $x = -\frac{2}{3}$; $-\frac{2}{3}$

**39b.**

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $-4$ |
| $-1$ | 1 |
| $-\frac{2}{3}$ | $\frac{4}{3}$ |
| 0 | 0 |
| 1 | $-7$ |

**39c.**

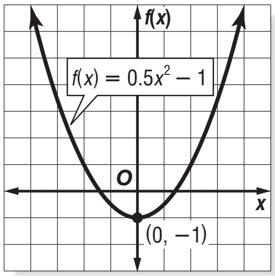



**41a.** $-1$; $x = 0$; 0

**41b.**

| $x$ | $f(x)$ |
|---|---|
| $-2$ | 1 |
| $-1$ | $-\frac{1}{2}$ |
| 0 | $-1$ |
| 1 | $-\frac{1}{2}$ |
| 2 | 1 |

**41c.**

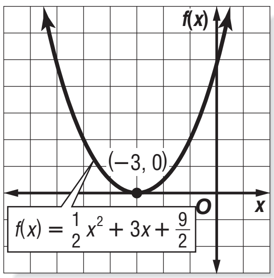



**43a.** $\frac{9}{2}$; $x = -3$; $-3$

**43b.**

| $x$ | $f(x)$ |
|---|---|
| $-5$ | 2 |
| $-4$ | 0.5 |
| $-3$ | 0 |
| $-2$ | 0.5 |
| $-1$ | 2 |

**43c.**

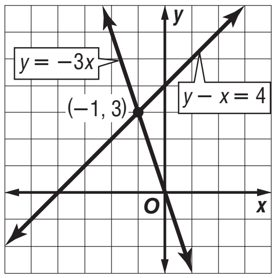



**45.** min.; $\frac{9}{2}$; D = all reals, R = $\left\{y \mid y \geq \frac{9}{2}\right\}$
**47.** max.; 5; D = all reals, R = $\{y \mid y \leq 5\}$
**49.** max.; 5; D = all reals, R = $\{y \mid y \leq 5\}$
**51.** $120 - 2x$ **53.** 60 ft by 30 ft **55.** 5 in. by 4 in. **57.** \$2645 **59.** 3.20 **61.** 3.38 **63.** 1.56 **65.** $c$; the $x$-coordinate of the vertex of $y = ax^2 + c$ is $-\frac{0}{2a}$ or 0, so the $y$-coordinate of the vertex, the minimum of the function, is $a(0)^2 + c$ or $c$; $-12.5$. **67.** C **69.** (1, 2)

**71.** $\begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}$ **73.** $\begin{bmatrix} -7 & 0 \\ 5 & 20 \end{bmatrix}$ **75.** $[10 \ \ -4 \ \ 5]$

**77.** $\begin{bmatrix} -28 & 20 & -44 \\ 8 & -16 & 36 \end{bmatrix}$ **79.** \$20, \$35 **81.** (3, $-5$)

**83.**

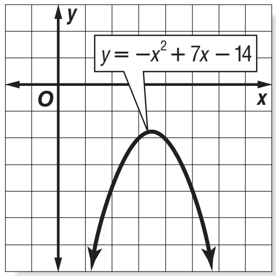



(−1, 3); consistent and independent

**85.** $-1$ **87.** $-5$ **89.** $-\frac{9}{5}$ **91.** $\{-5, 1\}$
**93.** 256 in$^2$ **95.** 8 **97.** $-1$

***Pages 249–251*** ***Lesson 5–2***

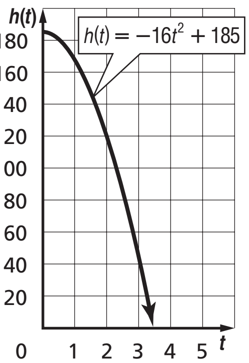
**1.** $-4$, 1 **3.** $-4$ **5.** $-4$, 6 **7.** 7 **9.** no real solutions **11.** between $-1$ and 0; between 1 and 2 **13.** 4 s **15.** 3 **17.** 0 **19.** no real solutions **21.** 0, 4 **23.** 3, 6 **25.** 6 **27.** no real solutions **29.** between $-1$ and 0; between 2 and 3 **31.** about 12 s **33.** $-\frac{1}{2}$, $2\frac{1}{2}$ **35.** $-2\frac{1}{2}$, 3 **37.** between 0 and 1; between 3 and 4 **39.** between $-3$ and $-2$; between 2 and 3 **41.** Let $x$ be the first number. Then, $7 - x$ is the other number. $x(7 - x) = 14$; $-x^2 + 7x - 14 = 0$;




Since the graph of the related function does not intersect the $x$-axis, this equation has no real solutions. Therefore no such numbers exist.

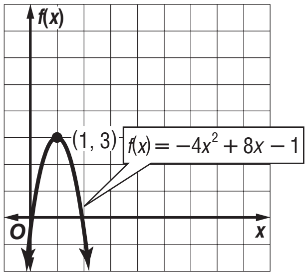
**43.** $-2$, 14 **45.** about 8 s **47.** The $x$-intercepts of the related function are the solutions to the equation. You can estimate the solutions by stating the consecutive integers between which the $x$-intercepts are located.

**49.**




Locate the positive $x$-intercept at about 3.4. This represents the time when the height of the ride is 0. Thus, if the ride were allowed to fall to the ground, it would take about 3.4 seconds. **51.** F

**53.** $-1$; $x = 1$; $1$;




**55.** $(1, 4)$ **57.** $-8$ **59.** \$500 **61.** $(x - 10)(x + 10)$ **63.** $(x - 9)^2$ **65.** $2(3x + 2)(x - 3)$

***Pages 256–258*** ***Lesson 5–3***

**1.** $x^2 - 3x - 28 = 0$ **3.** $15x^2 + 14x + 3 = 0$ **5.** $4x(y + 2)(y - 2)$ **7.** $\{0, 11\}$ **9.** $\left\{-\frac{3}{4}, 4\right\}$ **11.** $\{3\}$ **13.** $x^2 - 9x + 20 = 0$ **15.** $x^2 + x - 20 = 0$ **17.** $(x - 6)(x - 1)$ **19.** $3(x + 7)(x - 3)$ **21.** $\{-8, 3\}$ **23.** $\{-5, 5\}$ **25.** $\{-6, 3\}$ **27.** $\{2, 4\}$ **29.** $\{6\}$ **31.** 14, 16 or $-14$, $-16$ **33.** $\left\{0, \frac{5}{3}\right\}$ **35.** $\left\{-2, \frac{1}{4}\right\}$ **37.** $\left\{-\frac{1}{2}, -\frac{3}{2}\right\}$ **39.** $\left\{-\frac{8}{3}, -\frac{2}{3}\right\}$ **41.** $0, -6, 5$ **43.** $2x^2 - 7x + 3 = 0$ **45.** $12x^2 - x - 6 = 0$ **47.** $\frac{1}{4}$ s

**49.** 4; The logs must have a diameter greater than 4 in. for the rule to produce positive board feet values. **51.** Sample answer: Roots 6 and $-5$; $x^2 - x - 30 = 0$; the sign of the linear term changes, but the others stay the same.

**53.** To use the Zero Product Property, the equation must be written as a product of factors equal to zero. Move all the terms to one side and factor (if possible). Then set each factor equal to zero and solve for the variable. To use the Zero Product Property, one side of the equation must equal zero, so the equation cannot be solved by setting each factor on the left side equal to 24. **55.** G **57.** $-\frac{1}{2}$

**59.** min.; $-19$ **61.** $y = -2x - 2$

**63.** Comm. $(+)$ **65.** Assoc. $(+)$

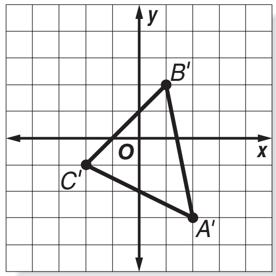
***Pages 264–266*** ***Lesson 5–4***

**1.** $2\sqrt{14}$ **3.** $\frac{4\sqrt{3}}{7}$ **5.** $6i$ **7.** 12 **9.** $i$ **11.** $\pm 3i$ **13.** $3, -3$ **15.** $10 + 3j$ amps **17.** $6 + 3i$ **19.** $-9 + 2i$ **21.** $\frac{7}{17} - \frac{11}{17}i$ **23.** $7\sqrt{3}$ **25.** $\frac{5\sqrt{14}}{9}$ **27.** $9i$ **29.** $10a^2|b|i$ **31.** $-75i$ **33.** 1 **35.** $-3$ **37.** 2 **39.** $6 - 7i$ **41.** $\frac{10}{17} - \frac{6}{17}i$ **43.** $\pm 4i$ **45.** $\pm 2i\sqrt{3}$ **47.** $4, -3$ **49.** $\frac{5}{3}, 4$ **51.** $4 + 2j$ amps **53.** $(5 - 2i)x^2 + (-1 + i)x + 7 + i$ **55.** $4i$ **57.** 6 **59.** $-8 + 4i$ **61.** $\frac{2}{5} + \frac{1}{5}i$ **63.** $20 + 15i$ **65.** $-\frac{1}{3} - \frac{2\sqrt{2}}{3}i$ **67.** $\pm 2i\sqrt{10}$ **69.** $\pm\frac{\sqrt{5}}{2}i$ **71.** $\frac{67}{11}, \frac{19}{11}$

**73.** Sample answer: $1 + 3i$ and $1 - 3i$ **75.** $(2i)(3i)(4i)$; The other three expressions represent real numbers, but $(2i)(3i)(4i) = -24i$, which is an imaginary number.

**77.** Some polynomial equations have complex solutions and cannot be solved using only the real numbers. $a$ and $c$ must have the same sign. The solutions are $\pm i$. **79.** H **81.** $12x^2 + 13x + 3 = 0$ **83.** $-4, -1\frac{1}{2}$ **85.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

**87.**




**89.** \$206.25 < $x$ < \$275.00 **91.** yes **93.** no **95.** no

***Pages 272–275*** ***Lesson 5–5***

**1.** $\{-10, -4\}$ **3.** $\{-8 \pm \sqrt{7}\}$ **5.** Jupiter

**7.** Yes; the acceleration due to gravity is significantly greater on Jupiter, so the time to reach the ground should be much less. **9.** $\frac{9}{4}$; $\left(x - \frac{3}{2}\right)^2$ **11.** $\{4 \pm \sqrt{5}\}$ **13.** $\{-2 \pm \sqrt{10}\}$ **15.** $\{3 \pm i\sqrt{3}\}$ **17.** $\{-2, 12\}$ **19.** $\left\{-\frac{11}{2}, -\frac{3}{2}\right\}$ **21.** $\{3 \pm 2\sqrt{2}\}$ **23.** $\left\{\frac{3}{2} \pm \sqrt{6}\right\}$ **25.** 81; $(x - 9)^2$ **27.** $\frac{49}{4}$; $\left(x + \frac{7}{2}\right)^2$ **29.** $\{-12, 10\}$ **31.** $\{2 \pm \sqrt{3}\}$ **33.** $\left\{\frac{1}{2}, 1\right\}$ **35.** $\left\{\frac{1 \pm \sqrt{5}}{3}\right\}$ **37.** $\{-3 \pm 2i\}$ **39.** $\{-4 \pm i\sqrt{2}\}$ **41.** $5\frac{1}{2}$ in. by $5\frac{1}{2}$ in. **43.** $\{-1.6, 0.2\}$ **45.** $\left\{\frac{-5 \pm \sqrt{11}}{3}\right\}$ **47.** 1.44; $(x - 1.2)^2$ **49.** $\frac{25}{16}$; $\left(x + \frac{5}{4}\right)^2$ **51.** $\{0.7, 4\}$ **53.** $\left\{\frac{3}{4} \pm \sqrt{2}\right\}$ **55.** $\left\{\frac{7 \pm i\sqrt{47}}{4}\right\}$ **57.** $\frac{x}{1}, \frac{1}{x - 1}$

**59.** Sample answers: The golden ratio is found in much of ancient Greek architecture, such as the Parthenon, as well as in modern architecture, such as in the windows of the United Nations building. Many songs have their climax at a point occurring 61.8% of the way through the piece, with 0.618 being about the reciprocal of the golden ratio. The reciprocal of the golden ratio is also used in the design of some violins. **61.** Sample answer: $x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{1}{4}$; $\left\{\frac{5}{6}, -\frac{1}{6}\right\}$ **63.** Never; the value of $c$ that makes $ax^2 + bx + c$ a perfect square trinomial is the square of $\frac{b}{2}$ and the square of a number can never be negative.

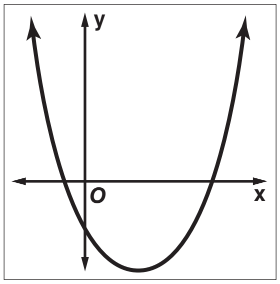
**65.** To find the distance traveled by the accelerating racecar in the given situation, you must solve the equation $t^2 + 22t + 121 = 246$ or $t^2 + 22t - 125 = 0$. Since the expression $t^2 + 22t - 125$ is prime, the solutions of $t^2 + 22t + 121 = 246$ cannot be obtained by factoring. Rewrite $t^2 + 22t + 121$ as $(t + 11)^2$. Solve $(t + 11)^2 = 246$ by applying the Square Root Property. Using a calculator, the two solutions are about 4.7 or $-26.7$. Since time cannot be negative, the driver takes about 4.7 seconds to reach the finish line. **67.** J **69.** $-1 + 3i$ **71.** $-2, 0$

**73.** $\frac{2}{3}$, 5 **75.** $\left(\frac{43}{21}, -\frac{6}{7}\right)$ **77.** greatest: −255°C; least: −259°C **79.** −16 **81.** 0

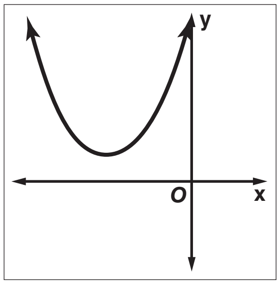
**Pages 281–283** **Lesson 5-6**

**1.** $\frac{1}{4}, -\frac{5}{2}$ **3.** $-\frac{1}{2}$ **5.** $\frac{2 \pm \sqrt{2}}{2}$ **7.** $\frac{-3 \pm i\sqrt{3}}{2}$
**9.** at about 0.7 s and again at about 4.6 s **11a.** 484 **11b.** 2 rational; yes, there were 2 rational roots **13a.** 8 **13b.** 2 irrational; yes, there were 2 irrational roots **15a.** 121 **15b.** 2 rational **15c.** $-\frac{1}{4}, \frac{2}{3}$ **17a.** 0 **17b.** 1 rational **17c.** $\frac{1}{3}$ **19a.** 21 **19b.** 2 irrational **19c.** $\frac{-3 \pm \sqrt{21}}{2}$ **21a.** 20 **21b.** 2 irrational **21c.** $-2 \pm \sqrt{5}$ **23a.** −16 **23b.** 2 imaginary **23c.** $1 \pm 2i$ **25.** −2, 32 **27.** $2 \pm i\sqrt{3}$ **29.** $\pm\sqrt{2}$ **31.** D: $0 \le t \le$ (current year − 1975), R: $73.7 \le A(t) \le 2.3(\text{current year} - 1975)^2 - 12.4(\text{current year} - 1975) + 73.7$ **33.** No; the fastest the car could have been traveling is about 67.2 mph, which is less than the Texas speed limit. **35a.** −31 **35b.** 2 complex **35c.** $\frac{9 \pm i\sqrt{31}}{8}$ **37a.** $\frac{28}{9}$ **37b.** 2 irrational **37c.** $\frac{2 \pm 4\sqrt{7}}{9}$ **39a.** −0.55 **39b.** 2 complex **39c.** $\frac{-0.1 \pm i\sqrt{0.55}}{0.4}$ **41.** $\frac{5 \pm \sqrt{46}}{3}$ **43.** $0, -\frac{3}{10}$ **45.** −2, 6 **47.** This means that the cables do not touch the floor of the bridge, since the graph does not intersect the $x$-axis and the roots are imaginary.

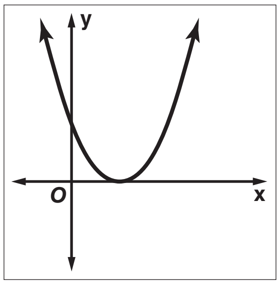
**49a.**




**49b.**

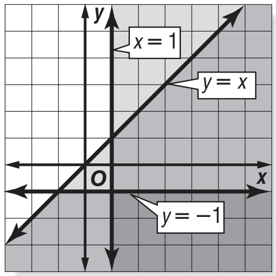



**49c.**




**51.** $-1, \frac{5}{2}$
**53.** The diver's height above the pool at any time $t$ can be determined by substituting the value of $t$ in the equation and evaluating. When the diver hits the water, her height above the pool is 0. Substitute 0 for $h$ and use the quadratic formula to find the positive value of $t$ which is a solution to the equation. This is the number of seconds that it will take for the diver to hit the water. **55.** G **57.** $4 \pm \sqrt{7}$ **59.** $\frac{1}{5} + \frac{3}{5}i$ **61.** $\frac{1}{13} + \frac{5}{13}i$

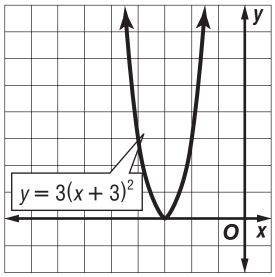
**63.**



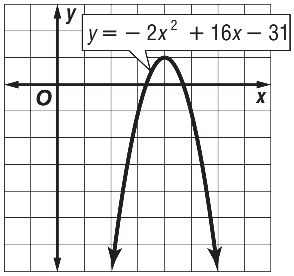

**65.** $y = -\frac{2}{3}x + 3$ **67.** no **69.** yes; $(2x + 3)^2$ **71.** no

**Pages 289–292** **Lesson 5-7**

**1.**

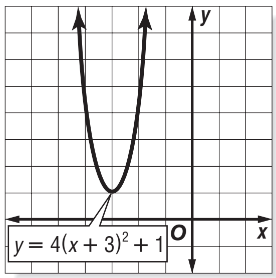


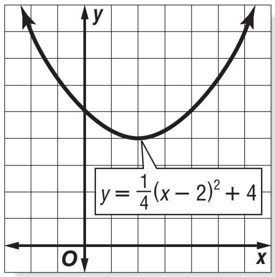

**3.**

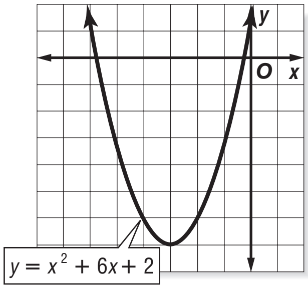



**5.** (−3, −1); $x = -3$; up **7.** $y = -3(x + 3)^2 + 38$; (−3, 38); $x = -3$; down **9.** $y = -(x + 3)^2 + 6$ **11.** $h(d) = -2d^2 + 4d + 6$ The graph opens downward and is narrower than the parent, and the vertex is at (1, 8).

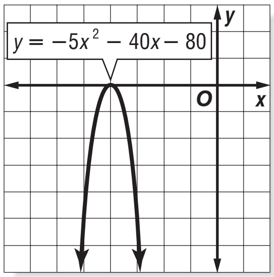
**13.**

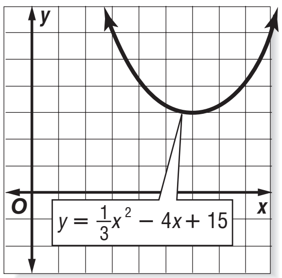


15.



17.



**19.** The graph is congruent to the original graph, and the vertex moves 7 units down the y-axis. **21.** $(-3, 0)$; $x = -3$; down **23.** $y = -(x + 2)^2 + 12$; $(-2, 12)$; $x = -2$; down **25.** $(0, -6)$; $x = 0$; up **27.** $y = 9(x - 6)^2 + 1$ **29.** $y = -\frac{2}{3}(x - 3)^2$ **31.** $y = \frac{1}{3}x^2 + 5$ **33.** Angle $A$; the graph of the equation for angle $A$ is higher than the other two since 3.27 is greater than 2.39 or 1.53. **35.** Angle $C$, Angle $A$

**37.**



**39.**



**41.** $y = 4(x + 3)^2 - 36$; $(-3, -36)$; $x = -3$; up
**43.** $y = -2(x - 5)^2 + 15$; $(5, 15)$; $x = 5$; down
**45.** $y = 4\left(x - \frac{3}{2}\right)^2 - 20$; $\left(\frac{3}{2}, -20\right)$; $x = \frac{3}{2}$; up
**47.** $y = \frac{4}{3}(x + 3)^2 - 4$ **49.** Sample answer: The graphs have the same shape, but the graph of $y = 2(x - 4)^2 - 1$ is 1 unit to the left and 5 units above the graph of $y = 2(x - 5)^2 + 4$. **51.** about 1.6 s **53.** about 2.0 s

**55.** $y = ax^2 + bx + c$

$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$

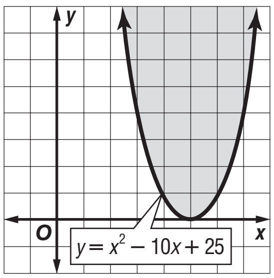
$$y = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] + c - a\left(\frac{b}{2a}\right)^2$$

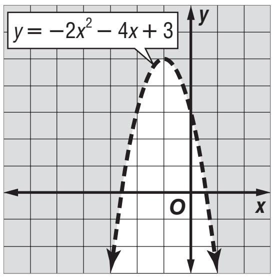
$$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

The axis of symmetry is $x = h$ or $-\frac{b}{2a}$.

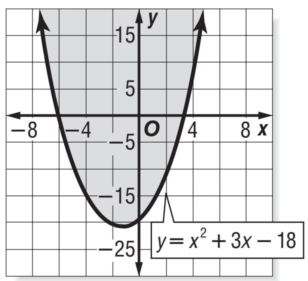
**57.** The equation of a parabola can be written in the form $y = ax^2 + bx + c$ with $a \neq 0$. For each of the three points, substitute the value of the $x$-coordinate for $x$ in the equation and substitute the value of the $y$-coordinate for $y$ in the equation. This will produce three equation in the three variables $a$, $b$, and $c$. Solve the system of equations to find the values of $a$, $b$, and $c$. These values determine the quadratic equation.
**59.** D **61.** 12; 2 irrational **63.** $-23$; 2 complex **65.** $\{3 \pm 3\boldsymbol{i}\}$
**67.** yes **69.** yes

***Pages 298–301*** ***Lesson 5-8***

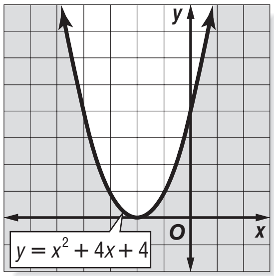
**1.**



**3.**


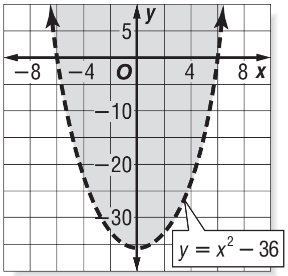

**5.** $x < 1$ or $x > 5$ **7.** $\{x | x < -3$ or $x > 4\}$
**9.** $\{x | -\sqrt{3} \leq x \leq \sqrt{3}\}$

**11.**


**13.**

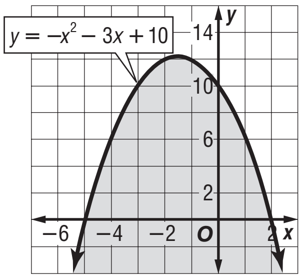



**15.**

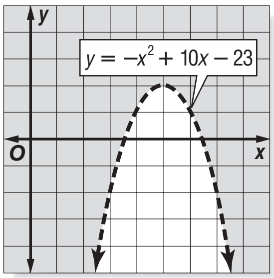



**17.** 5 **19.** $x < -3$ or $x > 3$ **21.** $\{x \mid x < -3 \text{ or } x > 6\}$ **23.** $\{x \mid -1 \leq x \leq 5\}$ **25.** $\{x \mid -4 \leq x \leq 3\}$ **27.** 0 to 10 ft or 24 to 34 ft

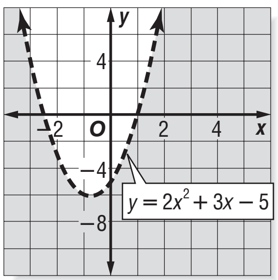
**29.**




**31.**




**33.**




**35.** $\{x \mid x = \frac{1}{3}\}$ **37.** ∅ **39.** all reals **41.** $\{x \mid -4 < x < 1$ or $x > 3\}$ **43.** $P(n) = n[15 + 1.5(60 - n)] - 525 = -1.5n^2 + 105n - 525$ **45.** \$1312.50; 35 passengers
**47.** Sample answer: −4, 0, and 6

**49.** $-16t^2 + 42t + 3.75 > 10$; One method of solving this inequality is to graph the related quadratic function $h(t) = -16t^2 + 42t + 3.75 - 10$. The interval in which the graph is above the $t$-axis represents the times when the trampolinist is above 10 feet. A second method of solving this inequality would be find the roots of the related quadratic equation $-16t^2 + 42t + 3.75 - 10 = 0$ and then test points in the three intervals determined by these roots to see if they satisfy the inequality. The interval in which the inequality is satisfied represent the times when the trampolinist is above 10 feet.
**51.** H **53.** $y = -2(x - 4)^2$; (4, 0), $x = 4$; down **55.** −4, −8
**57.** $\frac{-3 \pm 2\sqrt{6}}{3}$ **59.** (−3, −2) **61.** (1, 3) **63.** [−54 6] **65.** C
**67a.** Sample answer using (2000, 143,590) and (2003, 174,629): $y = 10{,}346x - 20{,}548{,}410$ **67b.** Sample answer: 247,050

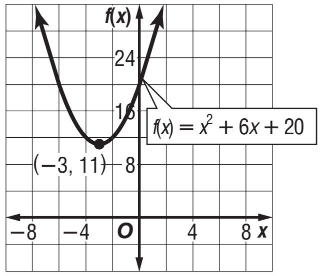
***Pages 302–306*** ***Chapter 5*** ***Study Guide and Review***

**1.** parabola **3.** axis of symmetry **5.** roots **7.** discriminant **9.** completing the square
**11a.** 20; $x = -3$; −3

**11b.**

| x | f(x) |
|---|---|
| −5 | 15 |
| −4 | 12 |
| −3 | 11 |
| −2 | 12 |
| −1 | 15 |

**11c.**

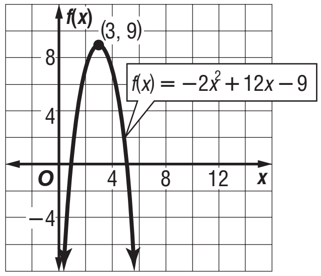



**13a.** −9; $x = 3$; 3

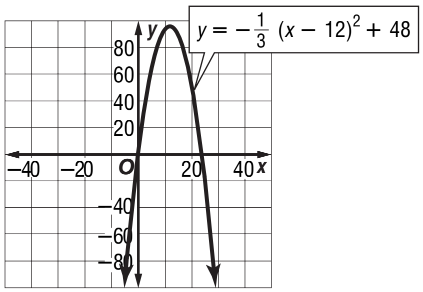
**13b.**

| x | f(x) |
|---|---|
| 1 | 1 |
| 2 | 7 |
| 3 | 9 |
| 4 | 7 |
| 5 | 1 |

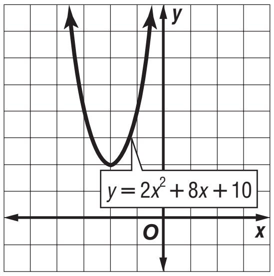
**13c.**



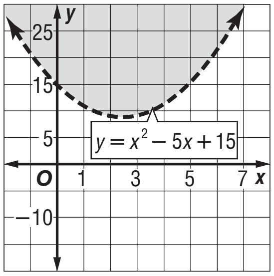

**15.** {−6, 6} **17.** between −2 and −1, between −1 and 0 **19.** 6.25 s **21.** $x^2 - 3x - 70 = 0$ **23.** {−4, 8} **25.** {−4, 4}
**27.** $\left\{\frac{1}{3}, -\frac{3}{2}\right\}$ **29.** base = 4 cm; height = 6 cm **31.** $8|n|\sqrt{m}$
**33.** $10 - 10i$ **35.** 7 **37.** $-\frac{2}{5} - \frac{11}{5}i$ **39.** 289; $(x + 17)^2$
**41.** $\left\{-\frac{3}{2}, 5\right\}$ **43.** 8 ft by 16 ft **45a.** 104 **45b.** 2 irrational
**45c.** $3 \pm \frac{\sqrt{26}}{2}$ **47.** about 204.88 ft **49.** $y = -\frac{1}{3}(x - 12)^2 + 48$; (12, 48); $x = 12$; down;



**51.** $y = 2(x + 2)^2 + 2$; $(-2, 2)$; $x = -2$; up;




**53.**




**55.** $\left\{x \mid x < -\frac{4}{3} \text{ or } x > \frac{1}{2}\right\}$

**57.** $\left\{x \,\middle|\, \frac{-1-\sqrt{10}}{2} \le x \le \frac{-1+\sqrt{10}}{2}\right\}$

**59.** $22.087 \le s \le 67.91$ mph

## Chapter 6 Polynomial Functions

### *Page 311* *Chapter 6* *Get Ready*

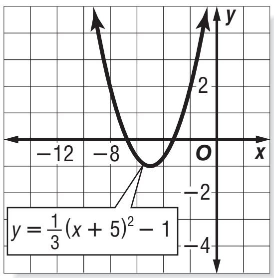
**1.** $2 + (-7)$ **3.** $x + (-y)$ **5.** $2xy + (-6yz)$ **7.** $\$4 + (-\$0.50x)$ **9.** $-x - 2$ **11.** $-6x^4 + 15x^2 + 6$ **13.** $-\frac{4}{3} - 4z$ **15.** $62.15 **17.** $-\frac{3}{2}, -\frac{1}{7}$ **19.** $1, \frac{2}{3}$

### *Pages 316–318* *Lesson 6-1*

**1.** $-15x^7y^9$ **3.** $-\frac{ab^4}{9}$ **5.** $\frac{1}{w^{12}z^6}$ **7.** 1 **9.** $\frac{1}{4x^6}$ **11.** $\frac{2}{3}a^{10}b^4$ **13.** $\frac{28x^4}{y^2}$ **15.** $an$ **17.** $-\frac{1}{4y^4}$ **19.** $n^{16}$ **21.** $16x^4$ **23.** $ab$ **25.** $\frac{cd^4}{5}$ **27.** $2 \times 10^{-7}$; $\pi \times 10^{-14}$ m$^2$ **29.** $24x^4y^4$ **31.** $\frac{a^4b^2}{2}$ **33.** $\frac{1}{x^2y^2}$ **35.** $\frac{a^4}{16b^4}$ **37.** $\frac{x^8}{16y^{14}}$ **39.** 7 **41.** about 330,000 times **43.** Alejandra; when Kyle used the Power of a Product property in his first step, he forgot to use the exponent $-2$ for both $-2$ and $a$. **45.** $100^{10} = (10^2)^{10}$ or $10^{20}$, and $10^{100} > 10^{20}$, so $10^{100} > 100^{10}$.

**47.** D **49.** $\{x \mid 2 < x < 6\}$ **51.** $\varnothing$

**53.**

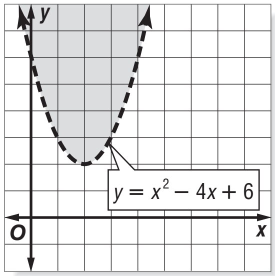



**55.** $-6$ **57.** $(2, 3, -1)$ **59.** A **61.** S **63.** Sample answer using $(0, 5.4)$ and $(25, 8.3)$: $y = 0.116x + 5.4$ **65.** 7 **67.** $2x + 2y$ **69.** $4x + 8$ **71.** $-5x + 10y$

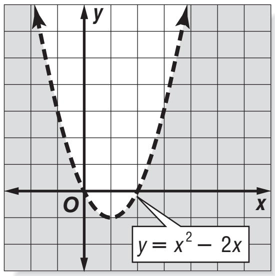
### *Pages 322–324* *Lesson 6-2*

**1.** yes, 1 **3.** no **5.** $-3x^2 - 7x + 8$ **7.** $10p^3q^2 - 6p^5q^3 + 8p^3q^5$ **9.** $x^2 + 9x + 18$ **11.** $4m^2 - 12mn + 9n^2$ **13.** $2x^3 - 9x^2 + 12x - 4$ **15.** yes, 2 **17.** no **19.** yes, 6 **21.** $4x^2 + 3x - 7$ **23.** $r^2 - r + 6$ **25.** $4b^2c - 4bdz$ **27.** $15a^3b^3 - 30a^4b^3 + 15a^5b^6$ **29.** $p^2 + 2p - 24$ **31.** $b^2 - 25$ **33.** $6x^2 + 34x + 48$ **35.** $27b^3 - 27b^2c + 9bc^2 - c^3$ **37.** $29.75 - 0.018x$ **39.** $5327.50 **41.** $-3x^3 - 16x^2 + 27x - 10$ **43.** $7x^2 - 8xy + 4y^2$ **45.** $2a^4 - 3a^3b + 4a^4b^4$ **47.** $xy^3 + y + \frac{1}{x}$ **49.** $2m^4 - 7m^2 - 15$ **51.** $1 + 8c + 16c^2$ **53.** Sample answer: $x^5 + x^4 + x^3$ **55.** 14; Sample answer: $(x^8 + 1)(x^6 + 1) = x^{14} + x^8 + x^6 + 1$ **57.** D **59.** $-64d^6$ **61.** $\frac{xz^2}{y^2}$

**63.**




**65.**




**67.** max.; 32 **69.** $\begin{bmatrix} -3 & 2 \\ 3 & -4 \\ -2 & 9 \end{bmatrix}$ **71.** $\begin{bmatrix} 29 & -8 \\ 8 & 9 \\ 16 & -16 \end{bmatrix}$

**73.** $y = \frac{2}{3}x - \frac{4}{3}$ **75.** $x^2$ **77.** $xy^2$

### *Pages 328–330* *Lesson 6-3*

**1.** $6y - 3 + 2x$ **3.** $-w + 16 + \frac{1000}{w}$ **5.** $3a^3 - 9a^2 + 7a - 6$ **7.** $x^2 - xy + y^2$ **9.** $b^3 + b - 1 + \frac{2}{b-2}$ **11.** $2y + 5$ **13.** $3ab - 6b^2$ **15.** $2c^2 - 3d + 4d^2$ **17.** $x^2$ **19.** $b^2 + 10b$ **21.** $y^2 - y - 1$ **23.** $t^4 + 2t^3 + 4t^2 + 5t + 10$ **25.** $2c^2 + c + 5 + \frac{6}{c-2}$ **27.** $x^4 - 3x^3 + 2x^2 - 6x + 19 - \frac{56}{x+3}$ **29.** $x^2 + x - 1 + \frac{2}{4x+1}$ **31.** $3t^2 - 2t + 3$ **33.** $x^3 - x - \frac{6}{2x+3}$ **35.** 5; Let $x$ be the number.

Multiplying by 4 results in $4x$. The sum of the number, 15, and the result of the multiplication is $x + 15 + 4x$ or $5x + 15$.

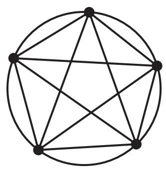
Dividing by the sum of the number and 3 gives $\frac{5x + 15}{x + 3}$ or 5. The end result is always 5.
**37.** about 2,423 subscriptions **39.** $x - 2s$
**41.** Sample answer: $(x^2 + x + 5) \div (x + 1)$ **43.** Jorge; Shelly is subtracting in the columns instead of adding. **45.** Division of polynomials can be used to solve for unknown quantities in geometric formulas that apply to manufacturing situations. $10x$ in. by $14x + 2f$ in. The area of a rectangle is equal to the length times the width. That is, $A = \ell w$. Substitute $140x^2 + 60x$ for $A$, $10x$ for $\ell$, and $14x + 2f$ for $w$. Solving for $f$ involves dividing $140x^2 + 60x$ by $10x$.

$$A = \ell w$$
$$140x^2 + 60x = 10x(14x + 2f)$$
$$\frac{140x^2 + 60x}{10x} = 14x + 2f$$
$$14x + 6 = 14x + 2f$$
$$6 = 2f$$
$$3 = f$$

The flaps are each 3 inches wide.
**47.** H **49.** $y^4z^4 - y^3z^3 + 3y^2z$ **51.** $a^2 - 2ab + b^2$ **53.** 20 **55.** $4a^2 - 10a + 6$

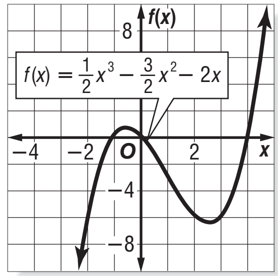
***Pages 335–338*** ***Lesson 6-4***

**1.** 6; 5 **3.** −21; 3 **5.** 109 lumens **7.** $100a^2 + 20$ **9. a.** $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; **b.** odd; **c.** 3 **11. a.** $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; **b.** odd; **c.** 1 **13.** 3; 1 **15.** No, this is not a polynomial because the term $\frac{1}{c}$ cannot be written in the form $c^n$, where $n$ is a nonnegative integer.
**17.** 3; −5 **19.** 12; 18 **21.** 1008; −36 **23.** $12a^2 - 8a + 20$ **25.** $12a^6 - 4a^3 + 5$ **27.** $3x^4 + 16x^2 + 26$ **29a.** $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; **29b.** even; **29c.** 4 **31a.** $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; **31b.** odd; **31c.** 5 **33a.** $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; **33b.** even; **33c.** 2 **35.** 10,345.5 joules
**37.** 86; 56 **39.** 7; 4 **41.** $-x^6 + x^3 + 2x^2 + 4x + 2$
**43.** odd **45.** Sample answer: Decrease; the graph appears to be turning at $x = 19$, indicating a maximum at that point. So attendance will decrease after 2005.
**47.** 16 regions;



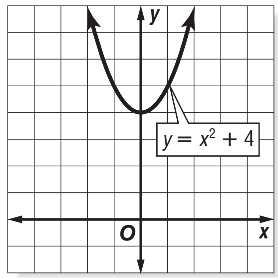
**49.** $4 = 4x^0$; $x = x^1$ **51.** Sometimes; a polynomial function with 4 real roots may be a sixth-degree polynomial function with 2 imaginary roots. A polynomial function that has 4 real roots is *at least* a fourth-degree polynomial. **53.** −1, 0, 4

**55.**

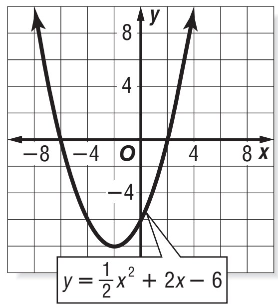



**57.** C **59.** $t^2 - 2t + 1$ **61.** $x^2 + 2$ **63.** $23{,}450(1 + p)$; $23{,}450(1 + p)^3$ **65.** $\left\{-\frac{7}{6}, \frac{5}{6}\right\}$ **67.** $|x| > 2$
**69.** $|x + 1| < 3$ **71.** Distributive

**73.**



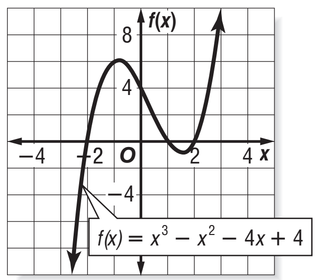

**75.**



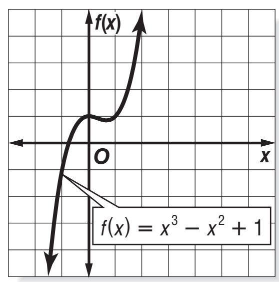

***Pages 343–345*** ***Lesson 6-5***

**1.**

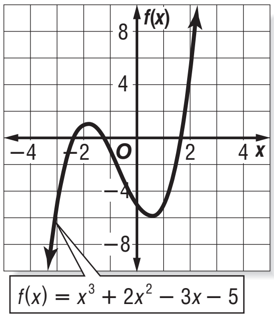
| $x$ | $f(x)$ |
|---|---|
| −3 | −20 |
| −2 | 0 |
| −1 | 6 |
| 0 | 4 |
| 1 | 0 |
| 2 | 0 |
| 3 | 10 |




**3.** between −1 and 0

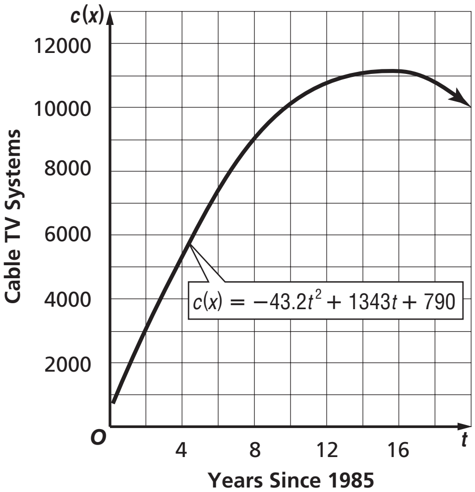


**5.**




Sample answer: rel. max. at $x \approx -2$, rel. min. at $x \approx 0.5$; domain: all real numbers, range: all real numbers

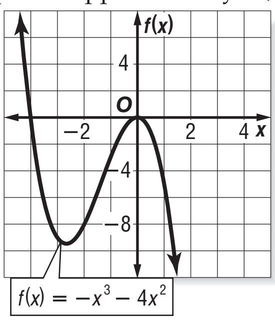
**7.**




**9.** The domain is all real numbers. The range is all real numbers less than or equal to approximately 11,225.

**11a.**

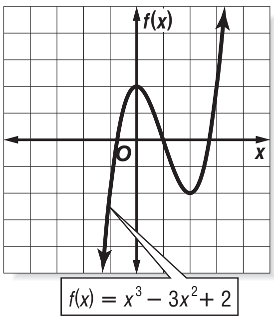
| $x$ | $f(x)$ |
|---|---|
| −5 | 25 |
| −4 | 0 |
| −3 | -9 |
| −2 | -8 |
| −1 | −3 |
| 0 | 0 |
| 1 | −5 |
| 2 | −24 |




**11b.** at $x = -4$ and $x = 0$ **11c.** Sample answer: rel. max. at $x \approx 0$, rel. min. at $x \approx -3$

**13a.**

| $x$ | $f(x)$ |
|---|---|
| −2 | −18 |
| −1 | −2 |
| 0 | 2 |
| 1 | 0 |
| 2 | −2 |
| 3 | 2 |
| 4 | 18 |

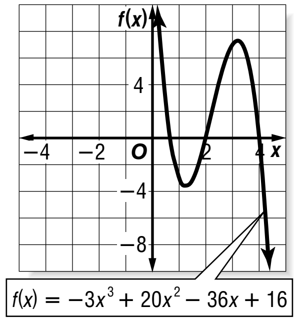



**13b.** at $x = 1$, between −1 and 0, and between 2 and 3

**13c.** Sample answer: rel. max. at $x \approx 0$, rel. min. at $x \approx 2$

**15a.**

| $x$ | $f(x)$ |
|---|---|
| −1 | 75 |
| 0 | 16 |
| 1 | −3 |
| 2 | 0 |
| 3 | 7 |
| 4 | 0 |
| 5 | −39 |

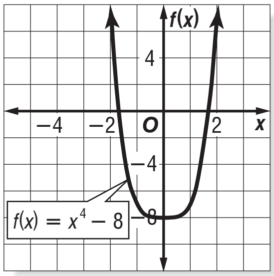



**15b.** between 0 and 1, at $x = 2$, and at $x = 4$

**15c.** Sample answer: rel. max. at $x \approx 3$, rel. min. at $x \approx 1$

**17a.**

| $x$ | $f(x)$ |
|---|---|
| −3 | 72 |
| −2 | 8 |
| −1 | −7 |
| 0 | −8 |
| 1 | −7 |
| 2 | 8 |
| 3 | 73 |



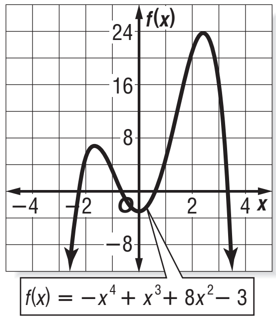

**17b.** between −2 and −1, and between 1 and 2

**17c.** Sample answer: no rel. max., rel. min. at $x = 0$

**19.** highest: 1982; lowest: 2000 **21.** 7 **23.** 0 s and about 5.3 s **25.** about 3.4 s

**27a.**

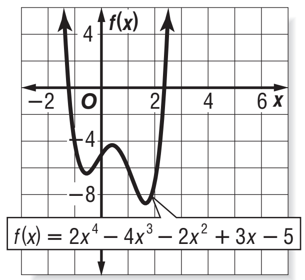
| $x$ | $f(x)$ |
|---|---|
| −3 | −39 |
| −2 | 5 |
| −1 | 3 |
| 0 | −3 |
| 1 | 5 |
| 2 | 21 |
| 3 | 15 |
| 4 | −67 |




**27b.** between −3 and −2, between −1 and 0, between 0 and 1, and between 3 and 4 **27c.** Sample answer: rel. max. at $x \approx -1.5$ and at $x \approx 2.5$, rel. min. at $x \approx 0$

**29a.**

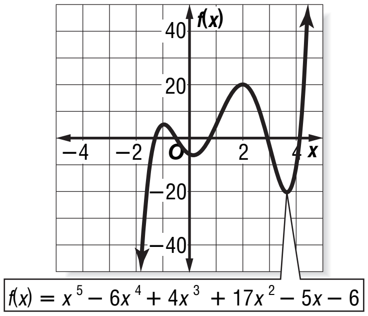
| $x$ | $f(x)$ |
|---|---|
| −2 | 45 |
| −1 | −4 |
| 0 | −5 |
| 1 | −6 |
| 2 | −7 |
| 3 | 40 |



**29b.** between −2 and −1, and between 2 and 3 **29c.** Sample answer: rel. max. at $x \approx 0.5$, rel. min. at $x \approx -0.5$ and at $x \approx 1.5$

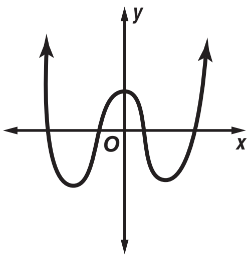
**31a.**

| x | f(x) |
|---|---|
| −2 | 88 |
| −1 | 5 |
| 0 | −6 |
| 1 | 5 |
| 2 | 20 |
| 3 | −3 |
| 4 | −10 |
| 5 | 269 |

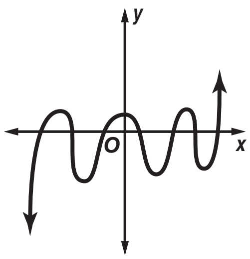



**b.** between −2 and −1, between −1 and 0, between 0 and 1, between 2 and 3, and between 4 and 5 **c.** Sample answer: rel. max. at $x \approx -1$ and at $x \approx 2$, rel. min. at $x \approx 0$ and at $x \approx 3.5$ **33.** The growth rate for both boys and girls increases steadily until age 18 and then begins to level off, with boys averaging a height of 71 in. and girls a height of 60 in. **35.** 3.41; 0.59 **37.** 0.52; −0.39, 1.62

**39.** Sample answer:




**41.** Sample answer:

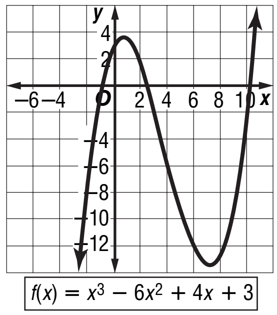



**43.** The turning points of a polynomial function that models a set of data can indicate fluctuations that may repeat. Polynomial functions best model data that contain turning points. To determine when the percentage of foreign-born citizens was at its highest, look for the rel. max. of the graph which is at about $t \approx 5$. The lowest percentage is found at $t \approx 74$, the rel. min. of the graph. **45.** H **47.** $10c^2 - 25c + 20$ **49.** $3x^3 - 10x^2 + 11x - 6$ **51.** $4x^4 - 9x^3 + 28x^2 - 33x + 20$ **53.** $x^3 + 9x^2 + 41x + 210 + \frac{1050}{x-5}$ **55.** $14x^2 + 26x - 4$ **57.** (−3, −2) **59.** (1, 3) **61.** 9 **63.** 4 **65.** 6

### *Pages 353–355 Lesson 6-6*

**1.** $-6x(2x + 1)$ **3.** $(x + 7)(3 - y)$ **5.** $(z - 6)(z + 2)$ **7.** $(4w + 13)(4w - 13)$ **9.** not possible **11.** −7, −1, 1, 7 **13.** 8 ft **15.** $6ab^2(a + 3b)$ **17.** prime **19.** $(3a + 1)(x - 5)$ **21.** $(2b - 1)(b + 7)$ **23.** $(t - 2)(t^2 + 2t + 4)$ **25.** not possible **27.** $b[7(b^2)^2 - 4(b^2) + 2)]$ **29.** $6\left(x^{\frac{1}{5}}\right)^2 - 4\left(x^{\frac{1}{5}}\right) - 16$ **31.** $-4, 4, -i, i$ **33.** $-4, 2 + 2i\sqrt{3}, 2 - 2i\sqrt{3}$ **35.** $\frac{3}{2}, \frac{-3 + 3i\sqrt{3}}{4}, \frac{-3 - 3i\sqrt{3}}{4}$ **37.** 3 in. × 3 in. **39.** $w = 4$ cm, $\ell = 8$ cm, $h = 2$ cm **41.** $(2y + 1)(y + 4)$ **43.** $(y^2 + z)(y^2 - z)$ **45.** $3(x + 3y)(x - 3y)$ **47.** $(a + 3b)(3a + 5)(a - 1)$ **49.** The height increased by 3, the width increased by 2, and the length increased by 4. **51.** yes **53.** no; $(2x + 1)(x - 3)$ **55.** Sample answer: $16x^4 - 12x^2 = 0$; $4[4(x^2)^2 - 3x^2] = 0$ **57.** Sample answer: If $a = 1$ and $b = 1$, then $a^2 + b^2 = 2$ but $(a + b)^2 = 4$. **59.** Solve the cubic equation $4x^3 + (-164x^2) + 1600x = 3600$ in order to determine the dimensions of the cut square if the desired volume is 3600 in$^3$. Solutions are 10 and $\frac{31 - \sqrt{601}}{2}$ in. There can be more than one square cut to produce the same volume because the height of the box is not specified and 3600 has many factors. **61.** G

**63.** Sample answer: rel. max. at $x \approx 0.5$, rel. min. at $x \approx 3.5$




**65.** 17; 27 **67.** $\frac{1715}{3}$; 135 **69.** yes **71.** $x^2 + 5x - 4$ **73.** $x^3 + 3x^2 - 2$

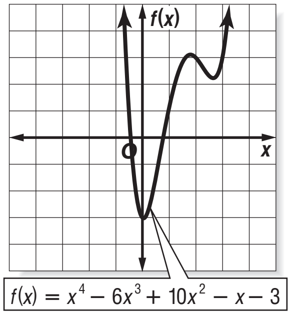
### *Pages 359–361 Lesson 6-7*

**1.** 7, −91 **3.** $3.236 billion **5.** Sample answer: Direct substitution, because it can be done quickly with a calculator. **7.** $x - 1, x + 2$ **9.** $x - 2, x^2 + 2x + 4$ **11.** 37, −19 **13.** 55, 272 **15.** 267, 680 **17.** 422, 3110 **19.** $x - 4, x + 2$ **21.** $x - 3, x - 1$ **23.** $x - 1, 3x + 4$ **25.** $x - 1, x + 6$ **27.** $2x - 3, 2x + 3, 4x^2 + 9$ **29.** $x - 2, x + 2, x^2 + 1$ **31.** $f(6) = 132.96$ ft/s. This means the boat is traveling at 132.96 ft/s when it passes the second buoy. **33.** Yes; 2-ft lengths; the binomial $x - 2$ is a factor of the polynomial since $f(2) = 0$. **35.** 8 **37.** −3 **39.** $16.70 **41.** No, he will still owe $4.40. **43.** dividend: $x^3 + 6x + 32$; divisor: $x + 2$; quotient: $x^2 - 2x + 10$; remainder: 12 **45.** Using the Remainder Theorem you can evaluate a polynomial for a value of $a$ by dividing the polynomial by $x - a$ using synthetic division. It is easier to use the Remainder Theorem when you have polynomials of degree 2 and lower or when you have access to a calculator. The estimated number of international travelers to the U.S. in 2006 is 65.9 million. **47.** G

**49.** $(a + 3)(b - 5)$ **51.** $(c - 6)(c^2 + 6c + 36)$

**53.**

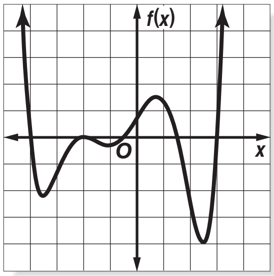
| $x$ | $f(x)$ |
|---|---|
| −1 | 15 |
| 0 | −3 |
| 1 | 1 |
| 2 | 3 |
| 3 | 3 |
| 4 | 25 |




**55.** $\frac{-7 \pm \sqrt{17}}{2}$ **57.** $\frac{-3 \pm i\sqrt{7}}{4}$

**Pages 366–368** **Lesson 6-8**

**1.** $2i$, $-2i$; 2 imaginary **3.** 2 or 0; 1; 2 or 0 **5.** $-4, 1 + 2i, 1 - 2i$ **7.** $2i, -2i, 3$ **9.** $f(x) = x^3 - 2x^2 + 16x - 32$ **11.** $-\frac{8}{3}$; 1 real **13.** $0, 3i, -3i$; 1 real, 2 imaginary **15.** $2, -2, 2i$, and $-2i$; 2 real, 2 imaginary **17.** 2 or 0; 1; 2 or 0 **19.** 3 or 1; 0; 2 or 0 **21.** 4, 2, or 0; 1; 4, 2, or 0 **23.** $-2, -2 + 3i, -2 - 3i$ **25.** $4 - i, 4 + i, -3$ **27.** $-\frac{3}{2}, 1 + 4i, 1 - 4i$ **29.** $2i, -2i, \frac{i}{2}, \frac{-i}{2}$ **31.** $3 - 2i, 3 + 2i, -1, 1$ **33.** $f(x) = x^3 - 2x^2 - 19x + 20$ **35.** $f(x) = x^4 + 7x^2 - 144$ **37.** $f(x) = x^3 - 11x^2 + 23x - 45$ **39.** 2 or 0; 1; 2 or 0 **41.** The company needs to produce no fewer than 4 and no more than 24 computers per day. **43.** radius = 4 m, height = 21 m **45.** 1 ft **47.** One root is a double root. Sample graph:




**49.** Sample answer: $f(x) = x^3 - 6x^2 + 5x + 12$ and $g(x) = 2x^3 - 12x^2 + 10x + 24$; each has zeros at $x = 4$, $x = -1$, and $x = 3$. **51.** C **53.** −127, 41 **55.** $5ab^2(3a - c^2)$ **57.** $4y(y + 3)^2$ **59.** $\pm\frac{1}{2}, \pm 1, \pm\frac{5}{2}, \pm 5$ **61.** $\pm\frac{1}{9}, \pm\frac{1}{3}, \pm 1, \pm 3$

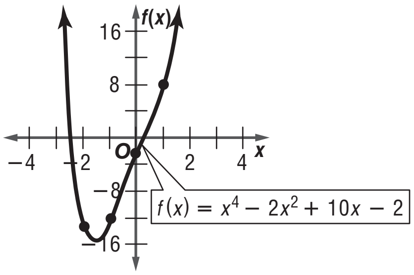
**Pages 371–373** **Lesson 6-9**

**1.** ±1, ±2, ±5, ±10 **3.** −4, 2, 7 **5.** 2, −2, 3, −3 **7.** 10 cm × 11 cm × 13 cm **9.** $2, -1, i, -i$ **11.** ±1, ±2, ±3, ±6 **13.** ±1, ±2, ±3, ±6, ±9, ±18 **15.** $\pm 1, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm 3, \pm 9, \pm 27$ **17.** −1, −1, 2 **19.** 0, 2, −2 **21.** −2, −4 **23.** $\frac{4}{5}, 0, \frac{5 \pm i\sqrt{3}}{2}$ **25.** −7, 1, 3 **27.** $-\frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}$ **29.** $3, \frac{2}{3}, -\frac{2}{3}, \frac{-3 \pm \sqrt{13}}{2}$ **31.** No; the dimensions of the space are $\ell = 36$ in., $w = 48$ in., $h = 32$ in., so the package is too tall to fit. **33.** $4, -5 \pm i\sqrt{15}$; 4 **35.** $V = \frac{1}{3}\ell^3 - 3\ell^2$ **37.** $\ell = 30$ in., $w = 30$ in., $h = 21$ in. **39.** $g$ **41.** Sample answer: $f(x) = 2x^2 - 8x + 3$ **43.** The polynomial equation that represents the volume of the compartment is $V = h^3 + 3h^2 - 40h$. Measures of the width of the compartment are, in inches, 1, 2, 3, 4, 6, 7, 9, 11, 12, 14, 18, 21, 22, 28, 33, 36, 42, 44, 63, 66, 77, and 84. The solution shows that $h = 14$ in., $\ell = 22$ in., and $w = 9$ in. **45.** G **47.** $-4, 2 + i, 2 - i$ **49.** $-7, 5 + 2i, 5 - 2i$ **51.** $x - 4, 3x^2 + 2$

**Pages 374–378** **Chapter 6** **Study Guide and Review**

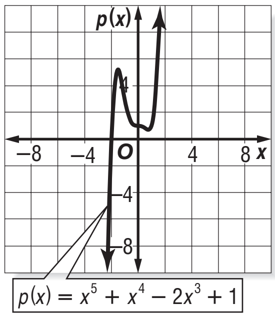
**1.** relative minimum **3.** quadratic form **5.** scientific notation **7.** depressed polynomial **9.** end behavior **11.** $\frac{1}{f^3}$ **13.** $8xy^4$ **15.** 10.48 times **17.** $4x^2 + 22x - 34$ **19.** $4a^4 + 24a^2 + 36$ **21.** $2x^3 + x - \frac{3}{x - 3}$ **23.** $16x^3 + 4x^2 - 12x + 8$ **25.** $8; -x - h + 4$ **27.** $21; x^2 + 2xh + h^2 + 5$ **29.** $-129; 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 1$

**31a.**



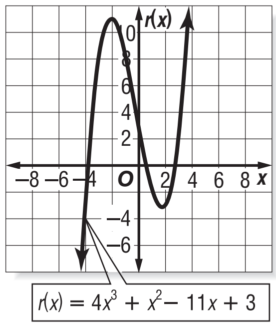

**31b.** at $x = 3$ **31c.** Sample answer: rel.max. at $x \approx -1.4$, rel. min. at $x \approx 1.4$

**33a.**




**33b.** between −2 and −3 **33c.** Sample answer: rel. max. at $x \approx -1.6$, rel. min. at $x \approx 0.8$

**35a.**




**35b.** between −2 and −1, between 0 and 1, and between 1 and 2 **35c.** Sample answer: rel. max. at $x \approx -1$, rel. min. at $x \approx 0.9$ **37.** 3: 2 rel. max. and 1 rel.

min. **39.** $(5w^2 + 3)(w - 4)$ **41.** prime **43.** $-8, 0, 5$ **45.** $4, -2 \pm 2i\sqrt{3}$ **47.** $4, -1$ **49.** $20, -20$ **51.** $x^2 + 2x + 3$ **53.** 3 or 1; 1; 2 or 0 **55.** 3 or 1; 1; 0 or 2 **57.** $(8 - x)(5 - x)(6 - x) = 72$ **59.** $-\frac{1}{2}, 3, 4$ **61.** $1, 2, 4, -3$ **63.** $\frac{1}{2}, 2$

# Chapter 7 Radical Equations and Inequalities

**Page 383 Chapter 7 Get Ready**

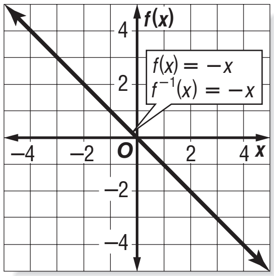
**1.** between 0 and 1, between 4 and 5 **3.** $3x + 4$
**5.** $170 - \dfrac{170}{t^2 + 1}$

**Pages 388–390 Lesson 7-1**

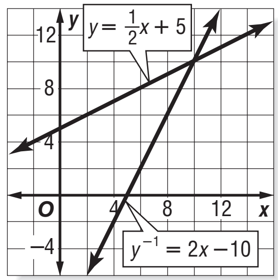
**1.** $4x + 9$; $2x - 1$; $3x^2 + 19x + 20$; $\dfrac{3x + 4}{x + 5}, x \neq -5$
**3.** $\{(-5, 7), (4, 9)\}$; $\{(4, 12)\}$ **5.** $6x - 8$; $6x - 4$ **7.** 30
**9.** 1 **11.** $\dfrac{3x}{4} - 5$; price of CD when 25% discount is taken and then the coupon is subtracted
**13.** Discount first, then coupon; $c[p(49.99)]$ gives a sale price of \$32.49, but $p[c(49.99)]$ gives \$33.74.
**15.** $6x + 6$; $-2x - 12$; $8x^2 + 6x - 27$; $\dfrac{2x - 3}{4x + 9}, x \neq -\dfrac{9}{4}$
**17.** $x^2 + 8x + 15$; $x^2 + 4x + 3$; $2x^3 + 18x^2 + 54x + 54$; $\dfrac{x + 3}{2}, x \neq -3$ **19.** $\dfrac{x^3 + x^2 - 7x - 15}{x + 2}, x \neq -2$; $\dfrac{x^3 + x^2 - 9x - 9}{x + 2}, x \neq -2$; $x^2 - 6x + 9, x \neq -2$; $x^2 + 4x + 4, x \neq -2, 3$ **21.** $(C - W)(x) = 2x^2 + 7x - 11$
**23.** $\{(2, 4), (4, 4)\}$; $\{(1, 5), (3, 3), (5, 3)\}$ **25.** $\{(4, 5), (2, 5), (6, 12), (8, 12)\}$; does not exist **27.** $\{(2, 3), (2, 2)\}$; $\{(-5, 6), (8, 6), (-9, -5)\}$ **29.** $15x - 5$; $15x + 1$
**31.** $3x^2 - 4$; $3x^2 - 24x + 48$ **33.** $2x^2 - 5x + 9$; $2x^2 - x + 5$ **35.** 50 **37.** 68 **39.** $-48$ **41.** 1.5
**43.** 104 **45.** 36 **47.** 1,085,000 **49.** $s[p(x)]$; The 30% would be taken off first, and then the sales tax would be calculated on this price. **51.** \$700, \$661.20, \$621.78, \$581.73, \$541.04 **53.** Danette is correct because $[g \circ f](x) = g[f(x)]$ which means you evaluate the $f$ function first and then the $g$ function. Marquan evaluated the functions in the wrong order. **55.** Using the revenue and cost functions, a new function that represents the profit is $p(x) = r(c(x))$. The benefit of combining two functions into one function is that there are fewer steps to compute and it is less confusing to the general population of people reading the formulas.
**57.** G **59.** $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm 6$
**61.** 2 or 0; 2 or 0; 4, 2, or 0 **63.** about 1830 times
**65.** $y = \dfrac{1 - 4x^2}{-5x}$ **67.** $t = \dfrac{I}{pr}$ **69.** $m = \dfrac{Fr^2}{GM}$

**Pages 394–396 Lesson 7-2**

**1.** $\{(4, 2), (1, -3), (8, 2)\}$
**3.** $f^{-1}(x) = -x$

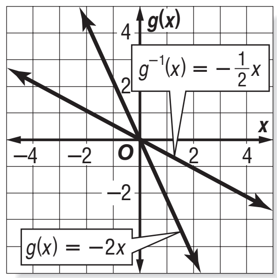



**5.** $y = 2x - 10$

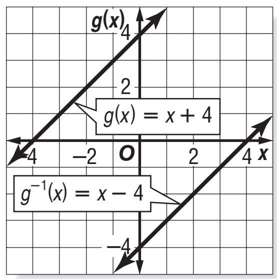



**7.** 15.24 m/s$^2$ **9.** no **11.** $\{(8, 3), (-2, 4), (-3, 5)\}$
**13.** $\{(-2, -1), (-2, -3), (-4, -1), (6, 0)\}$ **15.** $\{(8, 2), (5, -6), (2, 8), (-6, 5)\}$

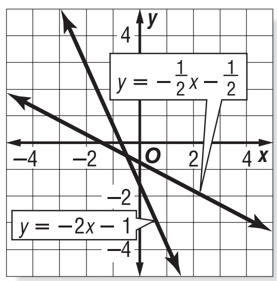
**17.** $g^{-1}(x) = -\frac{1}{2}x$



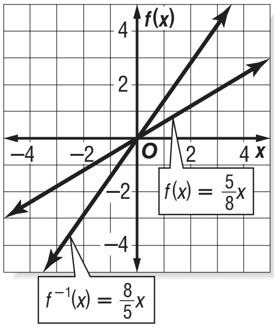

**19.** $g^{-1}(x) = x - 4$



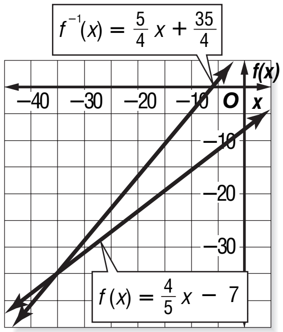

**21.** $y = -\frac{1}{2}x - \frac{1}{2}$



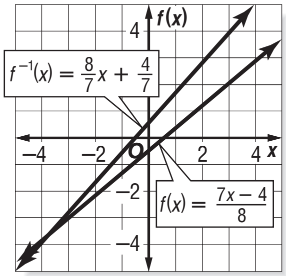
**23.** $f^{-1}(x) = \frac{8}{5}x$




**25.** $f^{-1}(x) = \frac{5}{4}x + \frac{35}{4}$




**27.** $f^{-1}(x) = \frac{8}{7}x + \frac{4}{7}$



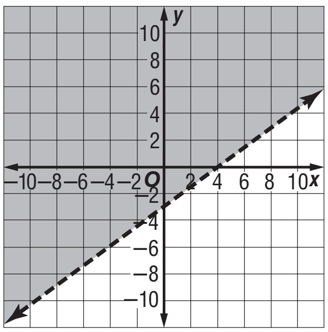

**29.** $\approx 3.39$ cm **31.** no **33.** yes **35.** yes
**37.** $y = 2x + 7$ **39.** $F^{-1}(x) = \frac{5}{9}(x - 32)$; $F[F^{-1}(x)] = F^{-1}[F(x)] = x$. **41.** $n$ is an odd whole number.
**43.** Sample answer: $f(x) = x$ and $f^{-1}(x) = x$ or $f(x) = -x$ and $f^{-1}(x) = -x$. **45.** A **47.** 6 **49.** 4

**51.** $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{5}{4}, \pm 2, \pm\frac{5}{2}, \pm 4, \pm 5, \pm 10, \pm 20$

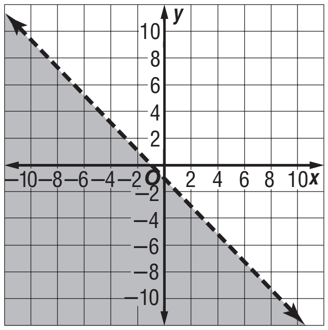
**53.** $\begin{bmatrix} 1 & 4 \\ -5 & -4 \end{bmatrix}$ **55.** consistent and independent

**57.** $-5$ **59.** $-2, 4$ **61.** $\{x \mid x > 6\}$

**63.**

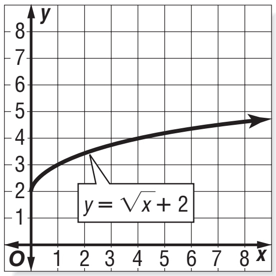


**65.**

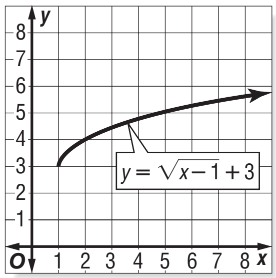


***Pages 399–401*** ***Lesson 7-3***

**1.** D: $x \geq 0$, R: $y \geq 2$

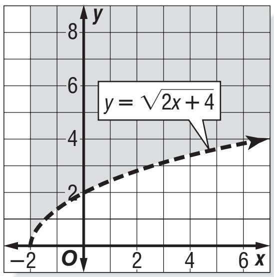



**3.** D: $x \geq 1$; R: $y \geq 3$

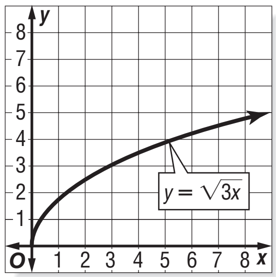



**5.** Yes; sample answer: the advertised pump will reach a maximum height of 87.9 ft.

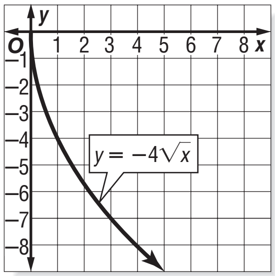
**7.**



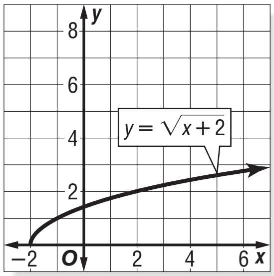

**9.** D: $x \geq 0$, R: $y \geq 0$



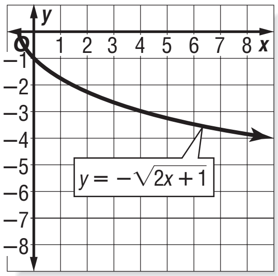
**11.** D: $x \geq 0$, R: $y \leq 0$



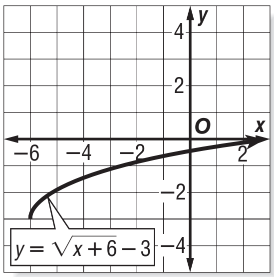

**13.** D: $x \geq -2$, R: $y \geq 0$



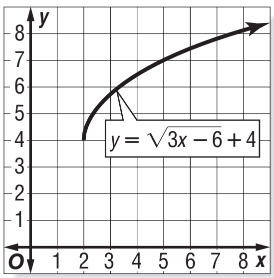

**15.** D: $x \geq -0.5$, R: $y \leq 0$




**17.** D: $x \geq -6$, R: $y \geq -3$

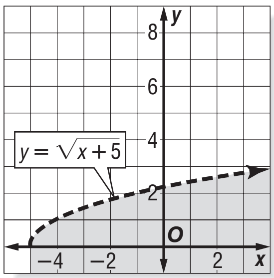



**19.** D: $x \geq 2$, R: $y \geq 4$

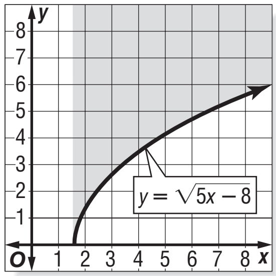



**21.** $h > 125$ ft **23.** ≈133.25 lb

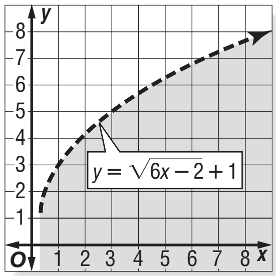
**25.**




**27.**




**29.**




**31.** If $a$ is negative, the graph is reflected over the $x$-axis. The larger the value of $a$, the less steep the graph. If $h$ is positive, the graph is translated to the right, and if $h$ is negative, the graph is translated to the left. When $k$ is positive, the graph is translated up, and when $k$ is negative, the graph is translated down.
**33.** Square root functions are used in bridge design because the engineers must determine what diameter of steel cable needs to be used to support a bridge based on its weight. Sample answer: when the weight to be supported is less than 8 tons; 13,608 tons **35.** G
**37.** no **39.** $(f + g)(x) = 2x + 2$; $(f - g)(x) = 2$;

$(f \cdot g)(x) = x^2 + 2x - 15$; $(f \div g)(x) = \dfrac{x+5}{x-3}$

**41.** $(f + g)(x) = \dfrac{8x^3 + 12x^2 - 18x - 26}{2x + 3}$;

$(f - g)(x) = \dfrac{8x^3 + 12x^2 - 18x - 28}{2x + 3}$;

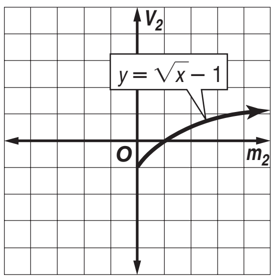
$(f \cdot g)(x) = 2x - 3$; $(f \div g)(x) = 8x^3 + 12x^2 - 18x - 27$

**43.** rational **45.** rational **47.** irrational

***Pages 405–406*** ***Lesson 7-4***

**1.** 4 **3.** −3 **5.** $x$ **7.** $6|a|b^2$ **9.** 8.775 **11.** 2.632
**13.** 15 **15.** not a real number **17.** −3 **19.** $\frac{1}{4}$ **21.** 0.5
**23.** $z^2$ **25.** $7|m^3|$ **27.** $3r$ **29.** $25g^2$ **31.** $5x^2|y^3|$
**33.** $13x^4y^2$ **35.** $2ab$ **37.** 11.358 **39.** 0.933 **41.** 3.893
**43.** 4.953 **45.** 4.004 **47.** 26.889 **49.** about $4088 \times 10^8$ m **51.** Sample answer: 64 **53.** $x = 0$ and $y \geq 0$, or $y = 0$ and $x \geq 0$ **55.** The radius and volume of a sphere can be related by an expression containing a cube root. As the value of V increases, the value of $r$ increases. **57.** G

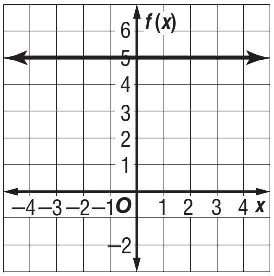
**59.**



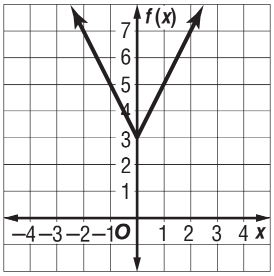

D = $\{x \mid x \geq 0\}$,
R = $\{y \mid y \geq -1\}$

**61.** no **63.** $29 - 28i$ **65.** $(3, -1)$ **67.** $(-4, 6)$

**69.**




**71.**



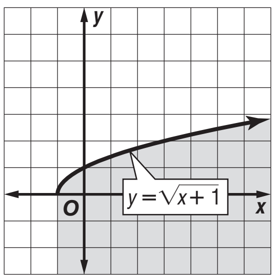

**73.** $y^2 + 3y - 10$ **75.** $a^2 + 3ab + 2b^2$
**77.** $6w^2 - 7wz - 5z^2$

### Pages 412–414 Lesson 7-5

**1.** $15\sqrt{7}$ **3.** $5|xy^3|\sqrt{3x}$ **5.** $\frac{|a^3|\sqrt{ab}}{|b^5|}$ **7.** $s = 2\sqrt{5\ell}$
**9.** $-24\sqrt{35}$ **11.** $\sqrt[3]{25}$ **13.** $22\sqrt[3]{2}$ **15.** 2 **17.** $\frac{19 - 7\sqrt{7}}{2}$
**19.** $6\sqrt{2}$ **21.** $2\sqrt[4]{6}$ **23.** $2y\sqrt[3]{2}$ **25.** $2|a|b^2\sqrt{10a}$
**27.** $4mn\sqrt[3]{3mn^2}$ **29.** $\frac{1}{2}wz\sqrt[5]{wz^2}$ **31.** $\frac{\sqrt[4]{54}}{3}$ **33.** $\frac{2r^4\mathrm{R}t}{|t^5|}$
**35.** $-60\sqrt{30}$ **37.** $40\sqrt{3}$ feet **39.** $5\sqrt{2}$ **41.** $4\sqrt{5} + 23\sqrt{6}$ **43.** $6 + 3\sqrt{6} + 2\sqrt{7} + \sqrt{42}$ **45.** $8 - 2\sqrt{15}$
**47.** $\frac{5\sqrt{6} - 3\sqrt{2}}{22}$ **49.** $\frac{12 + 7\sqrt{2}}{23}$ **51.** $\sqrt{x} + 1$ **53.** $\frac{\sqrt{10}}{5}$
**55.** 0 ft/s **57.** about 18.18 m **59.** Sample answer: $2\sqrt{2} + \sqrt{3} + 3\sqrt{27}$; Simplify the term $3\sqrt{27}$ to $9\sqrt{3}$. Then combine $\sqrt{3}$ and $9\sqrt{3}$. The simplified expression is $2\sqrt{2} + 10\sqrt{3}$.

**61.** The ratio of the lengths of the sides of the rectangle around the face is $\frac{2}{\sqrt{5} - 1}$. You can simplify this expression by multiplying the numerator and denominator by the conjugate of the denominator. The new expression is $\frac{\sqrt{5} + 1}{2}$. **63.** G **65.** $6ab^3$

**67.**




**69.** $-\frac{1}{2}\begin{bmatrix} 5 & -6 \\ -7 & 8 \end{bmatrix}$ **71.** does not exist **73.** $\frac{1}{2}$ **75.** $\frac{13}{12}$
**77.** $\frac{19}{30}$ **79.** $-\frac{5}{12}$

### Pages 419–421 Lesson 7-6

**1.** $\sqrt[3]{7}$ **3.** $26^{\frac{1}{4}}$ **5.** 5 **7.** 9 **9.** \$5.11 **11.** $x^{\frac{2}{3}}$ **13.** $\sqrt{3x}$
**15.** $\frac{x + 2x^{\frac{1}{2}} + 1}{x - 1}$ **17.** $\sqrt[5]{6}$ **19.** $\sqrt[5]{c^2}$ or $(\sqrt[5]{c})^2$ **21.** $23^{\frac{1}{2}}$
**23.** $2z^{\frac{1}{2}}$ **25.** 2 **27.** $\frac{1}{5}$ **29.** 81 **31.** $\frac{4}{3}$ **33.** about 4.62 in.
**35.** $y^4$ **37.** $b^{\frac{1}{5}}$ **39.** $\frac{w^{\frac{1}{5}}}{w}$ **41.** $\frac{a^{\frac{5}{12}}}{6a}$ **43.** $\sqrt{5}$ **45.** $17\sqrt[6]{17}$
**47.** $\frac{xy\sqrt{z}}{z}$ **49.** $2\sqrt{6} - 5$ **51.** $6r^{\frac{3}{4}}s^{\frac{3}{4}}$ **53.** $2^{\frac{3}{2}} + 3^{\frac{1}{2}}$

**55.** about 336 **57.** In radical form, the expression would be $\sqrt{-16}$, which is not a real number because the index is even and the radicand is negative.
**59.** Always; in exponential form $\sqrt[n]{b^m}$ equals $(b^m)^{\frac{1}{n}}$. By the Power of a Power Property, $(b^m)^{\frac{1}{n}} = b^{\frac{m}{n}}$. But, $b^{\frac{m}{n}}$ is also equal to $\left(\frac{1}{b^n}\right)^m$ by the Power of a Power Property. This last expression is equal to $\left(\sqrt[n]{b}\right)^m$. Thus, $\sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$. **61.** B **63.** $2|xy|\sqrt{x}$ **65.** $2\sqrt{2}$
**67.** $\sqrt{x - 5}$ **69.** $[K \circ C](F) = \frac{5}{9}(F - 32) + 273$
**71.** 2.5 s **73.** $2x - 3$ **75.** $4x - 12\sqrt{x} + 9$

### Pages 425–427 Lesson 7-7

**1.** 2 **3.** no solution **5.** 18 **7.** 9 **9.** $0 \le b < 4$ **11.** 16
**13.** no solution **15.** $-1$ **17.** no solution
**19.** no solution **21.** 3 **23.** 9 **25.** $-20$ **27.** $\frac{1}{3}$
**29.** about 1.82 ft **31.** $x > 1$ **33.** $x \le -11$
**35.** $0 \le x \le 2$ **37.** $b \ge 5$ **39.** 34 ft

**41.**
$$\frac{\sqrt{(x^2)^2}}{-x} = x$$
$$\frac{\cancel{\sqrt{}}(x^2)^{\cancel{2}}}{-x} = x$$
$$\frac{x^2}{-x} = x$$
$$x^2 = (x)(-x)$$
$x^2 \neq -x^2$, Never.
**43.** Sample answer: $\sqrt{x} + \sqrt{x + 3} = 3$
**45.** If a company's cost and number of units manufactured are related by an equation involving radicals or rational exponents, then the production level associated with given cost can be found by solving a radical equation.

| | | |
|---|---|---|
| $C = 10\sqrt[3]{n^2} + 1500$ | | |
| $10{,}000 = 10n^{\frac{2}{3}} + 1500$ | $C = 10{,}000$ | |
| $8500 = 10n^{\frac{2}{3}}$ | Subtract 1500 from each side. | |
| $850 = n^{\frac{2}{3}}$ | Divide each side by 10. | |
| $850^{\frac{3}{2}} = n$ | Raise each side to the $\frac{3}{2}$ power. | |
| $24{,}781.55 \approx n$ | Use a calculator. | |

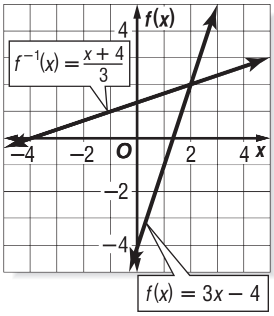
Round down so that the cost does not exceed \$10,000. The company can make at most 24,781 chips. **47.** G

**49.** $5^{\frac{3}{7}}$ **51.** $(x^2 + 1)^{\frac{2}{3}}$ **53.** $\frac{\sqrt[3]{100}}{10}$ **55.** $I(m) = 320 + 0.04m$; \$4500 **57.** $(f + g)(x) = x^2 + x - 2$; $(f - g)(x) = x^2 - x - 6$; $(f \cdot g)(x) = x^3 + 2x^2 - 4x - 8$; $(f \div g)(x) = x - 2$; $x \neq -2$
**59.** 4; If $x$ is your number, you can write the expression $\frac{3x + x + 8}{x + 2}$, which equals 4 after dividing the numerator and denominator by the GCF, $x + 2$.
**61.** $6p^2 - 2p - 20$ **63.** Sample answer: $y = 0.79x + 4.93$

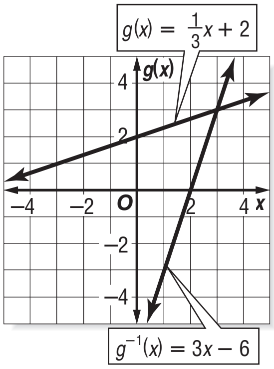
### *Pages 430–434* *Chapter 7* *Study Guide and Review*

**1.** radical equation **3.** like radical expressions **5.** inverse functions **7.** one-to-one **9.** inverse relations **11.** $x^2 - 1$; $x^2 - 6x + 11$ **13.** $-15x - 5$; $-15x + 25$ **15.** $|x + 4|$; $|x| + 4$

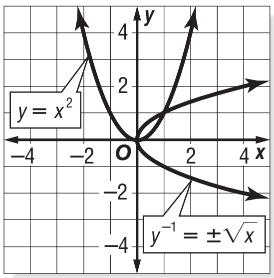
**17.** $f^{-1}(x) = \frac{x + 4}{3}$




**19.** $g^{-1}(x) = 3x - 6$

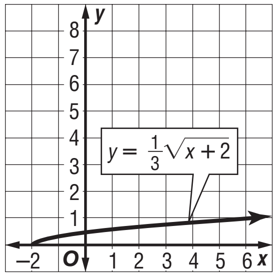



**21.** $y^{-1}(x) = \pm\sqrt{x}$

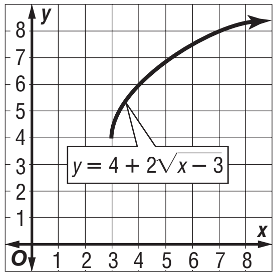



**23.** $I(m) = 400 + 0.1m$; \$6000

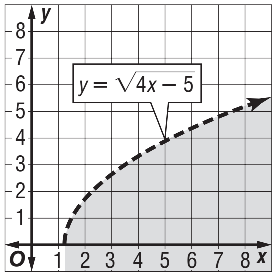
**25.** D: $x \geq -2$, R: $y \geq 0$




**27.** D: $x \geq 3$, R: $y \geq 4$




**29.**




**31.** $\pm16$ **33.** 8 **35.** $|x^4 - 3|$ **37.** $2m^2$ **39.** 10 meters per second **41.** $-5\sqrt{3}$ **43.** $20 + 8\sqrt{6}$ **45.** 9

**47.** $\frac{2\sqrt{10} - \sqrt{5}}{7}$ **49.** $\frac{1}{9}$ **51.** $\frac{9}{4}$ **53.** $\frac{xyz^{\frac{2}{3}}}{z}$ **55.** 6.3 amps

**57.** 343 **59.** 4 **61.** 5 **63.** 8 **65.** $x > \frac{11}{5}$ **67.** $x \geq 21$

**69.** $d > -\frac{3}{4}$ **71.** 1 m

# Chapter 8 Rational Expressions and Equations

### *Page 441* *Chapter 8* *Get Ready*

**1.** $\frac{1}{6}$ **3.** $\frac{5}{8}$ **5.** 16 **7.** $2\frac{1}{2}$ **9.** \$17.50 **11.** 12 **13.** 15 **15.** 15 **17.** 6 **19.** $7\frac{1}{2}$ **21.** \$5250

### *Pages 446–449* *Lesson 8-1*

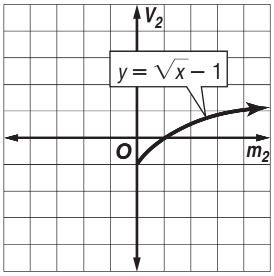
**1.** $\frac{9m}{4n^4}$ **3.** $x + 3$ **5.** D **7.** $\frac{-b^2 - ab - a^2}{a + b}$ **9.** $\frac{6}{5}$
**11.** $\frac{8y}{9x^2}$ **13.** $\frac{3x + 9}{4x + 24}$ **15.** $\frac{2y(y - 2)}{3(y + 2)}$ **17.** $-\frac{n}{7m}$ **19.** $\frac{1}{2}$
**21.** $-\frac{t + 3}{t + 4}$ **23.** $-\frac{4bc}{27a}$ **25.** $-2p^2$ **27.** $\frac{4}{3}$ **29.** $\frac{p - 7}{p + 7}$
**31.** $-2p$ **33.** $\frac{2x + y}{2x - y}$ **35.** $d = -2, -1$, or 2

**37.** $\frac{1}{2}(8x^2 + 18x - 5)\ \text{m}^2$ **39.** $\frac{s}{3}$ **41.** $\frac{y+2}{3y-1}$
**43.** $\frac{3x+4}{3(x+2)}$ **45.** $-\frac{b-3}{b+3}$ **47.** $\frac{5by}{3ax}$ **49.** $\frac{xz}{8y}$
**51.** $\frac{3(r+4)}{r+3}$ **53.** $a = -b$ or $b$ **55.** $\frac{5422+m}{12{,}138+a}$

**57.** $\frac{25}{27}$; Sample answer: the second airplane travels a bit further than the first airplane. **59.** $-3x + 2$; The expression defines the function $g(x)$. **61.** The tables are the same except for $f(x)$ the value $f(0)$ is undefined.

**63.** Sample answer: $\frac{x-4}{2}$, $\frac{3x-12}{6}$ **65.** $\frac{x+1}{\sqrt{x+3}}$ does not belong with the other three. The other three expressions are rational expressions. Since the denominator of $\frac{x+1}{\sqrt{x+3}}$ is not a polynomial, $\frac{x+1}{\sqrt{x+3}}$ is not a rational expression. **67.** A rational expression can be used to express the fraction of a nut mixture that is peanuts. The expression $\frac{8+x}{13+x+y}$ could be used to represent the fraction that is peanuts if $x$ pounds of peanuts and $y$ pounds of cashews were added to the original mixture. **69.** F

**71.**




D = $\{x \mid x \geq 0\}$,
R = $\{y \mid y \geq -1\}$

**73.** no **75.** odd; 3 **77.** about $4.99 \times 10^2$ s or about 8 min 19 s

**79.** $\left\{-\frac{1}{6}, \frac{1}{3}\right\}$ **81.** $\varnothing$ **83.** $-1\frac{1}{9}$ **85.** $1\frac{4}{15}$ **87.** $-\frac{11}{18}$

### Pages 453–456 Lesson 8-2

**1.** $12x^2y^2$ **3.** $x(x-2)(x+2)$ **5.** $\frac{2-x^3}{x^2y}$ **7.** $\frac{37}{42m}$

**9.** $\frac{5d+16}{(d+2)^2}$ **11.** $\frac{x^2-2x+1}{(x+2)(x-2)}$ **13.** $\frac{8}{5}$ **15.** $\frac{2}{x+2}$

**17.** $4x + 8$ **19.** $180x^2yz$ **21.** $x^2(x-y)(x+y)$

**23.** $\frac{31}{12v}$ **25.** $\frac{25-7ab}{5a^2b}$ **27.** $\frac{a+3}{a-4}$ **29.** $\frac{y(y-9)}{(y+3)(y-3)}$

**31.** $\frac{-8d+20}{(d-4)(d+4)(d-2)}$ **33.** $-1$ **35.** $-1$ **37.** $36p^3q^4$

**39.** $(n-4)(n-3)(n+2)$ **41.** $\frac{2x+15y}{3y}$ **43.** $\frac{110w-423}{90w}$

**45.** $\frac{x^2-6}{(x+2)^2(x+3)}$ **47.** $\frac{2y^2+y-4}{(y-1)(y-2)}$ **49.** $\frac{a+7}{a+2}$

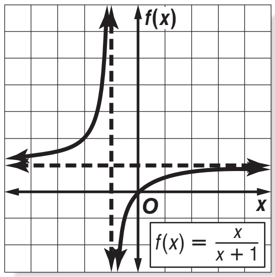
**51.** $\frac{3x-4}{2x(x-2)}$ **53.** $\frac{24}{x}$ h **55.** $\frac{48(x-2)}{x(x-4)}$ h **57.** Sample answer: $d^2 - d$, $d + 1$ **59.** Sample answer: $\frac{1}{x+1}$, $\frac{1}{x-2}$

**61.** Subtraction of rational expressions can be used to determine the distance between the lens and the film if the focal length of the lens and the distance between the lens and the object are known. $\frac{1}{q} = \frac{1}{50} - \frac{1}{1000}$ could be used to determine the distance between the lens and the film if the focal length of the lens is 50 mm and the distance between the lens and the object is 1000 mm. **63.** F **65.** $\frac{a(a+2)}{a+1}$ **67.** $\pm i$, $\pm 3$

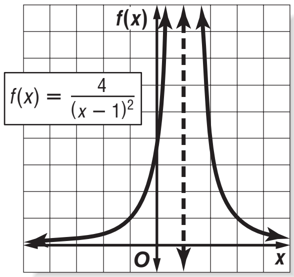
**69.** 5.0 ft **71.** $(x+1)(x+2)$ **73.** $(x+12)(x-1)$
**75.** $3(x-5)(x+5)$

### Pages 460–463 Lesson 8-3

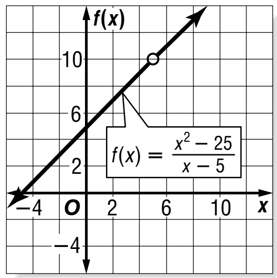
**1.** asymptote: $x = 2$ **3.**



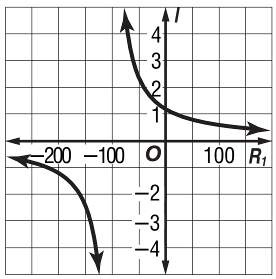

**5.**




**7.**

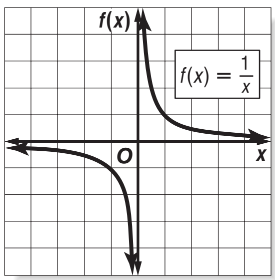



**9.**

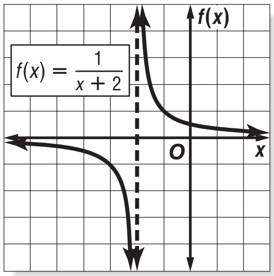



**11.** 0.5 amperes
**13.** asymptotes: $x = 2$, $x = 3$ **15.** asymptote: $x = -4$; hole: $x = -3$

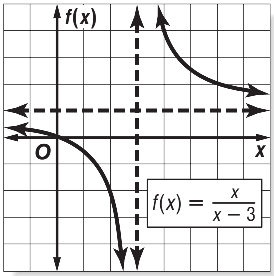
**17.**



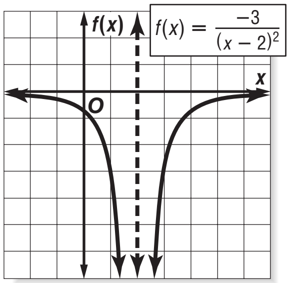
**19.**



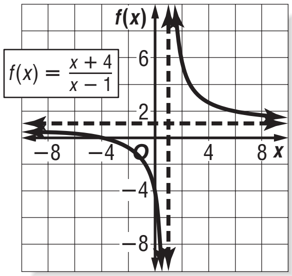

**21.**



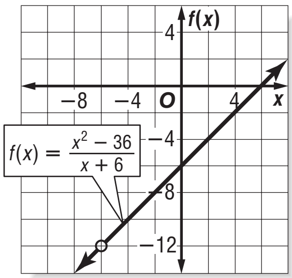

**23.**



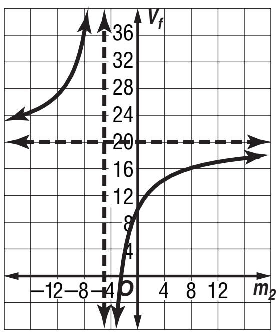

**25.**




**27.**

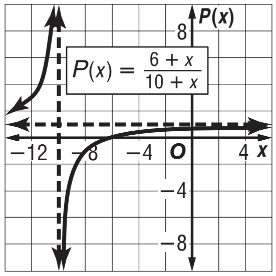



**29.**



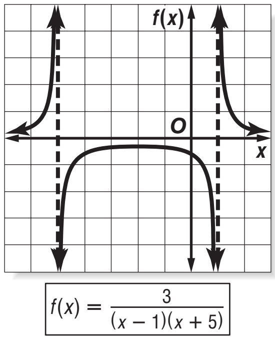

**31.** $m_2 = -5$, $V_f = 20$; $-2.5$; $10$

**33.**

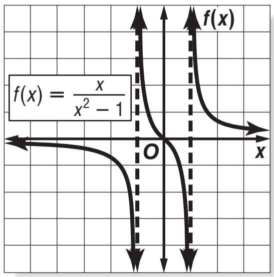



**35.** It represents her original free-throw percentage of 60%. **37.** hole: $x = 4$

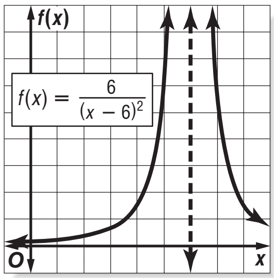
**39.**



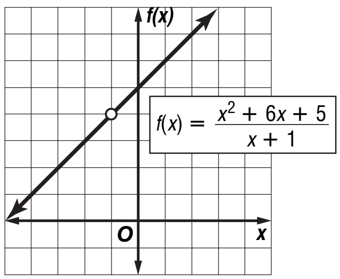

**41.**



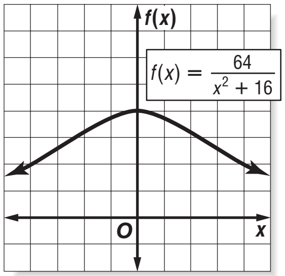

**43.**




**45.**



**47.**



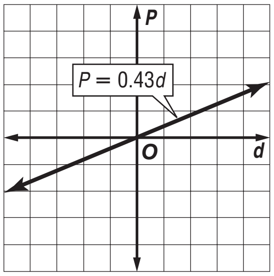

**49.** Since $\frac{-64}{x^2+16} = -\left(\frac{64}{x^2+16}\right)$, the graph of $f(x) = \frac{-64}{x^2+16}$ is the reflection image of the graph of $f(x) = \frac{64}{x^2+16}$ over the $x$-axis. **51.** Each of the graphs is a straight line passing through $(-5, 0)$ and $(0, 5)$. However, the graph of $f(x) = \frac{(x-1)(x+5)}{x-1}$ has a hole at $(1, 6)$, and the graph of $g(x) = x + 5$ does not have a hole. **53.** Sample answers: $f(x) = \frac{x+2}{(x+2)(x-3)}$, $f(x) = \frac{2(x+2)}{(x+2)(x-3)}$, $f(x) = \frac{5(x+2)}{(x+2)(x-3)}$ **55.** A **57.** $\frac{3m+4}{m+n}$ **59.** $\frac{5(w-2)}{(w+3)^2}$ **61.** $-\frac{1}{2}$, 2, 3 **63.** $\{-4 \pm 2i\}$ **65.** $\left\{\frac{-7 \pm 3\sqrt{13}}{2}\right\}$ **67.** 4.5 **69.** 20

***Pages 468–471*** ***Lesson 8-4***

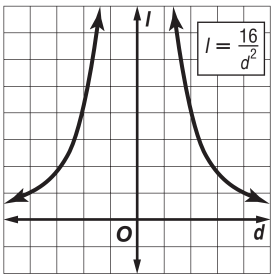
**1.** 24 **3.** −8 **5.** 25.8 psi

**7.**

| Depth (ft) | Pressure (psi) |
|---|---|
| 0 | 0 |
| 1 | 0.43 |
| 2 | 0.86 |
| 3 | 1.29 |
| 4 | 1.72 |




**9.** 20 **11.** 64 **13.** 4 **15.** about 359.6 mi **17.** direct; 3 **19.** joint; $\frac{1}{3}$ **21.** inverse; 2.5 **23.** direct; −7 **25.** −12.6 **27.** $2\frac{1}{4}$ **29.** 1.25 **31.** $V = \frac{k}{P}$ **33.** $\ell = 15md$ **37.** joint **39.** 30 mph **43.**

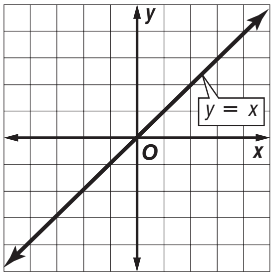



**45.** about $2 \times 10^{20}$ Newtons **47.** $6.67 \times 10^{-3}$ Newtons **49.** Sample answer: If the average student spends \$2.50 for lunch in the school cafeteria, write an equation to represent the amount $s$ students will spend for lunch in $d$ days. How much will 30 students spend in a week? $a = 2.50sd$; \$375 **51.** C **53.** asymptote: $x = 1$; hole: $x = -1$ **55.** hole: $x = -3$ **57.** $\frac{t^2 - 2t - 2}{(t+2)(t-2)}$ **59.** $1 \times 10^{14}$ **61.** 3; 7 **63.** C **65.** S **67.** A

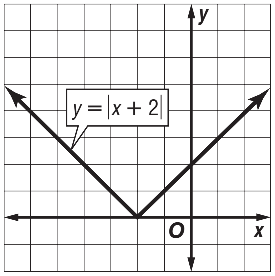
***Pages 476–478*** ***Lesson 8-5***

**1.** greatest integer **3.** constant **5.** b

**7.** direct variation

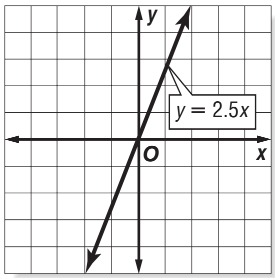



**9.** absolute value

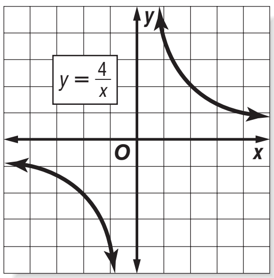



**11.** square root **13.** direct variation **15.** constant

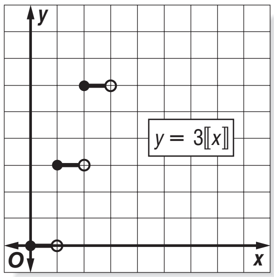
**17.** direct variation



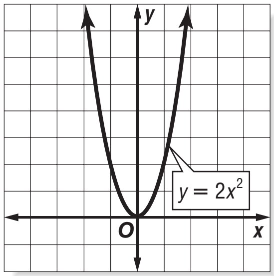

**19.** inverse variation or rational




**21.** greatest integer

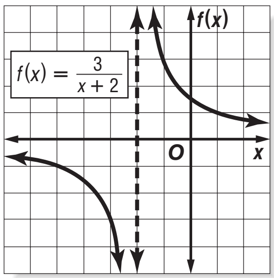


**23.** quadratic

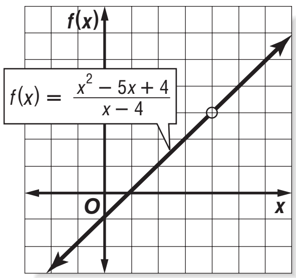



**25.** e **27.** a **29.** direct variation **31.** parabola **33.** The graph is similar to the graph of the greatest integer function because both graphs look like a series of steps. In the graph of the postage rates, the solid dots are on the right and the circles are on the left. However, in the greatest integer function, the circles are on the right and the solid dots are on the left. **35a.** absolute value **35b.** quadratic **35c.** greatest integer **35d.** square root **37.** There are several types of functions. Each type of function has features which distinguish it from other types. Knowing which features are characteristic of each type of graph can help you determine which type of function best describes the relationship between two quantities. **39.** G

**41.**




**43.**




**45.** $-7, \frac{3}{2}$ **47.** 120 m **49.** $45x^3y^3$ **51.** $3(x - y)(x + y)$ **53.** $(t - 5)(t + 6)(2t + 1)$

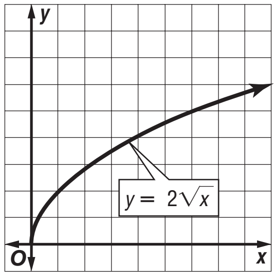
***Pages 484–486*** ***Lesson 8-6***

**1.** 3 **3.** $\frac{2}{3}$ **5.** 3 **7.** $2\frac{2}{9}$ h; The answer is reasonable. The time to complete the job when working together must be less than the time it would take either person working alone. **9.** $v < 0$ or $v > 1\frac{1}{6}$ **11.** $-\frac{4}{3}$ **13.** $-3, 2$ **15.** 2 **17.** $\varnothing$ **19.** $-1 < m < 1$ **21.** $0 < b < 1$

**23.** 2 or 4 **25.** $\frac{7}{3}$ **27.** $\frac{1 \pm \sqrt{145}}{4}$ **29.** $p < 0$ or $p > 2\frac{1}{2}$ **31.** 4.8 cm/g **33.** 15 km/h; With the wind, Alfonso's speed would be 18 km/h, and his 36-km trip would take 2 hours. Against the wind, his speed would be 12 km/h, and his 24-km trip would take 2 hours. The answer makes sense. **35.** $\{x \mid x < -2 \text{ or } x > 1\}$

**37.** $\frac{80}{13}$ **39.** Jeff; when Dustin multiplied by $3a$, he forgot to multiply the 2 by $3a$. **41.** If something has a general fee and cost per unit, rational equations can be used to determine how many units a person must buy in order for the actual unit price to be a given number. Since the cost is \$1.00 per download plus \$15.00 per month, the actual cost per download could never be \$1.00 or less. **43.** J

**45.** square root




**47.** 36 **49.** $\{x \mid 0 \le x \le 4\}$ **51.** $\begin{bmatrix} -25 & 23 & -54 \\ 66 & -26 & 57 \end{bmatrix}$

**53.** 196 beats per min **55.** 12

***Pages 489–192*** ***Chapter 8*** ***Study Guide and Review***

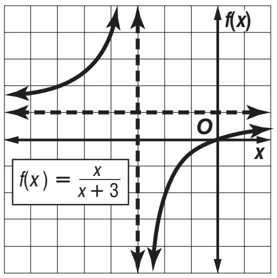
**1.** false; point discontinuity **3.** false; rational

**5.** true **7.** true **9.** $-\frac{4bc}{33a}$ **11.** $\frac{x+2}{x-3}$ **13.** $(y + 3)(y - 6)$

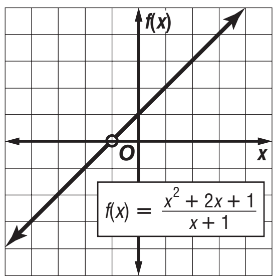
**15.** $4(x + 4)$ m **17.** $\frac{7}{5(x+1)}$ **19.** $\frac{18}{y-2}$

**21.** $\frac{3(3m^2 - 14m + 27)}{(m+3)(m-3)^2}$ **23.** $\approx 5.8$

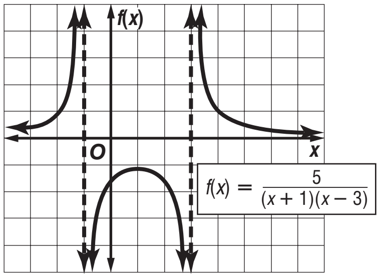
**25.**




**27.**



**29.**



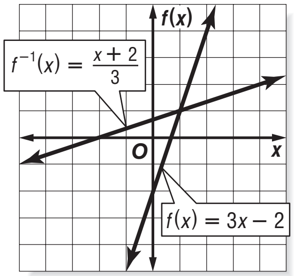

**31.** $-1\frac{2}{3}$ **33.** $\frac{32}{121}$ **35.** 17.6 **37.** square root **39.** $1\frac{1}{9}$
**41.** 0 **43.** $-2\frac{1}{2} < b < 0$

## Chapter 9 Exponential and Logarithmic Relations

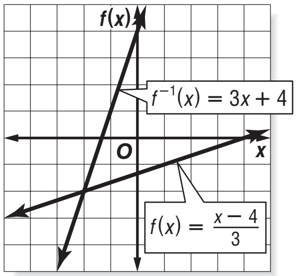
***Page 497*** ***Chapter 9*** ***Get Ready***

**1.** $x^{12}$ **3.** $-\frac{12x^3}{7y^5z}$ **5.** $\approx 1.9 \times 10^4$ kg/m$^3$

**7.** $f^{-1}(x) = \frac{x + 2}{3}$



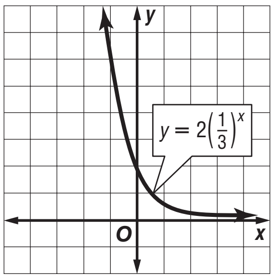

**9.** $f^{-1}(x) = 3x + 4$




**11.** \$275.77

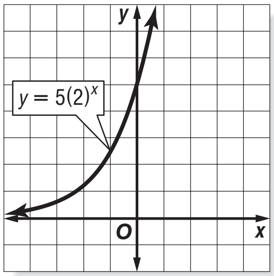
***Pages 503–506*** ***Lesson 9-1***

**1.** c **3.** b **5.** D = all real numbers, R = $\{y|y > 0\}$

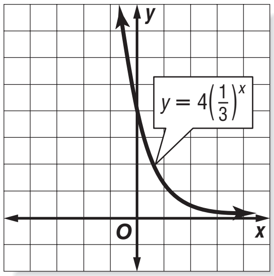



**7.** growth **9.** $y = -18(3)^x$ **11.** about \$2,578,760; Yes, the money is continuing to grow at a faster rate each year. In the first 10 years it grew by \$678,000, and in the next ten years it grew about \$900,000. **13.** 2 **15.** $x \leq 0$ **17.** $a \leq -3$

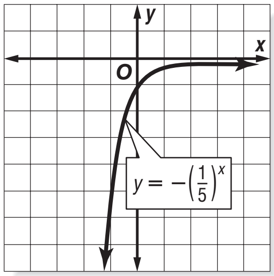
**19.** D = all real numbers, R = $\{y|y > 0\}$




**21.** D = all real numbers, R = $\{y|y > 0\}$

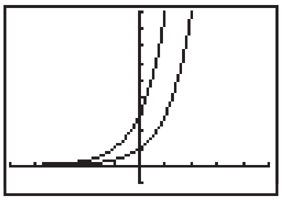



**23.** growth **25.** growth **27.** decay **29.** $y = 3(5)^x$ **31.** $y = -5\left(\frac{1}{3}\right)^x$ **33.** $y = -0.3(2)^x$ **35.** about 1,008,290 **37.** $A(t) = 1000(1.01)^{4t}$ **39.** $\frac{2}{3}$ **41.** $-\frac{8}{3}$ **43.** $\frac{5}{3}$ **45.** $n > 5$ **47.** $n < 3$ **49.** D = all real numbers. R = $\{y|y > 0\}$




**51.** $s \cdot 4^x$ **53.** $y = 3.93(1.35)^x$ **55.** 2144.87 million; 281.42 million; No, the growth rate has slowed considerably. The population in 2000 was much smaller than the equation predicts it would be.

**57.**



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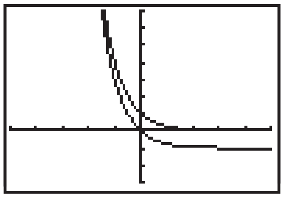
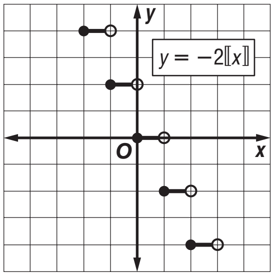
The graphs have the same shape. The graph of $y = 3^{x+1}$ is the graph of $y = 3^x$ translated one unit to the left. The asymptote for the graphs $y = 3^x$ and for $y = 3^{x+1}$ is the line $y = 0$. The graphs have the same domain, all real numbers, and range, $y > 0$. The $y$-intercept of the graph of $y = 3^x$ is 1 and of the graph of $y = 3^{x+1}$ is 3.

**59.** 

[−5, 5] scl: 1 by [−3, 7] scl: 1

The graphs have the same shape. The graph of $y = \left(\frac{1}{4}\right)^x - 1$ is the graph of $y = \left(\frac{1}{4}\right)^x$ translated one unit down. The asymptote for the graph of $y = \left(\frac{1}{4}\right)^x$ is the line $y = 0$ and for the graph of $y = \left(\frac{1}{4}\right)^x - 1$ is the line $y = -1$. The graphs have the same domain, all real numbers, but the range of $y = \left(\frac{1}{4}\right)^x$ is $y > 0$ and of $y = \left(\frac{1}{4}\right)^x - 1$ is $y > -1$. The $y$-intercept of the graph of $y = \left(\frac{1}{4}\right)^x$ is 1 and for the graph of $y = \left(\frac{1}{4}\right)^x - 1$ is 0.
**61.** Sample answer: 0.8 **63.** Sometimes; true when $b > 1$, but false when $b < 1$. **65.** A **67.** 1, 15 **69.** $-\frac{13}{3}$, 3

**71.** greatest integer



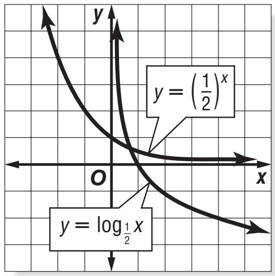

**73.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **75.** $\frac{1}{51}\begin{bmatrix} 3 & -6 \\ 11 & -5 \end{bmatrix}$ **77.** $g[h(x)] = 2x - 6$; $h[g(x)] = 2x - 11$ **79.** $g[h(x)] = -2x - 2$; $h[g(x)] = -2x + 11$

## Pages 514–517 Lesson 9-2

**1.** $\log_5 625 = 4$ **3.** $\log_3 243 = 5$ **5.** $36^{\frac{1}{2}} = 6$ **7.** 4 **9.** 3
**11.** 1000 **13.** $10^{13}$ **15.** $10^{5.5}$ or about 316,228 times
**17.** $\{x \mid \frac{1}{2} < x \leq 5\}$ **19.** $\frac{1}{2}$, 1 **21.** $x > 6$ **23.** $5^3 = 125$
**25.** $4^{-1} = \frac{1}{4}$ **27.** $8^{\frac{2}{3}} = 4$ **29.** $\log_8 512 = 3$
**31.** $\log_5 \frac{1}{125} = -3$ **33.** $\log_{100} 10 = \frac{1}{2}$ **35.** 4 **37.** $\frac{1}{2}$
**39.** −5 **41.** −3 **43.** $3x$ **45.** 125 **47.** ±3 **49.** 11 **51.** $10^{10.67}$
**53.** $0 < y \leq 8$ **55.** $x \geq 24$ **57.** ±8 **59.** $a > 3$

**61.**

| | |
|---|---|
| $\log_{16} 2 \cdot \log_2 16 \stackrel{?}{=} 1$ | Original equation |
| $\log_{16} 16^{\frac{1}{4}} \cdot \log_2 2^4 \stackrel{?}{=} 1$ | $2 = 16^{\frac{1}{4}}$ and $16 = 2^4$ |
| $\frac{1}{4}(4) \stackrel{?}{=} 1$ | Inverse Property of Exponents and Logarithms |
| $1 = 1$ ✓ | |

**63.** 


The graphs are reflections of each other over the line $y = x$. **65.** $10^3$ or about 1000 times as great
**67.** $10^{1.7}$ or about 50 times **69.** D = $\{x \mid x > 0\}$, D = $\{x \mid x > 0\}$, D = $\{x \mid x > 1\}$, D = $\{x \mid x > -2\}$, respectively; R = {all reals} **71.** $\log_2 16 = 4$; all other choices are equal to 2 **73.** All powers of 1 are 1, so the inverse of $y = 1^x$ is not a function. **75.** B **77.** $x^{2\sqrt{6}}$
**79.** ∅ **81.** $\pm\frac{7}{3}$ **83.** \$4000, CD; \$4000, savings **85.** $8a^6b^3$
**87.** 1

## Pages 524–526 Lesson 9-3

**1.** 2.6309 **3.** 1.1403 **5.** Mt. Everest: 26,855.44 pascals; Mt. Trisuli: 34,963.34 pascals; Mt. Bonete: 36,028.42 pascals; Mt. McKinley: 39,846.22 pascals; Mt. Logan: 41,261.82 pascals **7.** 2.5840 **9.** 2 **11.** 4 **13.** 2.1133
**15.** −0.2519 **17.** 0.1788 **19.** 1.2921 **21.** 2 **23.** 4 **25.** 2
**27.** 4 **29.** 14 **31.** $\frac{x^3}{4}$ **33.** 2 **35.** $\frac{3}{2}$ **37.** 10 **39.** 3 **41.** About 95 decibels; $L = 10 \log_{10} R$, where $L$ is the loudness of the sound in decibels and $R$ is the relative intensity of the sound. Since the crowd increased by a factor 3, we assume that the intensity also increases by a factor of 3. Thus, we need to find the loudness of $3R$. $L = 10 \log_{10} 3R$; $L = 10 (\log_{10} 3 + \log_{10} R)$; $L = 10 \log_{10} 3 + 10 \log_{10} R$; $L \approx 10(0.4771) + 90$; $L \approx 4.771 + 90$ or about 95
**43.** 7.5

**45.**

| | |
|---|---|
| $m^p = m^p$ | Reflexive property |
| $\left(b^{\log_b m}\right)^p = b^{\log_b (m^p)}$ | $m = b^{\log_b m}$ and $m^p = b^{\log_b (m^p)}$. |
| $b^{\log_b mp} = b^{\log_b (m^p)}$ | Use Property of Powers on the left hand side of the equation. Power of a Power: $a^{mn} = (a^m)^n$ |
| $\log_b mp = \log_b (m^p)$ | Exponents must be equal by the Property of Equality for Exponential Functions. |
| $p \log_b m = \log_b (m^p)$ | Reverse the order of multiplication on the left hand side. |

**47.** $\log_{\sqrt{a}}(a^2) = \dfrac{\log_x a^2}{\log_x \sqrt{a}}$

$$= \log_x a^2 - \log_x \sqrt{a}$$
$$= \log_x \left(a^{\frac{1}{2}}\right)^4 - \log_x \left(a^{\frac{1}{2}}\right)$$
$$= 4\log_x \left(a^{\frac{1}{2}}\right) - \log_x \left(a^{\frac{1}{2}}\right)$$
$$= \frac{4\log_x \left(a^{\frac{1}{2}}\right)}{\log_x \left(a^{\frac{1}{2}}\right)} = 4$$

**49.** False; $\log_2 (2^2 + 2^3) = \log_2 12$, $\log_2 2^2 + \log_2 2^3 = 2 + 3$, or 5, and $\log_2 12 \neq 5$, since $2^5 \neq 12$.

**51.** Let $b^x = m$ and $b^y = n$. Then $\log_b m = x$ and $\log_b n = y$.

| | |
|---|---|
| $\dfrac{b^y}{b^x} = \dfrac{m}{n}$ | |
| $b^{x-y} = \dfrac{m}{n}$ | Quotient Property |
| $\log_b b^{x-y} = \log_b \dfrac{m}{n}$ | Property of Equality for Logarithmic Equations |
| $x - y = \log_b \dfrac{m}{n}$ | Inverse Property of Exponents and Logarithms |
| $\log_b m - \log_b n = \log_b \dfrac{m}{n}$ | Replace $x$ with $\log_b m$ and $y$ with $\log_b n$. |

**53.** A **55.** 4 **57.** $2x$ **59.** $-8$ **61.** $\approx 3.06$ s **63.** 5

**65.** $-\frac{3}{4} < x < 2$

### Pages 531–533 Lesson 9-4

**1.** 0.6021 **3.** $-0.3010$ **5.** 1.7325 **7.** $\pm 1.1615$

**9.** $n > 0.4907$ **11.** $\frac{\log 5}{\log 7}$; 0.8271 **13.** $\frac{\log 9}{\log 2}$; 3.1699

**15.** 1.0792 **17.** 0.3617 **19.** $-1.5229$ **21.** 8 **23.** 0.5537 **25.** 4.8362 **27.** 8.0086 **29.** $\{a \mid a < 1.1590\}$

**31.** $\{n \mid n < -1.0178\}$ **33.** $\{y \mid y \leq 0.4275\}$

**35.** $\frac{\log 20}{\log 5} \approx 1.8614$ **37.** $\frac{\log 8}{\log 5} \approx 1.8928$

**39.** $\frac{0.5 \log 5}{\log 6} \approx 0.4491$ **41.** 2.2 **43.** 3.5 **45.** $\pm 2.6281$

**47.** 3.7162 **49.** 4.7095 **51.** 2.7674 **53.** 113.03 cents **55.** about 11.19 years

**57.**

| | |
|---|---|
| $\log_{\sqrt{a}} 3 = \log_a x$ | Original equation |
| $\dfrac{\log_a 3}{\log_a \sqrt{a}} = \log_a x$ | Change of Base Formula |
| $\log_a 3 = \log_a (a)^{\frac{1}{2}} = \log_a x$ | Quotient Property of Logarithms |
| $\log_a \left(\dfrac{3}{\sqrt{a}}\right) = \log_a x$ | Quotient Property of Logarithms |
| $x = \dfrac{3}{\sqrt{a}}$ | Property of Equality for Logarithmic Functions |
| $x = \dfrac{3\sqrt{a}}{a}$ | Rationalize the denominator |

**59a.** $\log_2 8 = 3$ and $\log_8 2 = \frac{1}{3}$

**59b.** $\log_9 27 = \frac{3}{2}$ and $\log_{27} 9 = \frac{2}{3}$

**59c.** Conjecture: $\log_a b = \frac{1}{\log_b a}$;

Proof:

| | |
|---|---|
| $\log_a b \stackrel{?}{=} \dfrac{1}{\log_b a}$ | Original statement |
| $\dfrac{\log_b b}{\log_b a} \stackrel{?}{=} \dfrac{1}{\log_b a}$ | Change of Base Formula |
| $\dfrac{1}{\log_b a} = \dfrac{1}{\log_b a}$ ✓ | Inverse Property of Exponents and Logarithms |

**61.** C **63.** 1.4248 **65.** 1.8416 **67.** $\left\{z \,\middle|\, 0 < z \leq \frac{1}{64}\right\}$ **69.** $-22$ **71.** $2^x = 3$ **73.** $5^3 = 125$

### Pages 540–542 Lesson 9-5

**1.** 403.4288 **3.** 1.4191 **5.** $-2.3026$ **7.** $x = \ln 4$ **9.** 1.0986 **11.** $h = -26{,}200 \ln \frac{P}{101.3}$ **13.** $\{x \mid x > 3.4012\}$ **15.** 2.4630 **17.** 54.5982 **19.** 0.3012 **21.** 1.0986 **23.** 1.6901 **25.** $-x = \ln 5$ **27.** $e^1 = e$ **29.** $x + 1 = \ln 9$ **31.** $e^{2x} = \frac{7}{3}$ **33.** 0.2877 **35.** 0.2747 **37.** 0 **39.** 0.3662 **41.** about 7.94 billion **43.** about 19.8 yr **45.** $100 \ln 2 \approx 70$ **47.** 27.2991 **49.** 1.7183 **51.** $x < 1.5041$ **53.** $x \geq 0.6438$ **55.** about 6065 people **57.** 232.9197 **59.** 2, 6 **61.** Sample answer: $e^x = 8$

**63.** Always;

| | |
|---|---|
| $\dfrac{\log x}{\log y} \stackrel{?}{=} \dfrac{\ln x}{\ln y}$ | Original statement |
| $\dfrac{\log x}{\log y} \stackrel{?}{=} \dfrac{\frac{\log x}{\log e}}{\frac{\log y}{\log e}}$ | Change of Base Formula |
| $\dfrac{\log x}{\log y} \stackrel{?}{=} \dfrac{\log x}{\log e} \cdot \dfrac{\log e}{\log y}$ | Multiply $\dfrac{\log x}{\log e}$ by the reciprocal of $\dfrac{\log y}{\log e}$. |
| $\dfrac{\log x}{\log y} = \dfrac{\log x}{\log y}$ | Simplify. |

**65.** B **67.** $\frac{\log 68}{\log 4} \approx 3.0437$ **69.** $\frac{\log 23}{\log 50} \approx 0.8015$ **71.** 4 **73.** joint, 1 **75.** 25 free throws and 17 field goals **77.** 1.54 **79.** 33.77 **81.** 9.32

### Pages 548–550 Lesson 9-6

**1.** about 5 h **3.** about 33.5 watts **5.** C **7.** about 284,618 people **9.** about 4.27 hr **11.** more than 44,000 years ago **13.** \$14,559 billion **15.** about 0.0347 **17.** after the year 2182 **19.** $t = \frac{20}{3}n^{0.585}$ **21.** Take the common logarithm of each side, use the Power Property to write $\log(1 + r)^t$ as $t\log(1 + r)$, and then divide each side by the quantity $\log(1 + r)$. **23.** Never; theoretically, the amount left will always be half of the amount that existed 1620 years before. **25.** D **27.** $\ln y = 3$ **29.** $4x^2 = e^8$ **31.** $p > 3.3219$ **33.** $\frac{0.5(0.08p)}{6} + \frac{0.5(0.08p)}{4}$ **35.** $\frac{0.5(0.08p)}{12}$ **37.** $5.0 \times 10^7$

***Pages 552–556*** ***Chapter 9*** ***Study Guide and Review***

**1.** true **3.** false, common logarithm **5.** true **7.** false, logarithmic function **9.** false; Property of Inequality for Logarithms **11.** growth **13.** $y = 7\left(\frac{1}{5}\right)^x$ **15.** $-1$ **17.** $x \leq -\sqrt{6}$ or $x \geq \sqrt{6}$ **19.** $\log_7 343 = 4$ **21.** $4^3 = 64$ **23.** 9 **25.** $\frac{1}{4}$ **27.** 2 **29.** $-4, 3$ **31.** 1000 **33.** 1.7712 **35.** 1.8856 **37.** 6 **39.** 10 decibels **41.** $\pm 2.2452$ **43.** $-0.6309$ **45.** 8.0086 **47.** $\frac{\log 15}{\log 2}$; 3.9069 **49.** $\ln 6 = x$ **51.** 0.9163 **53.** 0.3466 **55.** 11.6487 **57.** 23.37 yr **59.** 5.05 days **61.** about 3.6%

## Chapter 10 Conic Selections

***Page 561*** ***Chapter 10*** ***Get Ready***

**1.** $\{-4, -6\}$ **3.** $\left(\frac{3}{2}, -4\right)$ **5.** 9 in. by 6 in.

**7a.** $\begin{bmatrix} -2 & 4 & -1 \\ 2 & 0 & -2 \end{bmatrix}$ **7b.** $\begin{bmatrix} 5 & 5 & 5 \\ -3 & -3 & -3 \end{bmatrix}$

**7c.** $\begin{bmatrix} 3 & 9 & 4 \\ -1 & -3 & -5 \end{bmatrix}$

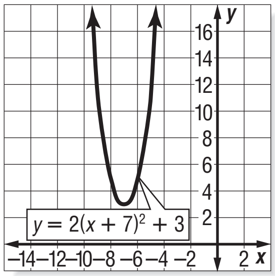
***Pages 564–566*** ***Lesson 10-1***

**1.** $\left(-2, \frac{13}{2}\right)$ **3.** (11.5, 5.3) **5.** 10 units **7.** $\sqrt{2.61}$ units **9.** $\sqrt{1885} \approx 43.4$ units **11.** $(-4, -2)$ **13.** $\left(\frac{17}{2}, \frac{27}{2}\right)$ **15.** around 8th St. and 10th Ave. **17.** 25 units **19.** $3\sqrt{17}$ units **21.** $\sqrt{70.25}$ units **23.** $\sqrt{130}$ units **25.** $\left(\frac{3}{10}, -\frac{1}{5}\right)$; 1 unit **27.** $\left(0, \frac{3\sqrt{5}}{8}\right)$; $\frac{\sqrt{813}}{12}$ units **29.** $6\sqrt{10}\pi$ units, $90\pi$ units$^2$ **31.** Sample answer: Draw several line segments across the U.S. One should go from the northeast corner to the southwest corner; another should go from the southeast corner to the northwest corner; another should go across the middle of the U.S. from east to west, and so on. Find the midpoints of these segments. Locate a point to represent all of these midpoints. **33.** about 85 mi **35.** 14 in. **37.** all of the points on the perpendicular bisector of the segment **39.** Most maps have a superimposed grid. Think of the grid as a coordinate system and assign approximate coordinates to the two cities. Then use the Distance Formula to find the distance between the points with those coordinates. **41.** G **43.** $-0.4055$ **45.** 146.4132 **47.** $y = 2(x + 5)^2$

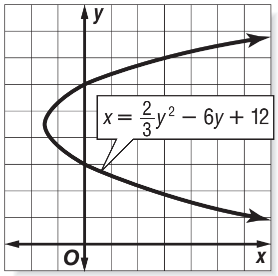
***Pages 571–573*** ***Lesson 10-2***

**1.** $y = 2(x - 3)^2 - 12$; vertex = $(3, -12)$; axis of symmetry: $x = 3$; opens upward

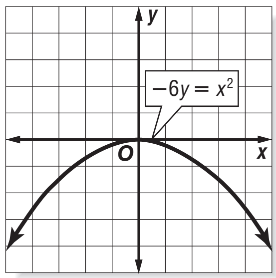
**3.**



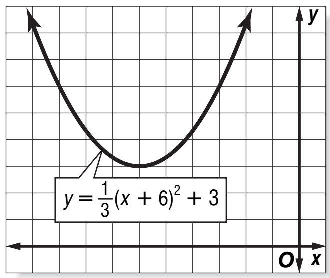

**5.**



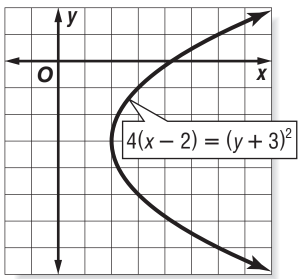

**7.** $y = (x - 3)^2 + 2$; vertex = $(3, 2)$; axis of symmetry: $x = 3$; opens upward **9.** $y = \frac{1}{2}(x + 12)^2 - 80$; vertex = $(-12, -80)$; axis of symmetry: $x = -12$; opens upward **11.**



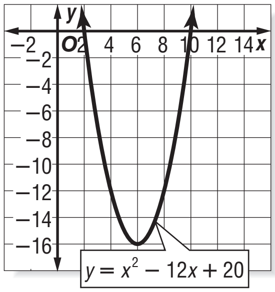

**13.**



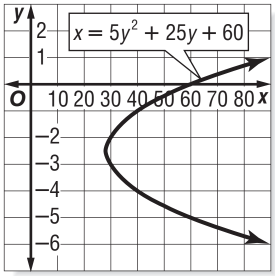

**15.**



**17.**



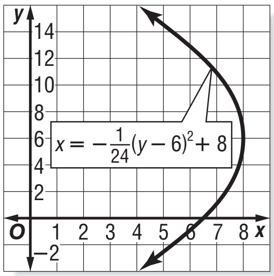

**19.**



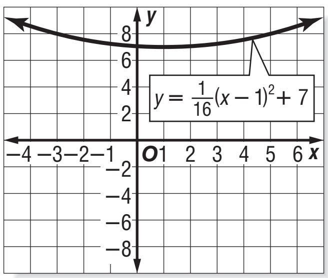

**21.** 0.75 cm **23.** $y = -\frac{1}{100}(x - 50)^2 + 25$ **25.** $y = -\frac{2}{3}$

**27.** The graph's vertex is shifted to the left $\frac{1}{3}$ unit and down $\frac{2}{3}$ unit.

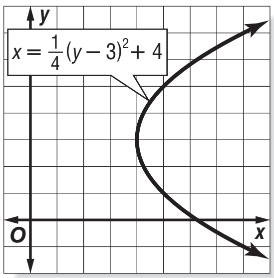
**29.** $x = -\frac{1}{24}(y-6)^2 + 8$



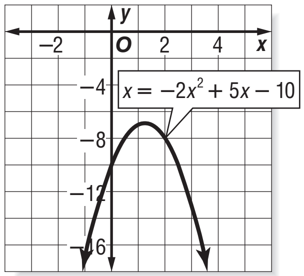

**31.** $y = \frac{1}{16}(x - 1)^2 + 7$




**33.** $x = \frac{1}{4}(y - 3)^2 + 4$

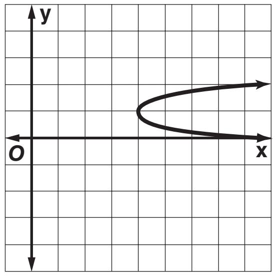



**35.**

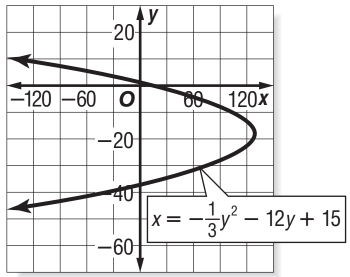



$\left(\frac{5}{4}, -\frac{55}{8}\right), \left(\frac{5}{4}, -7\right)$
$x = \frac{5}{4}, y = -\frac{27}{4}$
downward, $\frac{1}{2}$ unit

**37.** $(4,1), \left(\frac{81}{20}, 1\right), y = 1, x = \frac{79}{20}$, right, $\frac{1}{5}$ unit



**39.**



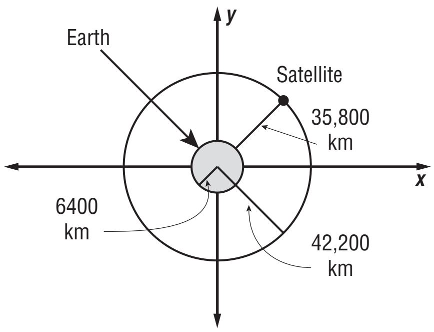

$(123, -18)$,
$(122\frac{1}{4}, -18)$,
$y = -18$,
$x = 123\frac{3}{4}$, left,
3 units

**41.** Rewrite it as $y = (x - h)^2$, where $h > 0$.
**43.** When she added 9 to complete the square, she forgot to also subtract 9. The standard form is $y = (x + 3)^2 - 9 + 4$ or $y = (x + 3)^2 - 5$.
**45.** A parabolic reflector can be used to make a car headlight more effective. Answers should include the following.

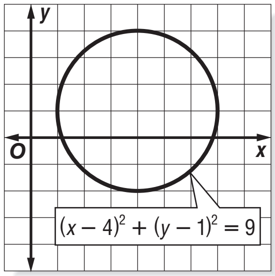
- Reflected rays are focused at that point.
- The light from an unreflected bulb would shine in all directions. With a parabolic reflector, most of the light can be directed forward toward the road. **47.** J **49.** 10 units **51.** about 3.82 days **53.** 4 **55.** 9 **57.** $2\sqrt{3}$ **59.** $4\sqrt{3}$

***Pages 577–579 Lesson 10–3***

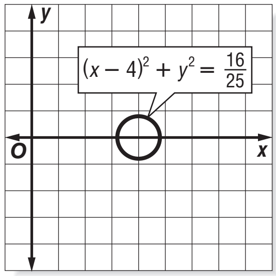
**1.** $(x - 3)^2 + (y + 1)^2 = 9$ **3.**



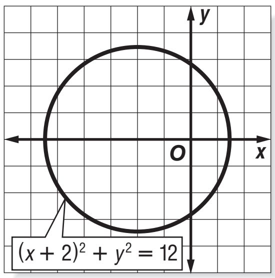
**5.** $x^2 + (y + 2)^2 = 25$ **7.** $(4, 1)$, 3 units




**9.** $(4, 0)$, $\frac{4}{5}$ unit

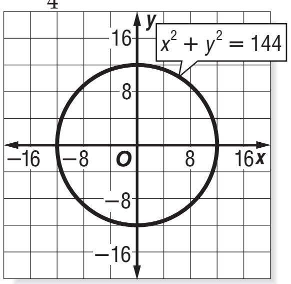



**11.** $(-4, 3)$, 5 units

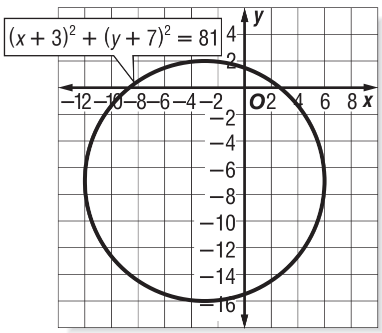



**13.** $(x + 1)^2 + (y - 1)^2 = 16$ **15.** $x^2 + y^2 = 18$ **17.** $(x + 8)^2 + (y - 7)^2 = \frac{1}{4}$ **19.** $(x + 1)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1945}{4}$ **21.** $(0, 0)$, 12 units

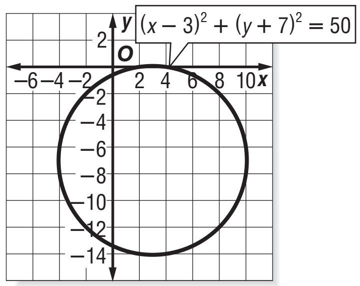
0



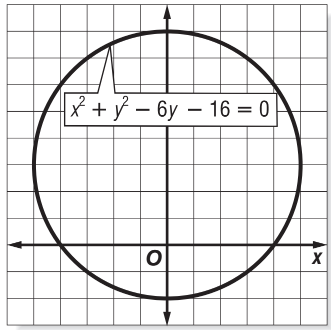

**23.** $(-3, -7)$, 9 units



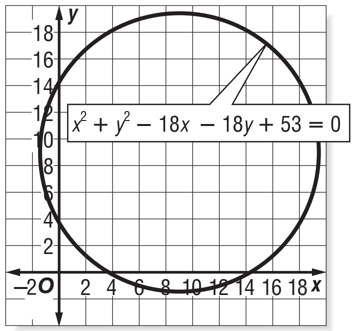

**25.** $(3, -7)$, $5\sqrt{2}$ units



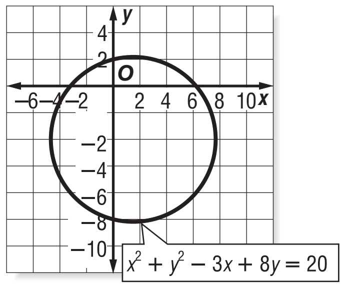

**27.** $(0, 3)$, 5 units




**29.** $(9, 9)$, $\sqrt{109}$ units



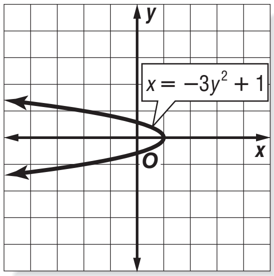

**31.** $\left(\frac{3}{2}, -4\right)$, $\frac{3\sqrt{17}}{2}$ units



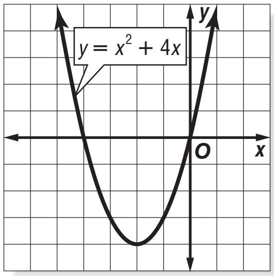

**33.** $(x + \sqrt{13})^2 + (y - 42)^2 = 1777$ **35.** $(x - 4)^2 + (y - 2)^2 = 4$ **37.** $(x + 5)^2 + (y - 4)^2 = 25$ **39.** about 109 mi **41.** $y = \sqrt{16 - (x + 3)^2}$, $y = -\sqrt{16 - (x + 3)^2}$

**43.** $x = -3 \pm \sqrt{16 - y^2}$; The equations $x = -3 - \sqrt{16 - y^2}$; and $x = -3 - \sqrt{16 - y^2}$ represent the right and left halves of the circle, respectively. **45.** $(x + 3)^2 + (y - 1)^2 = 64$; left 3 units, up 1 unit **47.** $(x + 1)^2 + (y + 2)^2 = 5$

**49.** D **51.** $(1, 0)$, $\left(\frac{11}{12}, 0\right)$, $y = 0$, $x = 1\frac{1}{12}$, left, $\frac{1}{3}$ unit




**53.** $(-2, -4)$, $\left(-2, -3\frac{3}{4}\right)$, $x = -2$, $y = -\frac{17}{4}$, upward 1 unit

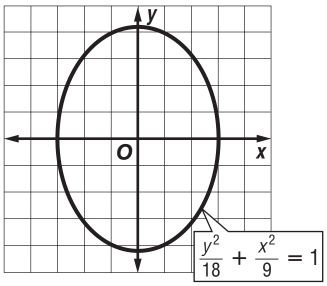



**55.** $(-1, -2)$ **57.** $-4, -2, 1$ **59.** 28 in. by 15 in. **61.** 6 **63.** $2\sqrt{5}$

***Pages 586–588*** ***Lesson 10–4***

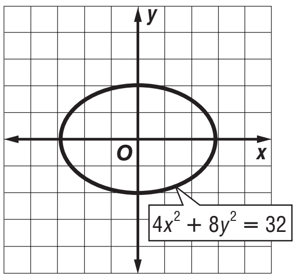
**1.** $\frac{x^2}{36} + \frac{y^2}{20} = 1$ **3.** $\frac{y^2}{100} + \frac{x^2}{36} = 1$

**5.**




$(0, 0)$; $(0, \pm 3)$; $6\sqrt{2}$; 6

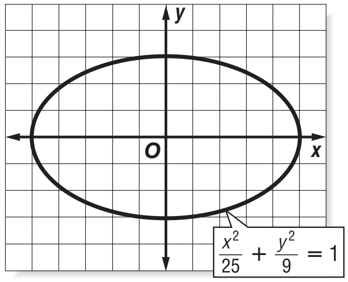
**7.**



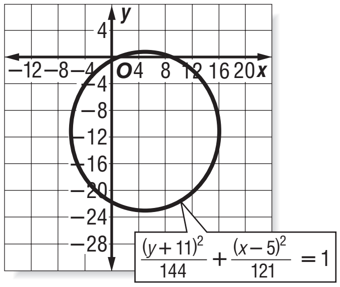

$(0, 0)$; $(\pm 2, 0)$; $4\sqrt{2}$; 4

**9.** $\frac{y^2}{64} + \frac{x^2}{36} = 1$ **11.** $\frac{(x-5)^2}{64} + \frac{(y-4)^2}{9} = 1$ **13.** $\frac{(x+2)^2}{81} + \frac{(y-5)^2}{16} = 1$ **15.** $\frac{(y-2)^2}{100} + \frac{(x-4)^2}{9} = 1$

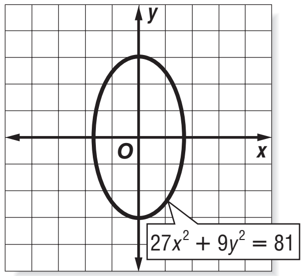
**17.** $\frac{x^2}{324} + \frac{y^2}{196} = 1$ **19.** $(0, 0)$; $(\pm 4, 0)$; 10; 6;



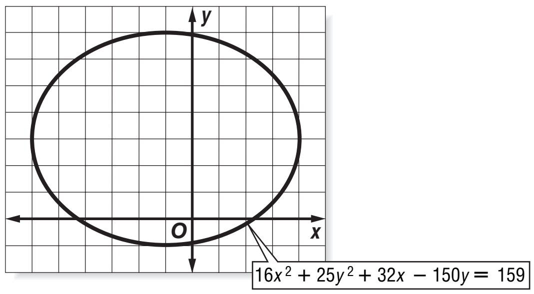

**21.** $(5, -11)$; $\left(5, -11 \pm \sqrt{23}\right)$; 24; 22;




**23.** $(0, 0)$; $\left(0, \pm \sqrt{6}\right)$; 6; $2\sqrt{3}$;



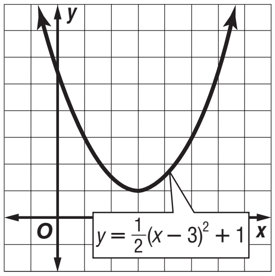

**25.** $(-1, 3)$; $(2, 3)$, $(-4, 3)$; 10; 8;




**27.** $\frac{(x-1)^2}{81} + \frac{(y-2)^2}{56} = 1$ **29.** $\frac{y^2}{20} + \frac{x^2}{4} = 1$

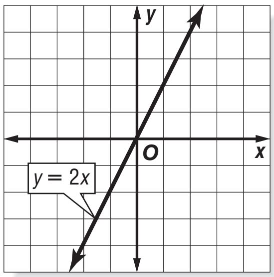
**31.** $\frac{y^2}{279{,}312.25} + \frac{x^2}{193{,}600} = 1$ **33.** Let the equation of a circle be $(x - h)^2 + (y - k)^2 = r^2$. Divide each side by $r^2$ to get $\frac{(x-3)^2}{r^2} + \frac{(y+k)^2}{r^2} = 1$. This is the equation of an ellipse with $a$ and $b$ both equal to $r$. In other words, a circle is an ellipse whose major and minor are both diameters. **35.** $\frac{x^2}{12} + \frac{y^2}{9} = 1$ **37.** C

**39.** $(x - 3)^2 + (y + 2)^2 = 25$ **41.** $y = \frac{1}{2}(x - 3)^2 + 1$

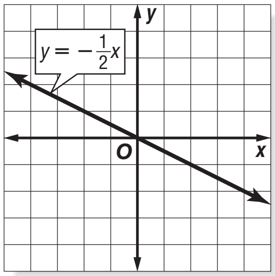



**43.** Sample answer using (0, 104.6) and (10, 112.6): $y = 0.8x + 104.6$

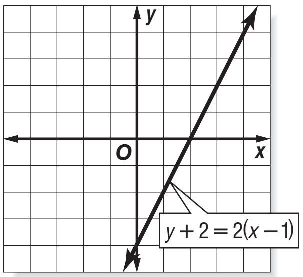
**45.**




**47.**

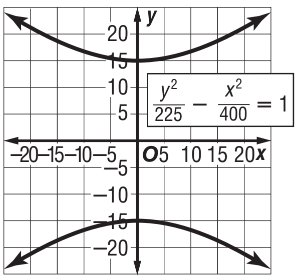



**49.**

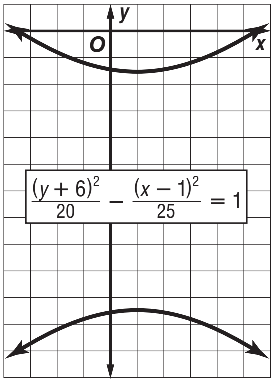



***Pages 594–597*** ***Lesson 10–5***

**1.** $\frac{y^2}{4} - \frac{x^2}{21} = 1$ **3.** $(0, \pm 15)$; $(0, \pm 25)$; $y = \pm\frac{3}{4}x$;

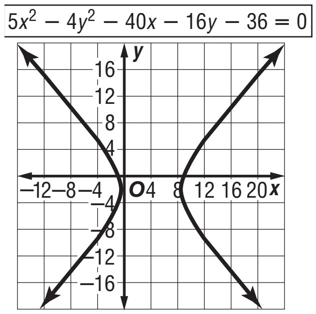



**5.**




$(1, -6 \pm 2\sqrt{5})$; $(1, -6 \pm 3\sqrt{5})$; $y + 6 = \pm\frac{2\sqrt{5}}{5}(x - 1)$

**7.**

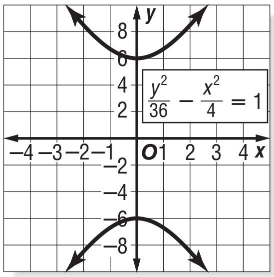



$(4 \pm 2\sqrt{5}, -2)$; $(4 \pm 3\sqrt{5}, -2)$; $y + 2 = \pm\frac{\sqrt{5}}{2}(x - 4)$

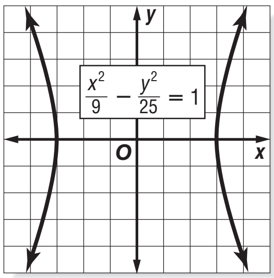
**9.** $\frac{(y-3)^2}{1} - \frac{(x+2)^2}{4} = 1$ **11.** $\frac{(x-3)^2}{4} - \frac{(y+5)^2}{9} = 1$

**13.** $\frac{y^2}{16} - \frac{x^2}{49} = 1$

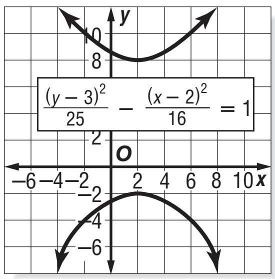
**15.** $\frac{(y-5)^2}{16} - \frac{(x+4)^2}{81} = 1$ **17.** $(0, \pm 6)$; $\left(0, \pm 2\sqrt{10}\right)$; $y = \pm 3x$



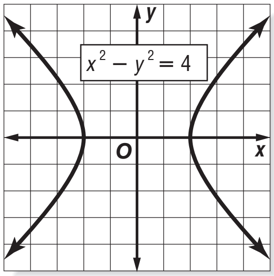

**19.** $(\pm 3, 0)$; $\left(\pm\sqrt{34}, 0\right)$; $y = \pm\frac{5}{3}x$



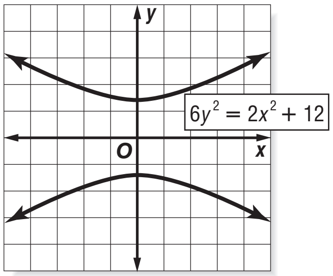

**21.** (2, –2), (2, 8); $\left(2, 3 \pm \sqrt{41}\right)$; $y - 3 = \pm\frac{5}{4}(x - 2)$



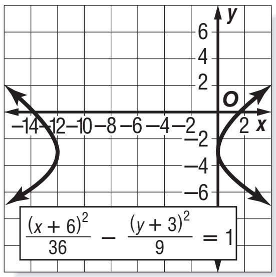
**23.** $(\pm 2, 0)$; $\left(\pm 2\sqrt{2}, 0\right)$; $y = \pm x$



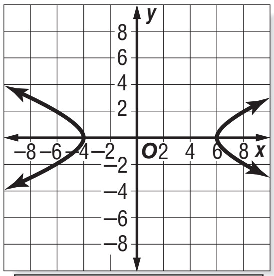

**25.** $\left(0, \pm\sqrt{2}\right)$; $\left(0, \pm 2\sqrt{2}\right)$; $y = \pm\frac{\sqrt{3}}{3}x$




**27.** $(-12, -3)$, $(0, -3)$; $\left(-6 \pm 3\sqrt{5}, -3\right)$; $y + 3 = \pm\frac{1}{2}(x + 6)$




**29.** $(-4, 0)$, $(6, 0)$; $\left(1 \pm \sqrt{29}, 0\right)$; $y = \pm\frac{2}{5}(x - 1)$

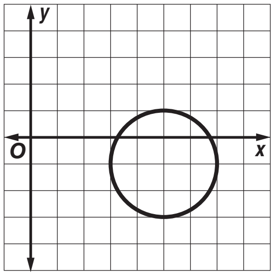


$4x^2 - 25y^2 - 8x - 96 = 0$

**31.** $\frac{y^2}{36} - \frac{x^2}{4} = 1$ **33.** about 47.32 ft
**35.** $\left(\sqrt{2}, \sqrt{2}\right)$, $\left(-\sqrt{2}, -\sqrt{2}\right)$ **37.** The graph of $xy = -2$ can be obtained by reflecting the graph of $xy = 2$ over the $x$-axis or over the $y$-axis. The graph of $xy = -2$ can also be obtained by rotationg the graph of $xy = 2$ by 90°. **39.** As $k$ increases, the branches of the hyperbola become wider. **41.** Hyperbolas and parabolas have different graphs and different reflective properties. Answers should include the following.

- Hyperbolas have two branches, two foci and two vertices. Parabolas have only one branch, one focus, and one vertex. Hyperbolas have asymptotes, but parabolas do not.
- Hyperbolas reflect rays directed at one focus toward the other focus. Parabolas reflect parallel incoming rays toward the only focus.

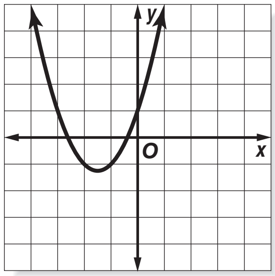
**43.** F **45.** $\frac{(x+3)^2}{9} + \frac{(y-1)^2}{16} = 1$ **47.** $(5, -1)$, 2 units



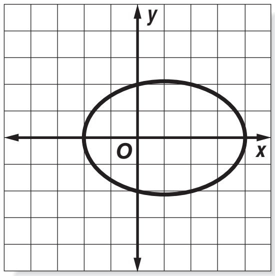
**49.** $-7, \frac{3}{2}$ **51.** $-5, 4$ **53.** $2, 3, -5$ **55.** $1, 0, 0$

***Pages 599–602*** ***Lesson 10–6***

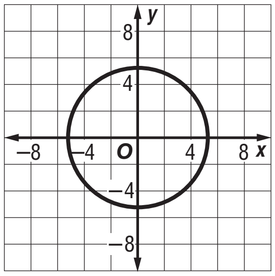
**1.** parabola $y = \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$



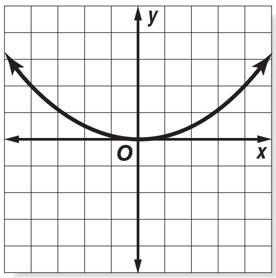
**3.** circle $\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{9}{4}$



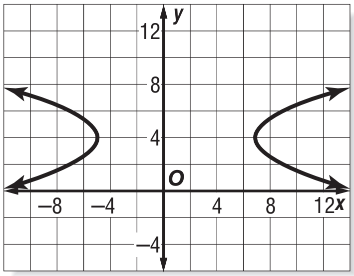
**5.** parabola **7.** hyperbola
**9.** circle $x^2 + y^2 = 27$



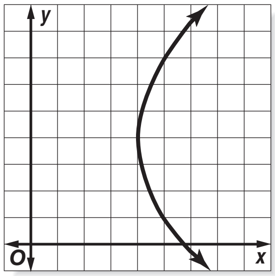
**11.** parabola $y = \frac{1}{8}x^2$



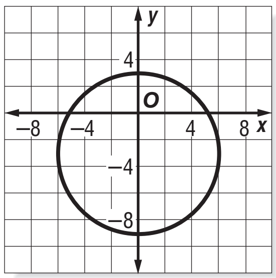
**13.** hyperbola $\dfrac{(x-1)^2}{36} - \dfrac{(y-4)^2}{4} = 1$




**15.** parabola $x = \frac{1}{9}(y-4)^2 + 4$

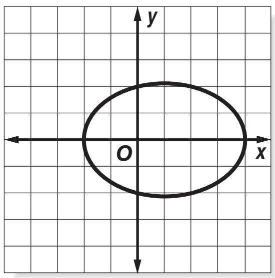



**17.** circle; $x^2 + (y+3)^2 = 36$

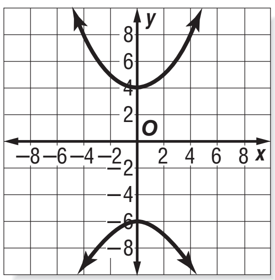



**19.** circle **21.** parabola **23.** b **25.** c **27.** about 1321 ft **29.** hyperbola **31.** hyperbola

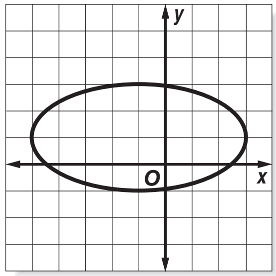
**33.** ellipse $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{\frac{9}{2}} = 1$



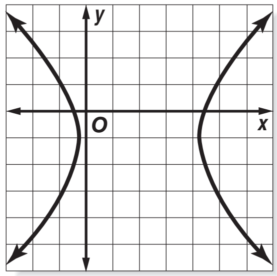

**35.** hyperbola $\dfrac{(y-1)^2}{25} - \dfrac{x^2}{9} = 1$



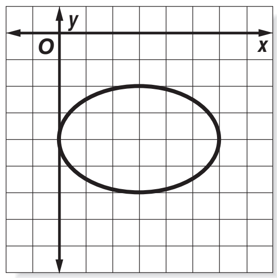

**37.** ellipse $\dfrac{(x+1)^2}{16} + \dfrac{(y-1)^2}{4} = 1$




**39.** hyperbola $\dfrac{(x-2)^2}{5} - \dfrac{(y+1)^2}{6} = 1$




**41.** ellipse **43.** parabola **45.** Sample answer: $2x^2 + 2y^2 - 1 = 0$ **47.** 2 intersecting lines **49.** $0 < e < 1$, $e > 1$ **51.** C **53.** $\dfrac{(y-4)^2}{36} - \dfrac{(x-5)^2}{16} = 1$ **55.** $(3, -4)$; $\left(3 \pm \sqrt{5}, -4\right)$, 6; 4;

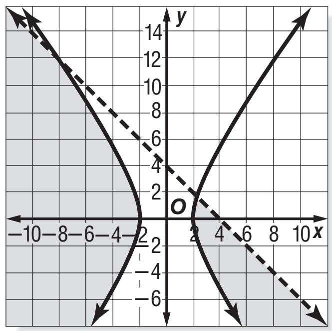



**57.** $m^{12}n$ **59.** 196 beats per min **61.** $y = -\frac{5}{3}x - \frac{4}{3}$ **63.** $(3, 2)$

***Pages 606–608*** ***Lesson 10-7***

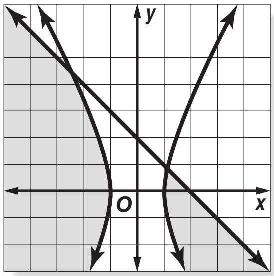
**1.** $(\pm 4, 5)$ **3.** no solution **5.** $(40, 30)$

**7.**

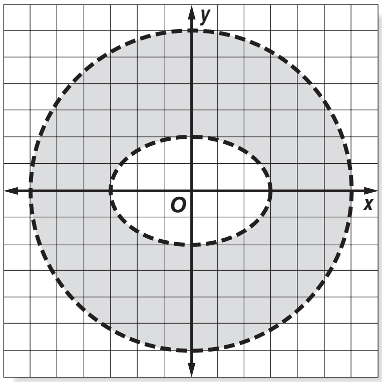



**9.** $\left(\frac{3}{2}, \frac{9}{2}\right)$, $(-1, 2)$ **11.** no solution **13.** no solution **15.** $(0, 3)$, $\left(\pm\frac{\sqrt{23}}{2}, -\frac{11}{4}\right)$ **17.** $(0, \pm 5)$

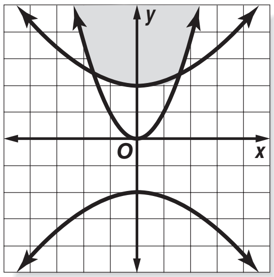
**19.** (4, ±3), (-4, ±3) **21.**



**23.**



**25.**

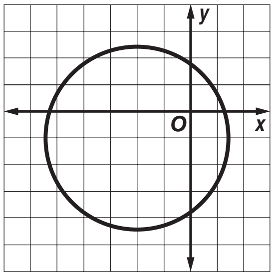


**27.** (39.2, ±4.4) **29.** $\left(-\frac{5}{3}, -\frac{7}{3}\right)$, (1, 3) **31.** 0.5 s

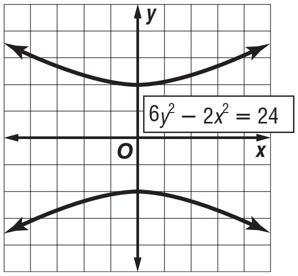
**33.** $y = \pm 900\sqrt{1 - \frac{x^2}{(300)^2}}$; $y = \pm 690\sqrt{1 - \frac{x^2}{(600)^2}}$

**35.** Sample answer: The orbit of the satellite modeled by the second equation is closer to a circle than the other orbit. The distance on the x-axis is twice as great for one satellite than the other. **37.** Sample answer: $x^2 + y^2 = 36, \frac{(x+2)^2}{16} - \frac{y^2}{4} = 1$ **39.** Sample answer: $x^2 + y^2 = 81, \frac{x^2}{4} + \frac{y^2}{100} = 1$ **41.** impossible **43.** Sample answer: $x^2 + y^2 = 40, y = x^2 + x$ **45.** none **47.** none **49.** $x^2 + y^2 = 20, \frac{x^2}{25} + \frac{y^2}{16} = 1$ This system has four solutions whereas the other three only have two solutions. **51.** C

**53.** $(x + 2)^2 + (y + 1)^2 = 11$, circle,

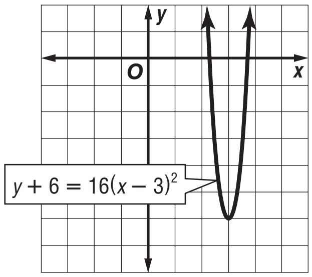


**55.** (0, ± 2), (0, ± 4), $y = \pm\frac{\sqrt{3}}{3}x$

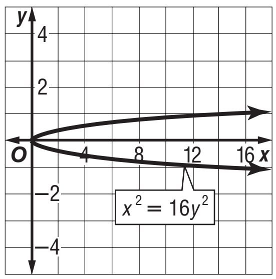



### *Pages 609–614 Chapter 10 Study Guide and Review*

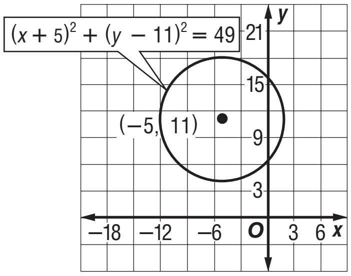
**1.** true **3.** False; a parabola is the set of all points that are the same distance from a given point called the focus and a given line called the directrix. **5.** False; the conjugate axis of a hyperbola is a line segment perpendicular to the transverse axis. **7.** true **9.** $\left(-5, \frac{3}{2}\right)$ **11.** (16, 26) **13.** $\sqrt{290}$ units **15.** $\sqrt{2}$ **17.** $\left(4, -\frac{5}{2}\right)$ **19.** (3, −6); $\left(3, -5\frac{63}{64}\right)$; $x = 3$; $y = -6\frac{1}{64}$; upward; $\frac{1}{16}$ unit



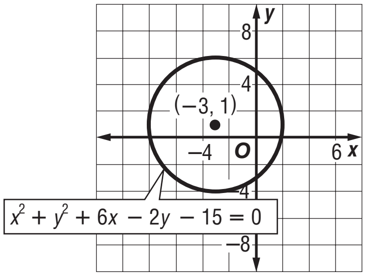

**21.** (0, 0); $\left(\frac{1}{64}, 0\right)$; $y = 0$; $x = -\frac{1}{64}$; right; $\frac{1}{16}$ unit




**23.** $y = -\frac{1}{400}x^2 + x$ **25.** $(x + 4)^2 + y^2 = \frac{9}{16}$
**27.** $(x + 1)^2 + (y - 2)^2 = 4$
**29.** (−5, 11), 7 units

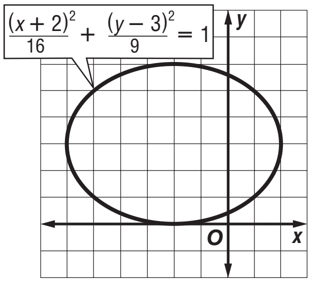



**31.** $(-3, 1)$, 5 units



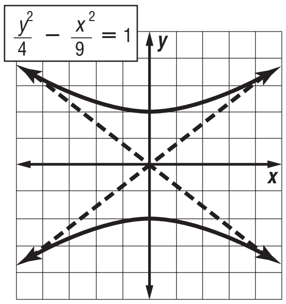

**33.** $\dfrac{(x+1)^2}{25}+\dfrac{(y-1)^2}{4}=1$

**35.** $(-2, 3)$; $(-2 \pm \sqrt{7}, 3)$; 8; 6

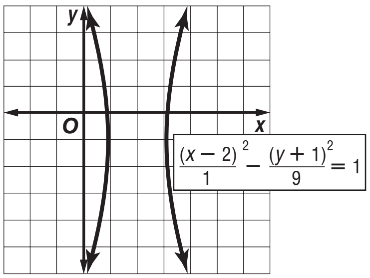



**37.** $\dfrac{x^2}{5.45^2}+\dfrac{y^2}{4.4^2}=1$

**39.** $(0, \pm 2)$; $(0 \pm \sqrt{13})$; $y = \pm \frac{2}{3}x$

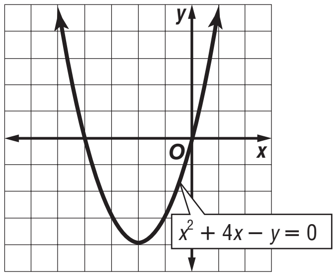



**41.** $(1, -1)$, $(3, -1)$; $(2 \pm \sqrt{10}, -1)$; $y + 1 = \pm 3(x - 2)$

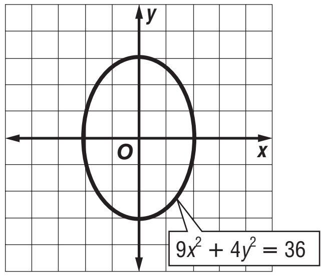



**43.** $\left(\dfrac{40-24\sqrt{5}}{5}, \dfrac{45-12\sqrt{5}}{5}\right)$

**45.** parabola, $y = (x + 2)^2 - 4$




**47.** ellipse $\dfrac{y^2}{9}+\dfrac{x^2}{4}=1$

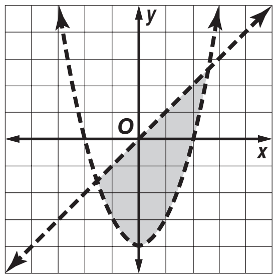



**49.** hyperbola

**51.** circle

**53.** $(6, -8)$, $(12, -16)$

**55.**



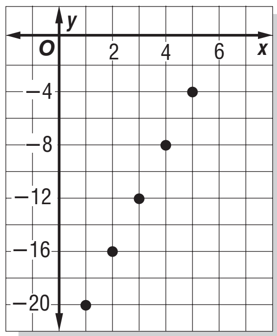
**57.** $(0, 10)$ and $(20, 10)$

## Chapter 11 Sequences and Series

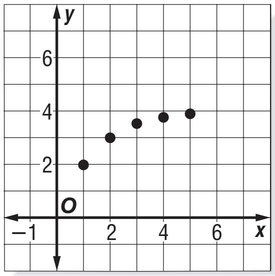
***Page 621*** ***Chapter 11*** ***Get Ready***

**1.** $-10$ **3.** $-5$ **5.** 6

**7.**



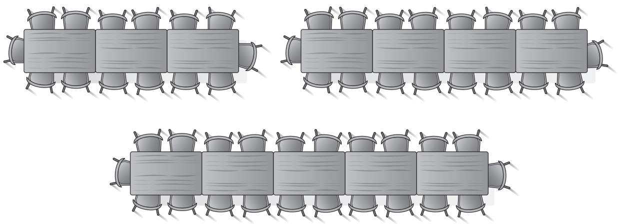
**9.**



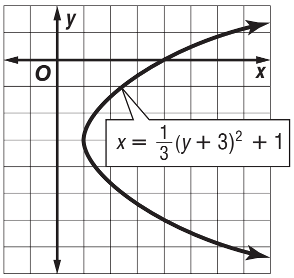

**11.** 17 **13.** $\frac{1}{32}$

***Pages 625–628*** ***Lesson 11-1***

**1.** 24, 28, 32, 36 **3.** 5, 8, 11, 14, 17 **5.** $\frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}$ **7.** 43 **9.** 79 **11.** 39.15 **13.** $a_n = 11n - 37$ **15.** 56, 68, 80 **17.** 30, 37, 44, 51 **19.** 6, 10, 14, 18 **21.** 2, 15, 28, 41, 54 **23.** 6, 2, −2, −6, −10 **25.** 28 **27.** 94 **29.** 335 **31.** 27 **33.** 176 ft **35.** 30th **37.** $a_n = 9n - 2$ **39.** $a_n = -2n - 1$ **41.** 70, 85, 100 **43.** −5, −2, 1, 4 **45.** $\frac{7}{3}, 3, \frac{11}{3}, \frac{13}{3}$ **47.** 5.5, 5.1, 4.7, 4.3 **49.** $\frac{4}{3}, 1, \frac{2}{3}, \frac{1}{3}, 0$ **51.** 29 **53.** 14, 18, 22



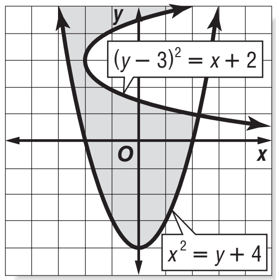
**55.** No; there is no whole number $n$ for which $4n + 2 = 100$. **57.** $-\frac{25}{2}$ **59.** 173 **61.** $a_n = 7n - 600$ **63.** $a_n = -6n + 615$ **65.** Sample answer: Maya has $50 in her savings account. She withdraws $5 each week to pay for music downloads. **67.** $z = 2y - x$ **69.** B **71.** (−1, ±4), (5, ±2) **73.** $x = \frac{1}{3}(y + 3)^2 + 1$; parabola




**75.** 15 **77.** 2 **79.** $-\frac{1}{2}, \frac{3}{2}, \frac{7}{2}$ **81.** $y = 3x + 57$ **83.** 5, 4, 3, 2 **85.** −2, −4, −6, −8, −10

***Pages 632–635*** ***Lesson 11-2***

**1.** 800 **3.** 260 **5.** 135h **7.** 230 **9.** 552 **11.** 19 **13.** −6, 0, 6 **15.** 95 **17.** 663 **19.** −88 **21.** 182 **23.** 225 **25.** 8 days **27.** 2 **29.** 18 **31.** −13, −8, −3 **33.** 13, 18, 23 **35.** 735 **37.** −204 **39.** −35 **41.** 510 **43.** 24,300 **45.** 2646 **47.** 119 **49.** $-\frac{245}{6}$ **51.** 166,833 **53.** $522,500 **55.** 3649 **57.** 6900.5 **59.** 600 **61.** True; for any series, $2a_1 + 2a_2 + 2a_3 + \cdots + 2a_n = 2(a_1 + a_2 + a_3 + \cdots + a_n)$. **63.** Arithmetic series can be used to find the seating capacity of an amphitheater. The sequence represents the numbers of seats in the rows. The sum of the first $n$ terms of the series is the seating capacity of the first $n$ rows. One method is to write out the terms and add them: 18 + 22 + 26 + 30 + 34 + 38 + 42 + 46 + 50 + 54 = 360. Another method is to use the formula $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$: $S_{10} = \frac{10}{2}[2(18) + (10 - 1)4]$ or 360. **65.** G **67.** −135 **69.**




**71.** $\log_7 x = 3$ **73.** $1\frac{5}{7}$ days **77.** $-\frac{9}{2}$ **79.** $\frac{3 \pm \sqrt{89}}{2}$ **81.** −25.21 **83.** $a = -2, b = 2$ **85.** $c = 9, d = 4$ **87.** −54

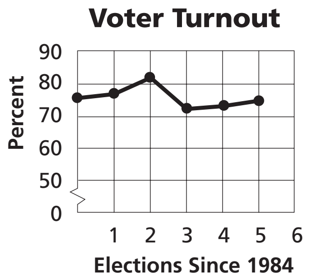
***Pages 639–641*** ***Lesson 11-3***

**1.** 67.5, 101.25 **3.** A **5.** 16 **7.** $\frac{1}{27}$ **9.** $a_n = 15\left(\frac{1}{3}\right)^{n-1}$ **11.** −4 **13.** 6, 18 **15.** 192, 256 **17.** 48, 32 **19.** 1, 4, 16, 64, 256 **21.** 2592 **23.** $46,794.34 **25.** 1024 **27.** 2 **29.** 192 **31.** $a_n = 64\left(\frac{1}{4}\right)^{n-1}$ **33.** $a_n = 4(-3)^{n-1}$ **35.** ±12, 36, ±108 **37.** 6, 12, 24, 48 **39.** $\frac{125}{24}, \frac{625}{48}$ **41.** −21.875, 54.6875 **43.** 576, −288, 144, −72, 36 **45.** 8 days **47.** 243 **49.** −8748 **51.** 800 **53.** Sample answer: $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \ldots$ **55.** The sequence 9, 16, 25, … does not belong with the other three. The other three sequences are geometric sequences, but 9, 16, 25, … is not. **57.** False; the sequence 1, 1, 1, 1, …, for example, is arithmetic $(d = 0)$ and geometric $(r = 1)$. **59.** C **61.** 632.5 **63.** 19, 23 **65.** $5\sqrt{2} + 3\sqrt{10}$ units **67.** $\frac{63}{32}$

***Pages 646–649*** ***Lesson 11-4***

**1.** 81,915 **3.** $\frac{1330}{9}$ **5.** 93 in. or 7 ft 9 in. **7.** $\frac{1093}{9}$ **9.** 32,552 **11.** $\frac{4921}{27}$ **13.** 39,063 **15.** −504 **17.** 3 **19.** −2 **21.** 765 **23.** 1,328,600 **25.** 1441 **27.** 300 **29.** $10,737,418.23 **31.** 206,668 **33.** −364 **35.** 1024 **37.** 6 **39.** $\frac{215}{4}$ **41.** 7.96875 **43.** −118,096 **45.** 156.248 **47.** $-\frac{182}{9}$ **49.** 3,145,725 **51.** $\frac{387}{4}$ **53.** 8 **55.** −1,048,575 **57.** 6.24999936 **59.** Sample answer: $4 + 2 + 1 + \frac{1}{2}$ **61.** If the first term and common ratio of a geometric series are integers, then all the terms of the series are integers. Therefore, the sum of the series is an integer. **63.** If the number of people that each person sends the joke to is constant, then the total number of people who have seen the joke is the sum of a geometric series. Increase the number of days that the joke circulates so that it is inconvenient to find and add all the terms of the series.

**65.** J **67.** $-3, -\frac{9}{2}, -\frac{27}{4}, -\frac{81}{8}$
**69.** 192 **71.** $-14$ **73.** even; 4 **75.** $(7p + 3)(6q - 5)$
**77.**




**79.** Sample answer: 70.4 **81.** 2 **83.** $\frac{2}{3}$ **85.** 0.6

***Pages 653–655*** ***Lesson 11-5***

**1.** 108 **3.** does not exist **5.** 96 cm **7.** 100 **9.** $\frac{4}{5}$ **11.** $\frac{73}{99}$ **13.** 14 **15.** 7.5 **17.** 64 **19.** does not exist **21.** 3 **23.** 144 **25.** does not exist **27.** does not exist **29.** $\frac{1}{9}$ **31.** $\frac{82}{99}$ **33.** 78 cm **35.** 1 **37.** 7.5 **39.** 6 **41.** $\frac{82}{333}$ **43.** $\frac{41}{90}$ **45.** 6 ft **47.** 75, 30, 12 **49.** $-8, -3\frac{1}{5}, -1\frac{7}{25}, -\frac{64}{125}$ **51.** 0.999999… can be written as the infinite geometric series $\frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots$. The first term of this series is $\frac{9}{10}$ and the common ratio is $\frac{1}{10}$, so the sum is $\frac{\frac{9}{10}}{1 - \frac{1}{10}}$ or 1.

**53.**
$$\begin{aligned} S &= a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots \\ (-)\ rS &= a_1r + a_1r^2 + a_1r^3 + a_1r^4 + \cdots \\ \hline S - rS &= a_1 + 0 + 0 + 0 + 0 + \cdots \\ S(1 - r) &= a_1 \\ S &= \frac{a_1}{1 - r} \end{aligned}$$

**55.** D **57.** $-182$ **59.** 32.768% **61.** $-\frac{3}{2}$ **63.** $\frac{-2a + 5b}{a^2b}$
**65.** $\frac{3x + 7}{(x + 4)(x + 2)}$ **67.** $x^2 + 9x + 14 = 0$
**69.** about $-46{,}037$ visitors per year **71.** 2 **73.** 2 **75.** 4

***Pages 660–662*** ***Lesson 11-6***

**1.** 12, 9, 6, 3, 0 **3.** 0, $-4$, 4, $-12$, 20 **5.** $b_n = 1.03b_{n-1} - 10$ **7.** 5, 11, 29 **9.** 3, 11, 123 **11.** 13, 18, 23, 28, 33 **13.** 6, 10, 15, 21, 28 **15.** 4, 6, 12, 30, 84 **17.** $-2.1$ **19.** 5, 17, 65 **21.** $-4, -19, -94$ **23.** $a_n = a_{n-2} + a_{n-1}$ **25.** 1, 3, 6, 10, 15 **27.** 20,100 **29.** \$75.77 **31.** $-1, -1, -1$ **33.** $\frac{4}{3}, \frac{10}{3}, \frac{76}{3}$ **35.** Sometimes; if $f(x) = x^2$ and $x_1 = 2$, then $x_2 = 2^2$ or 4, so $x_2 \neq x_1$. But, if $x_1 = 1$, then $x_2 = 1$, so $x_2 = x_1$. **37.** Under certain conditions, the Fibonacci sequence can be used to model the number of shoots on a plant. The 13th term of the sequence is 233, so there are 233 shoots on the plant during the 13th month. **39.** F **41.** $\frac{1}{6}$ **43.** $-5208$ **45.** $3x + 7$ units **47.** 6

***Pages 667–669*** ***Lesson 11-7***

**1.** $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$ **3.** $x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$ **5.** 40,320 **7.** 17,160 **9.** $56a^5b^3$ **11.** $a^3 - 3a^2b + 3ab^2 - b^3$ **13.** $r^8 + 8r^7s + 28r^6s^2 + 56r^5s^3 + 70r^4s^4 + 56r^3s^5 + 28r^2s^6 + 8rs^7 + s^8$ **15.** $x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$ **17.** 362,880 **19.** 72 **21.** $-126x^4y^5$ **23.** $280x^4$ **25.** 10 **27.** $16b^4 - 32b^3x + 24b^2x^2 - 8bx^3 + x^4$ **29.** $243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$ **31.** $\frac{a^5}{32} + \frac{5a^4}{8} + 5a^3 + 20a^2 + 40a + 32$ **33.** 495 **35.** $1{,}088{,}640a^6b^4$ **37.** $\frac{35}{27}x^4$ **39.** 500 **41.** Sample answer: $(5x + y)^4$ **43.** The coefficients in a binomial expansion give the numbers of sequences of births resulting in given numbers of boys and girls. $(b + g)^5 = b^5 + 5b^4g + 10b^3g^2 + 10b^2g^3 + 5bg^4 + g^5$; There is one sequence of births with all five boys, five sequences with four boys and one girl, ten sequences with three boys and two girls, ten sequences with two boys and three girls, five sequences with one boy and four girls, and one sequence with all five girls. **45.** H **47.** 3, 5, 9, 17, 33 **49.** hyperbola **51.** $\frac{\log 5}{\log 2}$; 2.3219 **53.** $\frac{\log 8}{\log 5}$; 1.2920 **55.** asymptotes: $x = -4$, $x = 1$ **57.** true **59.** true **61.** true

***Pages 672–673*** ***Lesson 11-8***

**1. Step 1:** When $n = 1$, the left side of the given equation is 1. The right side is $\frac{1(1 + 1)}{2}$ or 1, so the equation is true for $n = 1$. **Step 2:** Assume $1 + 2 + 3 + \cdots + k = \frac{k(k + 1)}{2}$ for some positive integer $k$. **Step 3:** $1 + 2 + 3 + \cdots + k + (k + 1) = \frac{k(k + 1)}{2} + (k + 1) = \frac{k(k + 1) + 2(k + 1)}{2} = \frac{(k + 1)(k + 2)}{2}$
The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$ for all positive integers $n$. **3. Step 1:** After the first guest has arrived, no handshakes have taken place. $\frac{1(1-1)}{2} = 0$, so the formula is correct for $n = 1$. **Step 2:** Assume that after $k$ guests have arrived, a total of $\frac{k(k - 1)}{2}$ handshakes have taken place, for some positive integer $k$. **Step 3:** When the $(k + 1)$st guest arrives, he or she shakes hands with the $k$ guests already there, so the total number of handshakes that have then taken place is $\frac{k(k - 1)}{2} + k$.
$$\begin{aligned} \frac{k(k - 1)}{2} + k &= \frac{k(k - 1) + 2k}{2} \\ &= \frac{k[(k - 1) + 2]}{2} \\ &= \frac{k(k + 1)}{2} \text{ or } \frac{(k + 1)k}{2} \end{aligned}$$
The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the formula is true for $n = k + 1$.
Therefore, the total number of handshakes is $\frac{n(n - 1)}{2}$ for all positive integers $n$. **5. Step 1:** $5^1 + 3 = 8$, which is divisible by 4. The statement is true for $n = 1$.

**Step 2:** Assume that $5^k + 3$ is divisible by 4 for some positive integer $k$. This means that $5^k + 3 = 4r$ for some positive integer $r$.

**Step 3:**
$$\begin{aligned} 5^k + 3 &= 4r \\ 5^k &= 4r - 3 \\ 5^{k+1} &= 20r - 15 \\ 5^{k+1} + 3 &= 20r - 12 \\ 5^{k+1} + 3 &= 4(5r - 3) \end{aligned}$$

Since $r$ is a positive integer, $5r - 3$ is a positive integer. Thus, $5^{k+1} + 3$ is divisible by 4, so the statement is true for $n = k + 1$. Therefore, $5^n + 3$ is divisible by 4 for all positive integers $n$. **7.** Sample answer: $n = 3$

**9. Step 1:** When $n = 1$, the left side of the given equation is 2. The right side is $\frac{1[3(1) + 1]}{2}$ or 2, so the equation is true for $n = 1$.**Step 2:** Assume $2 + 5 + 8 + \ldots + (3k - 1) = \frac{k(3k + 1)}{2}$ for some positive integer $k$.

**Step 3:** $2 + 5 + 8 + \ldots + (3k - 1) + [3(k + 1) - 1] = \frac{k(3k + 1)}{2} + [3(k + 1) - 1]$
$$\begin{aligned} &= \frac{k(3k + 1) + 2[3(k + 1) - 1]}{2} \\ &= \frac{3k^2 + k + 6k + 6 - 2}{2} \\ &= \frac{3k^2 + 7k + 4}{2} \\ &= \frac{(k + 1)(3k + 4)}{2} \\ &= \frac{(k + 1)[3(k + 1) + 1]}{2} \end{aligned}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n + 1)}{2}$ for all positive integers $n$. **11. Step 1:** When $n = 1$, the left side of the given equation is $1^2$ or 1. The right side is $\frac{1[2(1) - 1][2(1) + 1]}{3}$ or 1, so the equation is true for $n = 1$. **Step 2:** Assume $1^2 + 3^2 + 5^2 + \cdots + (2k - 1)^2 = \frac{k(2k - 1)(2k + 1)}{3}$ for some positive integer $k$.

**Step 3:** $1^2 + 3^2 + 5^2 + \ldots + (2k - 1)^2 + [2(k + 1) - 1]^2$
$$\begin{aligned} &= \frac{k(2k - 1)(2k + 1)}{3} + [2(k + 1) - 1]^2 \\ &= \frac{k(2k - 1)(2k + 1) + 3(2k + 1)^2}{3} \\ &= \frac{(2k + 1)[k(2k - 1) + 3(2k + 1)]}{3} \\ &= \frac{(2k + 1)(2k^2 - k + 6k + 3)}{3} \\ &= \frac{(2k + 1)(2k^2 + 5k + 3)}{3} \\ &= \frac{(2k + 1)(k + 1)(2k + 3)}{3} \\ &= \frac{(k + 1)[2(k+1) - 1][2(k + 1) + 1]}{3} \end{aligned}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}$ for all positive integers $n$.

**13. Step 1:** $9^1 - 1 = 8$, which is divisible by 8. The statement is true for $n = 1$. **Step 2:** Assume that $9^k - 1$ is divisible by 8 for some positive integer $k$. This means that $9^k - 1 = 8r$ for some whole number $r$.

**Step 3:**
$$\begin{aligned} 9^k - 1 &= 8r \\ 9^k &= 8r + 1 \\ 9^{k+1} &= 72r + 9 \\ 9^{k+1} - 1 &= 72r + 8 \\ 9^{k+1} - 1 &= 8(9r + 1) \end{aligned}$$

Since $r$ is a whole number, $9r + 1$ is a whole number. Thus, $9^{k+1} - 1$ is divisible by 8, so the statement is true for $n = k + 1$. Therefore, $9^n - 1$ is divisible by 8 for all positive integers $n$. **15. Step 1:** When $n = 1$, the left side of the given equation is $f_1$. The right side is $f_3 - 1$. Since $f_1 = 1$ and $f_3 = 2$ the equation becomes $1 = 2 - 1$ and is true for $n = 1$. **Step 2:** Assume $f_1 + f_2 + \ldots + f_k = f_{k+2} - 1$ for some positive integer $k$.

**Step 3:** $f_1 + f_2 + \ldots + f_k + f_{k+1}$
$= f_{k+2} - 1 + f_{k+1} = f_{k+1} + f_{k+2} - 1$
$= f_{k+3} - 1$, since Fibonacci numbers are produced by adding the two previous Fibonacci numbers. The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $f_1 + f_2 + \ldots + f_n = f_{n+2} - 1$ for all positive integers $n$.

**17.** Sample answer: $n = 4$ **19.** Sample answer: $n = 3$
**21.** Sample answer: $n = 41$

**23. Step 1:** When $n = 1$, the left side of the given equation is $\frac{1}{4}$. The right side is $\frac{1}{3}\left(1 - \frac{1}{4}\right)$ or $\frac{1}{4}$, so the equation is true for $n = 1$. **Step 2:** Assume $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots + \frac{1}{4^k} = \frac{1}{3}\left(1 - \frac{1}{4^k}\right)$ for some positive integer $k$.

**Step 3:** $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^k} + \frac{1}{4^{k+1}}$
$$\begin{aligned} &= \frac{1}{3}\left(1 - \frac{1}{4^k}\right) + \frac{1}{4^{k+1}} \\ &= \frac{1}{3} - \frac{1}{3 \cdot 4^k} + \frac{1}{4^{k+1}} \\ &= \frac{4^{k+1} - 4 + 3}{3 \cdot 4^{k+1}} \\ &= \frac{4^{k+1} - 1}{3 \cdot 4^{k+1}} \\ &= \frac{1}{3}\left(\frac{4^{k+1} - 1}{4^{k+1}}\right) \\ &= \frac{1}{3}\left(1 - \frac{1}{4^{k+1}}\right) \end{aligned}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots + \frac{1}{4^n} = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$ for all positive integers $n$.

**25. Step 1:** $13^1 + 11 = 24$, which is divisible by 12. The statement is true for $n = 1$. **Step 2:** Assume that $13^k + 11$ is divisible by 12 for some positive integer $k$. This means that $13^k + 11 = 12r$ for some positive integer $r$.

**Step 3:** $13^k + 11 = 12r$

$$13^k = 12r - 11$$
$$13^{k+1} = 156r - 143$$
$$13^{k+1} + 11 = 156r - 132$$
$$13^{k+1} + 11 = 12(13r - 11)$$

Since $r$ is a positive integer, $13r - 11$ is a positive integer. Thus, $13^{k+1} + 11$ is divisible by 12, so the statement is true for $n = k + 1$. Therefore, $13^n + 11$ is divisible by 12 for all positive integers $n$.

**27. Step 1:** When $n = 1$, the left side of the given equation is $a_1$.

The right side is $\frac{a_1(1-r^1)}{1-r}$ or $a_1$, so the equation is true for $n = 1$. **Step 2:** Assume $a_1 + a_1r + a_1r^2 + \ldots + a_1r^{k-1} = \frac{a_1(1-r^k)}{1-r}$ for some positive integer $k$.

**Step 3:** $a_1 + a_1r + a_1r^2 + \ldots + a_1r^{k-1} + a_1r^k$

$$= \frac{a_1(1-r^k)}{1-r} + a_1r^k$$
$$= \frac{a_1(1-r^k) + (1-r)a_1r^k}{1-r}$$
$$= \frac{a_1 - a_1r^k + a_1r^k - a_1r^{k+1}}{1-r}$$
$$= \frac{a_1(1-r^{k+1})}{1-r}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $a_1 + a_1r + a_1r^2 + \ldots + a_1r^{n-1} = \frac{a_1(1-r^n)}{1-r}$ for all positive integers $n$. **29.** Sample answer: $3^n - 1$ **31.** An analogy can be made between mathematical induction and a ladder with the positive integers on the steps. Showing that the statement is true for $n = 1$ (Step 1) corresponds to stepping on the first step. Assuming that the statement is true for some positive integer $k$ and showing that it is true for $k + 1$ (Steps 2 and 3) corresponds to knowing that you can climb from one step to the next. **33.** H **35.** $a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$ **37.** 4, 10, 28 **39.** 12 h

### *Pages 674–678* *Chapter 11* *Study Guide and Review*

**1.** partial sum **3.** sigma notation **5.** Binomial Theorem **7.** arithmetic series **9.** 38 **11.** –11 **13.** –3, 1, 5 **15.** 6, 3, 0, –3 **17.** 48 **19.** 990 **21.** 282 **23.** 32 **25.** 3 **27.** 6, 12 **29.** \$5796.37 **31.** $\frac{21}{8}$ **33.** $\frac{11}{16}$ **35.** 72 **37.** $-\frac{16}{13}$ **39.** –2, 3, 8, 13, 18 **41.** 1, 1, 1 **43.** \$13,301 **45.** $243r^5 + 405r^4s + 270r^3s^2 + 90r^2s^3 + 15rs^4 + s^5$ **47.** $-13{,}107{,}200x^9$

**49. Step 1:** When $n = 1$, the left side of the given equation is 1. The right side is $2^1 - 1$ or 1, so the equation is true for $n = 1$.

**Step 2:** Assume $1 + 2 + 4 + \ldots + 2^{k-1} = 2^k - 1$ for some positive integer $k$. **Step 3:** $1 + 2 + 4 + \ldots + 2^{k-1} + 2^{(k+1)-1} = 2^k - 1 + 2^{(k+1)-1} = 2^k - 1 + 2^k = 2 \cdot 2^k - 1 = 2^{k+1} - 1$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $1 + 2 + 4 + \ldots + 2^{n-1} = 2^n - 1$ for all positive integers $n$. **51. Step 1:** $3^1 - 1 = 2$ which is divisible by 2. The statement holds true for $n = 1$.

**Step 2:** Assume that $3^k - 1$ is divisible by 2 for some positive integer $k$. This means that $3^k - 1 = 2r$ for some whole number $r$.

**Step 3:** $3^k - 1 = 2r$

$$3^k = 2r + 1$$
$$3(3^k) = 3(2r + 1)$$
$$3^{k+1} = 6r + 3$$
$$3^{k+1} - 1 = 6r + 2$$
$$3^{k+1} - 1 = 2(3r + 1)$$

Since $r$ is a whole number, $3r + 1$ is a whole number. Thus, $3^{k+1} - 1$ is divisible by 2, so the statement is true for $n = k + 1$. Therefore, $3^n - 1$ is divisible by 2 for all positive integers $n$. **53.** $n = 2$ **55.** $n = 2$

## Chapter 12 Probability and Statistics

### *Page 683* *Chapter 12* *Get Ready*

**1.** $\frac{1}{6}$ **3.** $\frac{1}{6}$ **5.** $\frac{1}{2}$ **7.** $\frac{5}{32}$ **9.** $a^3 + 3a^2b + 3ab^2 + b^3$
**11.** $m^5 - 5m^4n + 10m^3n^2 - 10m^2n^3 + 5mn^4 - n^5$
**13.** $(h + t)^5 = h^5 + 5h^4t + 10h^3t^2 + 10h^2t^3 + 5ht^4 + t^5$

### *Pages 687–689* *Lesson 12-1*

**1.** independent **3.** 30 **5.** 256 **7.** 20 **9.** independent **11.** dependent **13.** 16 **15.** 30 **17.** 1024 **19.** 6 **21.** 160 **23.** 240 **25.** 27,216. **27.** Sample answer: buying a shirt that comes in 3 sizes and 6 colors. **29.** 17 **31.** A

**33. Step 1:** When $n = 1$, the left side of the given equation is 4. The right side is $\frac{1[3(1) + 5]}{2}$ or 4, so the equation is true for $n = 1$.

**Step 2:** Assume $4 + 7 + 10 + \ldots + (3k + 1) = \frac{k(3k + 5)}{2}$ for some positive integer $k$.

**Step 3:** $4 + 7 + 10 + \ldots + (3k + 1) + [k(3 + 1) + 1] + [3(k \ldots$

$$= \frac{k(3k + 5)}{2} + [3(k + 1) + 1]$$
$$= \frac{k(3k + 5) + 2[3(k + 1) + 1]}{2}$$
$$= \frac{3k^2 + 5k + 6k + 6 + 2}{2}$$
$$= \frac{3k^2 + 11k + 8}{2}$$
$$= \frac{(k + 1)(3k + 8)}{2}$$
$$= \frac{(k + 1)[3(k + 1) + 5]}{2}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$. Therefore, $4 + 7 + 10 + \ldots + (3n + 1) = \frac{n(3n + 5)}{2}$ for all positive integers $n$.

**35.** $280a^3b^4$ **37.** $\{-4, 4\}$ **39.** $\left\{-3, \frac{1}{3}\right\}$ **41.** (1, 3) **43.** 30 **45.** 720 **47.** 15 **49.** 1

### *Pages 693–695* *Lesson 12-2*

**1.** 60 **3.** 6 **5.** permutation; 5040 **7.** permutation; 1260 **9.** 84 **11.** 56 **13.** 2520 **15.** 10 **17.** 792 **19.** 27,720 **21.** permutation; 5040 **23.** permutation; 2520 **25.** combination; 28 **27.** combination; 45 **29.** 111,540 **31.** 267,696 **33.** 60 **35.** 80,089,128 **37.** Sample answer: There are six people in a contest. How many ways can the first, second, and third prizes be awarded? **39.** Sometimes; the statement is true when $r = 1$.

**41.** Permutations and combinations can be used to find the number of different lineups. There are 9! different 9-person lineups available: 9 choices for the first player, 8 choices for the second player, 7 for the third player, and so on. So, there are 362,880 different lineups. There are $C(16, 9)$ ways to choose 9 players from 16: $C(16, 9) = \frac{16!}{7!9!}$ or 11,440. **43.** J **45.** 80 **47.** Sample answer: $n = 2$ **49.** $x > 0.8047$ **51.** 20 days **53.** $\frac{(y-4)^2}{9} + \frac{(x-4)^2}{4} = 1$ **55.** $\frac{1}{2}$ **57.** $\frac{1}{3}$

### *Pages 700–702* *Lesson 12-3*

**1.** $\frac{1}{7}$ **3.** $\frac{4}{7}$ **5.** $\frac{1}{210}$ **7.** $\frac{3}{8}$ **9.** $\frac{11}{115}$ **11.** $\frac{6}{115}$ **13.** $\frac{24}{115}$ **15.** 0 **17.** $\frac{1}{56}$ **19.** $\frac{1}{70}$ **21.** $\frac{1}{70}$ **23.** $\frac{9}{20}$ **25.** $\frac{1}{20}$ **27.** $\frac{9}{20}$ **29.** 0.007 **31.** 0.109 **33.** $\frac{1}{120}$ **35.** theoretical; $\frac{1}{36}$ **37.** theoretical; $\frac{1}{17}$ **39.** C **41.** permutation; 120 **43.** combination; 35 **45.** direct variation **47.** $\frac{6}{35}$ **49.** $\frac{1}{4}$ **51.** $\frac{9}{20}$

### *Pages 706–709* *Lesson 12-4*

**1.** $\frac{1}{36}$ **3.** $\frac{1}{4}$ **5.** $\frac{1}{16}$ **7.** $\frac{11}{850}$ **9.** dependent; $\frac{5}{204}$ or about 0.025 **11.** $\frac{4}{663}$ **13.** $\frac{25}{36}$ **15.** $\frac{1}{6}$ **17.** $\frac{5}{6}$ **19.** $\frac{25}{49}$ **21.** $\frac{1}{42}$ **23.** 0 **25.** $\frac{10}{171}$ or about 0.058 **27.** $\frac{20}{171}$ or about 0.117 **29.** $\frac{1}{20}$ **31.** independent; $\frac{1}{9}$ or about 0.111 **33.** dependent; $\frac{1}{21}$ **35.** independent; $\frac{8}{27}$ or about 0.296 **37.** $\frac{31{,}213}{3{,}200{,}000}$ or about 0.0098

**39.**

| Second Spin \ First Spin | | blue $\frac{1}{3}$ | yellow $\frac{1}{3}$ | red $\frac{1}{3}$ |
|---|---|---|---|---|
| | blue $\frac{1}{3}$ | BB $\frac{1}{9}$ | BY $\frac{1}{9}$ | BR $\frac{1}{9}$ |
| | yellow $\frac{1}{3}$ | YB $\frac{1}{9}$ | YY $\frac{1}{9}$ | YR $\frac{1}{9}$ |
| | red $\frac{1}{3}$ | RB $\frac{1}{9}$ | RY $\frac{1}{9}$ | RR $\frac{1}{9}$ |

**41.** $\frac{1}{3}$ **43.** $\frac{19}{1{,}160{,}054}$ **45.** $\frac{6327}{20{,}825}$ **47.** no **49.** Sample answer: putting on your socks, and then your shoes **51.** 21 **53.** D **55.** $\frac{1}{204}$ **57.** 1440 ways **59.** 36 **61.** $x$, $x - 4$ **63.** $\frac{5}{6}$ **65.** $\frac{11}{12}$ **67.** $1\frac{5}{12}$

### *Pages 713–715* *Lesson 12-5*

**1.** $\frac{1}{3}$ **3.** $\frac{1}{3}$ **5.** $\frac{1}{2}$ **7.** mutually exclusive; $\frac{2}{13}$ **9.** $\frac{13}{16}$ **11.** $\frac{11}{56}$ **13.** $\frac{55}{56}$ **15.** $\frac{128}{1001}$ **17.** $\frac{202}{429}$ **19.** $\frac{227}{429}$ **21.** inclusive; $\frac{1}{2}$ **23.** mutually exclusive; $\frac{4}{13}$ **25.** $\frac{2}{3}$ **27.** $\frac{11}{221}$ **29.** $\frac{63}{221}$ **31.** $\frac{19}{36}$ **33.** $\frac{4}{15}$ **35.** $\frac{1}{30}$ **37.** 0.42 **39.** $\frac{53}{108}$ **41.** $\frac{17}{162}$ **43.** Sample answer: mutually exclusive events: tossing a coin and rolling a die; inclusive events: drawing a 7 or a diamond from a standard deck of cards **45.** Probability can be used to estimate the percents of what teens do online. The events are inclusive because some people send/read email and buy things online. Also, you know that the events are inclusive because the sum of the percents is not 100%. **47.** G **49.** $\frac{125}{216}$ **51.** $\frac{1}{8}$ **53.** 254 **55.** $(x + 1)^2(x - 1)(x^2 + 1)$ **57.** direct variation **59.** 35.4, 34, no mode, 72 **61.** 63.75, 65, 50 and 65, 30 **63.** 12.98, 12.9, no mode, 4.7

### *Pages 720–723* *Lesson 12-6*

**1.** \$7912.50, \$6460.75 **3.** 40, 6.3 **5.** 424.3, 20.6 **7.** 1.6, 1.3 **9.** 4.8, 2.2 **11.** 569.4, 23.9 **13.** 43.6, 6.6 **15.** The median seems to represent the center of the data. **17.** Mode; it is the least expensive price. **19.** Mean; it is highest. **21.** 2,290,403; 2,150,000; 2,000,000 **23.** Mean; it is highest. **25.** 64% **27.** 19.3 **29.** 19.5 **31.** Different scales are used on the vertical axes. **33.** Sample answer: The second graph might be shown by the company owner to a prospective buyer of the company. It looks like there is a dramatic rise in sales. **35.** Sample answer: The variance of the set {0, 1} is 0.25, and the standard deviation is 0.5. **37.** The first histogram is lower in the middle and higher on the ends, so it represents data that are more spread out. Since set B has the greater standard deviation, set B corresponds to the first histogram and set A corresponds to the second. **39.** The statistic(s) that best represent a set of test scores depends on the distribution of the particular set of scores. Answers should include the following. The mean, median, and mode of the data set are 73.9, 76.5, 94. The mode is not representative at all because it is the highest score. The median is more representative than the mean because it is influenced less than the mean by the two very low scores of 34 and 19. Each measure is increased by 5.

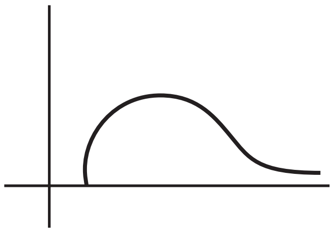
**41.** J **43.** mutually exclusive; $\frac{3}{7}$ **45.** $\frac{4}{663}$ **47.** $\frac{1}{16}$

**49.** 136 **51.** 380 **53.** 396

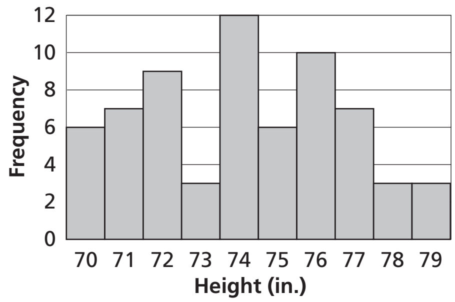
***Pages 726–728*** ***Lesson 12-7***

**1.** normally distributed **3.** 13.5% **5.** 13,600 **7.** 3200 **9.** positively skewed **11.** Negatively skewed; the histogram is high at the right and has a tail to the left. **13.** 733 **15.** 50% **17.** 50% **19.** 25 **21.** 34% **23.** 16% **25.** 4.81

**27.**



Sample answer: the use of cassettes since CDs were introduced **29.** If a large enough group of athletes is studied, some of the characteristics may be normally distributed; others may have skewed distributions. Since the histogram goes up and down several times, the data may not be normally distributed. This may be due to players who play certain positions tending to be of similar large sizes while players who play the other positions tend to be of similar smaller sizes.




**31.** J **33.** 42.5, 6.5 **35.** $\frac{4}{13}$ **37.** 0.0183 **39.** 0.6065

***Pages 731–733*** ***Lesson 12-8***

**1.** 0.22 **3.** 0.31 **5.** 0.105 **7.** 0.25 **9.** 0.05 **11.** 0.61 **13.** 0.49 **15.** 0.37 **17.** App. 10 **19.** 0.53 **21.** 0.98 **23.** 9 **25.** Never; The probability that $x$ will be greater than the mean is always 36.8% for exponential distributions. **27.** The poll will give you a percent of people supporting the science wing addition. The percent of supporters represents the probability of success. You can use the formula for the expected number of success in a binomial distribution with the total number of students in the school to predict the number that will support the science wing addition.

**29.** G **31.** 97.5% **33.** $\frac{1}{3}$ **35.** $x^2 + 2x - 63$

**37.** $-8$ **39.** $56c^5d^3$

***Pages 737–739*** ***Lesson 12-9***

**1.** $\frac{3}{8}$ **3.** $\frac{7}{8}$ **5.** $\frac{48}{28{,}561}$ **7.** about 0.075 **9.** $\frac{1}{32}$ **11.** $\frac{5}{16}$

**13.** $\frac{3}{16}$ **15.** $\frac{125}{324}$ **17.** $\frac{425}{432}$ **19.** $\frac{4096}{15{,}625}$ **21.** $\frac{48}{3125}$

**23.** $\frac{2816}{3125}$ **25.** $\frac{105}{512}$ **27.** $\frac{319}{512}$ **29.** $\frac{560}{2187}$ **31.** about 0.002

**33.** $\frac{1}{64}, \frac{3}{32}, \frac{15}{64}, \frac{5}{16}, \frac{15}{64}, \frac{3}{32}, \frac{1}{64}$ **35.** normal distribution

**37a.** Each trial has more than two possible outcomes. **37b.** The number of trials is not fixed. **37c.** The trials are not independent. **39.** Getting a right answer and a wrong answer are the outcomes of a binomial experiment. The probability is far greater that guessing will result in a low grade than in a high grade. Use $(r + w)^5 = r^5 + 5r^4w + 10r^3w^2 + 10r^2w^3 + 5rw^4 + w^5$ and the chart on page 729 to determine the probabilities of each combination of right and wrong.

$P$(5 right): $r^5 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ or about 0.098%;

$P$(4 right, 1 wrong): $\frac{15}{1024}$ or about 1.5%;

$P$(3 right, 2 wrong): $10r^3w^2 = 10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{45}{512}$ or about 8.8%;

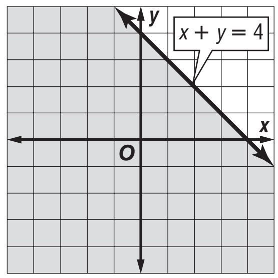
$P$(3 wrong, 2 right): $10r^2w^3 = 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 = \frac{135}{512}$ or about 26.4%;

$P$(4 wrong, 1 right): $5rw^4 = 5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 = \frac{405}{1024}$ or about 39.6%;

$P$(5 wrong): $w^5 = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$ or about 23.7%.

**41.** G **43.** 10 **45.** Mean; it is highest.

**47.**




**49.** 0.1 **51.** 0.039 **53.** 0.041

***Pages 737–738*** ***Lesson 12-10***

**1.** Yes; the last digits of social security numbers are random. **3.** 9% **5.** 5% **7.** 1089 **9.** Yes; all seniors would have the same chance of being selected. **11.** No; basketball players are more likely to be taller than the average high school student, so a sample of basketball playetrs would not give representative heights for the whole school. **13.** 4% **15.** 3% **17.** 2% **19.** 4% **21.** 2% **23.** 36 or 64 **25.** 3% **27.** The margin of sampling error decreases when the size of the sample $n$ increases. As $n$ increases, $\frac{p\,(1 - p)}{n}$ decreases. **29.** A **31.** $\frac{1}{32}$ **33.** $\frac{1}{2}$ **35.** 97.5%

**Pages 740–744** **Chapter 12** **Study Guide and Review**

**1.** probability **3.** dependent events **5.** mutually exclusive events **7.** sample space **9.** 46,656 passwords **11.** 4 **13.** $\frac{4}{7}$ **15.** independent; $\frac{1}{36}$ **17.** dependent; $\frac{1}{7}$ **19.** inclusive; $\frac{1}{3}$ **21.** mutually exclusive; $\frac{1}{2}$ **23.** 8 **25.** 125 **27.** 34% **29.** 24 **31.** 0.12 **33.** $\frac{1}{2,176,782,336}$ **35.** $\frac{1}{64}, \frac{3}{32}, \frac{15}{64}, \frac{5}{16}, \frac{15}{64}, \frac{3}{32}, \frac{1}{64}$ **37.** about 4%

# Chapter 13 Trigonometric Functions

**Page 757** **Chapter 13** **Getting Ready**

**1.** 10 **3.** 16.7 **5** 10.44 ft **7.** $x = 4\sqrt{3}, y = 8$

**Pages 764–767** **Lesson 13-1**

**1.** $\sin\theta = \frac{8}{17}$; $\cos\theta = \frac{15}{17}$; $\tan\theta = \frac{8}{15}$; $\csc\theta = \frac{17}{8}$; $\sec\theta = \frac{17}{15}$; $\cot\theta = \frac{15}{8}$ **3.** $\sin\theta = \frac{5}{6}$; $\cos\theta = \frac{\sqrt{11}}{6}$; $\tan\theta = \frac{5\sqrt{11}}{11}$; $\csc\theta = \frac{6}{5}$; $\sec\theta = \frac{6\sqrt{11}}{11}$; $\cot\theta = \frac{\sqrt{11}}{5}$ **5.** $\cos 23° = \frac{32}{x}$; $x \approx 34.8$ **7.** $B = 45°, a = 6, c \approx 8.5$ **9.** $a \approx 16.6, A \approx 67°, B \approx 23°$ **11.** 25.6 m **13.** $\sin\theta = \frac{4}{11}$; $\cos\theta = \frac{\sqrt{105}}{11}$; $\tan\theta = \frac{4\sqrt{105}}{105}$; $\csc\theta = \frac{11}{4}$; $\sec\theta = \frac{11\sqrt{105}}{105}$; $\cot\theta = \frac{\sqrt{105}}{4}$ **15.** $\sin\theta = \frac{\sqrt{7}}{4}$; $\cos\theta = \frac{3}{4}$; $\tan\theta = \frac{\sqrt{7}}{3}$; $\csc\theta = \frac{4\sqrt{7}}{7}$; $\sec\theta = \frac{4}{3}$; $\cot\theta = \frac{3\sqrt{7}}{7}$ **17.** $\cos 60° = \frac{3}{x}$, $x = 6$ **19.** $\tan 17.5° = \frac{x}{23.7}$; $x \approx 7.5$ **21.** $\sin x° = \frac{16}{22}$, $x \approx 47$ **23.** $A = 63°, a \approx 13.7, c \approx 15.4$ **25.** $A = 75°, a \approx 24.1, b \approx 6.5$ **27.** $B = 45°, a = 7, b = 7$ **29.** about 142.8 ft **31.** $\sin\theta = \frac{\sqrt{5}}{5}$; $\cos\theta = \frac{2\sqrt{5}}{5}$; $\tan\theta = \frac{1}{2}$; $\csc\theta = \sqrt{5}$; $\sec\theta = \frac{\sqrt{5}}{2}$; $\cot\theta = 2$ **33.** $A = 72°, b \approx 1.3, c \approx 4.1$ **35.** $A \approx 63°, B \approx 27°, a \approx 11.5$ **37.** $A \approx 41°, B \approx 49°, b = 8, c \approx 10.6$

**39a.**

| | | |
|---|---|---|
| $\sin 30° = \frac{\text{opp}}{\text{hyp}}$ | | sine ratio |
| $\sin 30° = \frac{x}{2x}$ | | Replace *opp* with $x$ and *hyp* with $2x$. |
| $\sin 30° = \frac{1}{2}$ | | Simplify. |

**39b.**

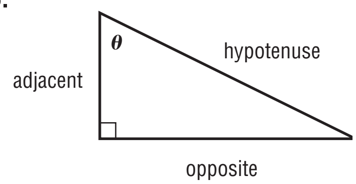
| | | |
|---|---|---|
| $\cos 30° = \frac{\text{adj}}{\text{hyp}}$ | | cosine ratio |
| $\cos 30° = \frac{\sqrt{3}x}{2x}$ | | Replace *adj* with $\sqrt{3}x$ and *hyp* with $2x$. |
| $\cos 30° = \frac{\sqrt{3}}{2}$ | | |

**39c.**

| | | |
|---|---|---|
| $\sin 60° = \frac{\text{opp}}{\text{hyp}}$ | | sine ratio |
| $\sin 60° = \frac{\sqrt{3}x}{2x}$ | | Replace *opp* with $\sqrt{3}x$ and *hyp* with $2x$. |
| $\sin 60° = \frac{\sqrt{3}}{2}$ | | Simplify. |

**41.** about 6° **43.** 93.53 units$^2$

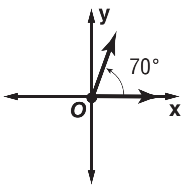
**45.**



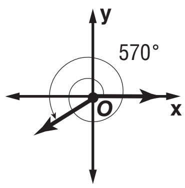

**47.** The sine and cosine ratios of acute angles of right triangles each have the longest measure of the triangle, the hypotenuse, as their denominator. A fraction whose denominator is greater than its numerator is less than 1. The tangent ratio of an acute angle of a right triangle does not involve the measure of the hypotenuse, $\frac{\text{opp}}{\text{adj}}$. If the measure of the opposite side is greater than the measure of the adjacent side, the tangent ratio is greater than 1. If the measure of the opposite side is less than the measure of the adjacent side, the tangent ratio is less than 1. **49.** C **51.** No; Band members may be more likely to like the same kinds of music. **53.** $\frac{3}{8}$ **55.** $\frac{15}{16}$ **57.** $\{-2, -1, 0, 1, 2\}$ **59.** 20 qt **61.** 12 m$^2$

**Pages 772–774** **Lesson 13-2**

**1.**

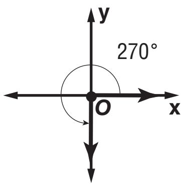



**3.**

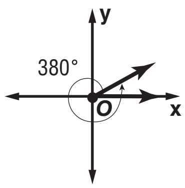



**5.** $\frac{13\pi}{18}$ **7.** $\frac{97\pi}{36}$ **9.** −30° **11.** 21 h **13.** Sample answer: 420°, −300° **15.** Sample answer: $\frac{7\pi}{3}, -\frac{5\pi}{3}$

**17.**

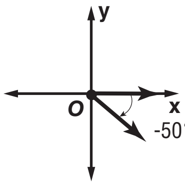



**19.**

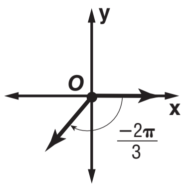


**21.** $\frac{\pi}{3}$ **23.** $-\frac{5\pi}{4}$ **25.** 495° **27.** −60° **29.** Sample answer: 390°, −330° **31.** Sample answer: $\frac{11\pi}{4}$, $-\frac{5\pi}{4}$ **33.** Sample answer: $\frac{3\pi}{4}$, $-\frac{13\pi}{4}$ **35.** about 188.5 $m^2$

**37.**



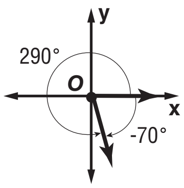

**39.**




**41.** $\frac{19\pi}{6}$ **43.** $\frac{13\pi}{9}$ **45.** 510° **47.** $\frac{540}{\pi} \approx 171.9°$

**49.** Sample answer: 8°, −352° **51.** Sample answer: $\frac{4\pi}{3}$, $-\frac{8\pi}{3}$ **53.** Sample answer: $\frac{25\pi}{4}$, $-\frac{7\pi}{4}$ **55.** number 17

**57.**

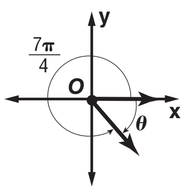



**59a.** $a^2 + (-b)^2 = a^2 + b^2 = 1$
**59b.** $b^2 + a^2 = a^2 + b^2 = 1$
**59c.** $b^2 + (-a)^2 = a^2 + b^2 = 1$
**61.** An angle with a measure of more than 180° gives an indication of motion in a circular path that ended at a point more than halfway around the circle from where it started. Negative angles convey the same meaning as positive angles, but in an opposite direction. The standard convention is that negative angles represent rotations in a clockwise direction. Rates over 360° per minute indicate that an object is rotating or revolving more than one revolution per minute. **63.** J **65.** $A = 22°$, $a \approx 5.9$, $c \approx 15.9$
**67.** $c = 0.8$, $A = 30°$, $B = 60°$ **69.** about 7.07%
**71.** combination, 35 **73.** $[g \circ h](x) = 4x^2 - 6x + 23$, $[h \circ g](x) = 8x^2 + 34x + 44$ **75.** $\frac{3\sqrt{5}}{5}$ **77.** $\frac{\sqrt{10}}{2}$
**79.** $\frac{\sqrt{10}}{4}$

***Pages 781–783*** ***Lesson 13-3***

**1.** $\sin\theta = \frac{8}{17}$, $\cos\theta = -\frac{15}{17}$, $\tan\theta = -\frac{8}{15}$, $\csc\theta = \frac{17}{8}$, $\sec\theta = -\frac{17}{15}$, $\cot\theta = -\frac{15}{8}$ **3.** $\sin\theta = \frac{\sqrt{2}}{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = 1$, $\csc\theta = \sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = 1$ **5.** −1

**7.** $-\frac{2\sqrt{3}}{3}$ **9.** ; $\frac{\pi}{4}$

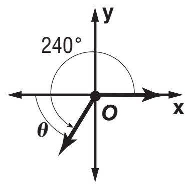



**11.** $\sin\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = -\sqrt{3}$, $\csc\theta = \frac{2\sqrt{3}}{3}$, $\sec\theta = -2$, $\cot\theta = -\frac{\sqrt{3}}{3}$ **13.** about 12.4 ft

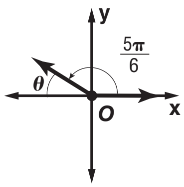
**15.** $\sin\theta = \frac{\sqrt{5}}{5}$, $\cos\theta = \frac{2\sqrt{5}}{5}$, $\tan\theta = \frac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = 2$ **17.** $\sin\theta = -\frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = -\frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

**19.** $\sin\theta = 0$, $\cos\theta = -1$, $\tan\theta = 0$, $\csc\theta =$ undefined, $\sec\theta = -1$, $\cot\theta =$ undefined **21.** $\sin\theta = -\frac{\sqrt{6}}{3}$, $\cos\theta = -\frac{\sqrt{3}}{3}$, $\tan\theta = \sqrt{2}$, $\csc\theta = -\frac{\sqrt{6}}{2}$, $\sec\theta = -\sqrt{3}$, $\cot\theta = \frac{\sqrt{2}}{2}$ **23.** −2 **25.** $-\sqrt{3}$ **27.** $\frac{1}{2}$ **29.** $\frac{\sqrt{2}}{2}$ **31.** 2

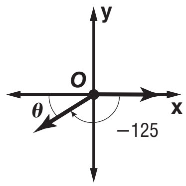
**33.** −1 **35.** ; 60°



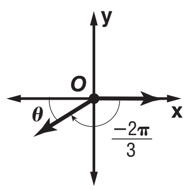

**37.** ; $\frac{\pi}{6}$




**39.** ; 55°




**41.** ; $\frac{\pi}{3}$




**43.** $\sin\theta = \frac{\sqrt{26}}{26}$, $\cos\theta = -\frac{5\sqrt{26}}{26}$, $\csc\theta = \sqrt{26}$, $\sec\theta = -\frac{\sqrt{26}}{5}$, $\cot\theta = -5$ **45.** $\sin\theta = -\frac{2\sqrt{5}}{5}$, $\cos\theta = -\frac{\sqrt{5}}{5}$, $\tan\theta = 2$, $\csc\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$ **47.** 45°; $2 \times 45°$ or 90° yields the greatest value for $\sin 2\theta$.

**49.** 0.2, 0, −0.2, 0, 0.2, 0, and −0.2; or about 11.5°, 0°, −11.5°, 0°, 11.5°, 0°, and −11.5°

**51.** Sample answer: 200°  **53.** Answers should include the following.

- The cosine of any angle is defined as $\frac{x}{r}$, where $x$ is the $x$-coordinate of any point on the terminal ray of the angle and $r$ is the distance from the origin to that point. This means that for angles with terminal sides to the left of the $y$-axis, the cosine is negative, and those with terminal sides to the right of the $y$-axis, the cosine is positive. Therefore the cosine function can be used to model real-world data that oscillate between being positive and negative.
- If we knew the length of the cable we could find the vertical distance from the top of the tower to the rider. Then if we knew the height of the tower we could subtract from it the vertical distance calculated previously. This will leave the height of the rider from the ground.

**55.** F  **57.** 300°  **59.** 635  **61.** (−4, 3)  **63.** 4.7  **65.** 2.7

***Pages 790–792  Lesson 13-4***

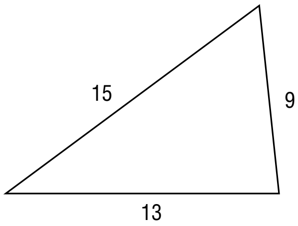
**1.** 57.5 in$^2$  **3.** $C = 30°$, $a \approx 2.9$, $c \approx 1.5$  **5.** $B \approx 20°$, $A \approx 20°$, $a \approx 20.2$  **7.** two; $B \approx 42°$, $C \approx 108°$, $c \approx 5.7$; $B \approx 138°$, $C \approx 12°$, $c \approx 1.2$  **9.** one; $B \approx 19°$, $C \approx 16°$, $c \approx 8.9$  **11.** 43.1 m$^2$  **13.** 572.8 ft$^2$  **15.** 4.2 m$^2$  **17.** $B = 101$, $c \approx 3.0$, $b \approx 3.4$  **19.** $B \approx 21$, $C \approx 37$, $b \approx 13.1$  **21.** $C = 97°$, $a \approx 5.5$, $b \approx 14.4$  **23.** no  **25.** two; $B \approx 72°$, $C \approx 75°$, $c \approx 3.5$; $B \approx 108°$, $C \approx 39°$, $c \approx 2.3$  **27.** one; $B = 90°$, $C = 60°$, $c \approx 24.2$  **29.** two; $B \approx 56°$, $C \approx 72°$, $c \approx 229.3$; $B \approx 124°$, $C \approx 4°$, $c \approx 16.8$  **31.** 4.6 and 8.5 mi  **33.** $C \approx 67°$, $B \approx 63°$, $b \approx 2.9$  **35.** 690 ft  **37.** Gabe; Dulce used the wrong angle. The Law of Sines must first be used to find $\angle B$. Then $m\angle C$ can be found. Once $m\angle C$ is found, $A = \frac{1}{2}ba \sin C$ will yield the area of the triangle.  **39.** If the height of the triangle is not given, but the measure of two sides and their included angle are given, then the formula for the area of a triangle using the sine function should be used. You might use this formula to find the area of a triangular piece of land, since it might be easier to measure two sides and use surveying equipment to measure the included angle than to measure the perpendicular distance from one vertex to its opposite side.

**41.** F  **43.** $\sqrt{3}$  **45.** 660°, −60°  **47.** $\frac{17\pi}{6}$, $-\frac{7\pi}{6}$  **49.** 5.6  **51.** 39.4°

***Pages 796–798  Lesson 13-5***

**1.** cosines; $A \approx 77°$, $B \approx 68°$, $c \approx 6.5$  **3.** sines; $C \approx 101°$, $B \approx 37°$, $c \approx 92.5$  **5.** 19.5 m  **7.** sines; $A = 60°$, $b \approx 14.3$, $c \approx 11.2$  **9.** cosines; $A \approx 47°$, $B \approx 74°$, $C \approx 60°$  **11.** cosines; $A \approx 57°$, $B \approx 82°$, $c \approx 11.5$  **13.** cosines; $A \approx 55°$, $C \approx 78°$, $b \approx 17.9$  **15.** no  **17.** cosines; $A \approx 103°$, $B \approx 49°$, $C \approx 28°$  **19.** 4.4 cm, 9.0 cm  **21.** cosines; $A \approx 15°$, $B \approx 130°$, $C \approx 35°$  **23.** sines; $C = 102°$, $b \approx 5.5$, $c \approx 14.4$  **25.** cosines; $A \approx 107°$, $B \approx 35°$, $c \approx 13.8$  **27.** about 159.7°  **29.** Since the step angle for the carnivore is closer to 180°, it appears as though the carnivore made more forward progress with each step than the herbivore did.  **31. 1a.** Use the Law of Cosines to find the measure of one angle. Then use the Law of Sines or the Law of Cosines to find the measure of a second angle. Finally, subtract the sum of these two angles from 180° to find the measure of the third angle. **1b.** Use the Law of Cosines to find the measure of the third side. Then use the Law of Sines or the Law of Cosines to find the measure of a second angle. Finally, subtract the sum of these two angles from 180° to find the measure of the third angle.

**33.** Sample answer:




**35.** Given the latitude of a point on the surface of Earth, you can use the radius of the Earth and the orbiting height of a satellite in geosynchronous orbit to create a triangle. This triangle will have two known sides and the measure of the included angle. Find the third side using the Law of Cosines and then use the Law of Sines to determine the angles of the triangle. Subtract 90 degrees from the angle with its vertex on Earth's surface to find the angle at which to aim the receiver dish.  **37.** F  **39.** $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{15}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$  **41.** $\sin\theta = -\frac{\sqrt{6}}{4}$, $\cos\theta = \frac{\sqrt{10}}{4}$, $\tan\theta = \frac{\sqrt{15}}{5}$, $\csc\theta = \frac{2\sqrt{6}}{3}$, $\sec\theta = \frac{2\sqrt{10}}{5}$, $\cot\theta = \frac{\sqrt{15}}{3}$  **43.** $\{x \mid x > -0.6931\}$  **45.** 405, −315°  **47.** 540°, −180°  **49.** $\frac{19\pi}{6}$, $-\frac{5\pi}{6}$

***Pages 803–805  Lesson 13-6***

**1.** $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$  **3.** $\frac{\sqrt{3}}{2}$  **5.** 2 s  **7.** $\sin\theta = \frac{4}{5}$; $\cos\theta = -\frac{3}{5}$  **9.** $\sin\theta = \frac{15}{17}$; $\cos\theta = \frac{8}{17}$  **11.** $\sin\theta = \frac{\sqrt{3}}{2}$; $\cos\theta = -\frac{1}{2}$  **13.** $-\frac{1}{2}$  **15.** −1  **17.** 1  **19.** 6  **21.** 2π  **23.** $\frac{1}{440}$ s  **25.** $\frac{1}{4}$  **27.** $\frac{1-\sqrt{3}}{2}$  **29.** $-3\sqrt{3}$  **31.** $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $(-1, 0)$, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  **33.** $\frac{y}{x}$  **35.** $-\frac{x}{y}$  **37.** $\sqrt{3}$

**39.** Sample answer: the motion of the minute hand on a clock; 60 s  **41.** sine: D = {all reals}, R = $\{-1 \le y \le 1\}$; cosine: D = {all reals}, R = $\{-1 \le y \le 1\}$  **43.** B  **45.** cosines: $c \approx 12.4$, $B \approx 59°$, $A \approx 76°$  **47.** 27.0 in$^2$  **49.** does not exist  **51.** 8  **53.** 110°

***Pages 809–811  Lesson 13-7***

**1.** 45°  **3.** 30°  **5.** $\pi \approx 3.14$  **7.** 0.75  **9.** 0.58  **11.** 30°  **13.** 30°  **15.** 90°  **17.** does not exist  **19.** 0.52  **21.** 0.66  **23.** 0.5  **25.** 60° south of west  **27.** 0.81  **29.** 3  **31.** 1.57  **33.** does not exist  **35.** 0.87  **37.** No; with this point on the terminal side of the throwing angle $\theta$, the measure of $\theta$ is found by solving the equation $\tan\theta = \frac{17}{18}$. Thus $\theta = \tan^{-1}\frac{17}{18}$ or about 43.3°, which is greater than the 40° requirement. **39.** $\text{Sin}^{-1}x + \text{Cos}^{-1}x = \frac{\pi}{2}$ for all values of $x$

**41.** Sample answer: $\text{Cos } 45° = \frac{\sqrt{2}}{2}$; $\text{Cos}^{-1}\frac{\sqrt{2}}{2} = 45°$

**43.** 102° **45.** Trigonometry is used to determine proper banking angles. Answers should include the following.

- Knowing the velocity of the cars to be traveling on a road and the radius of the curve to be built, then the banking angle can be determined. First find the ratio of the square of the velocity to the product of the acceleration due to gravity and the radius of the curve. Then determine the angle that had this ratio as its tangent. This will be the banking angle for the turn.
- If the speed limit were increased and the banking angle remained the same, then in order to maintain a safe road the curvature would have to be decreased. That is, the radius of the curve would also have to increase, which would make the road less curved.

**47.** J **49.** −1 **51.** sines; $B \approx 69°$, $C \approx 81°$, $c \approx 6.1$ or $B \approx 111°$, $C \approx 39°$, $c \approx 3.9$ **53.** 46, 39 **55.** 11, 109

***Pages 812–816 Chapter 13 Study Guide and Review***

**1.** false; coterminal. **3.** true **5.** true **7.** $A \approx 26°$, $B \approx 64°$, $b \approx 14.4$ **9.** $A = 45°$, $a \approx 8.5$, $b \approx 8.5$ **11.** $A = 41°$, $b \approx 10.4$, $c \approx 13.7$ **13.** 587.6 ft **15.** $-\frac{7\pi}{6}$

**17.** −720° **19.** 320°, −400° **21.** $\frac{\pi}{4}$; $-\frac{15\pi}{4}$ **23.** $\sin\theta = \frac{5\sqrt{29}}{29}$, $\cos\theta = \frac{2\sqrt{29}}{29}$, $\tan\theta = \frac{5}{2}$, $\csc\theta = \frac{\sqrt{29}}{5}$, $\sec\theta = \frac{\sqrt{29}}{2}$, $\cot\theta = \frac{2}{5}$ **25.** −1 **27.** about 86.2 ft **29.** two; $B \approx 53°$, $C \approx 87°$, $c \approx 12.4$; $B \approx 127°$, $C \approx 13°$, $c \approx 3.0$ **31.** no **33.** 107 mph **35.** sines; $C = 105°$, $a \approx 28.3$, $c \approx 38.6$ **37.** sines; $B \approx 52°$, $C \approx 92°$, $c \approx 10.2$; $B \approx 128°$; $C \approx 16°$, $c \approx 2.7$ **39.** about 1148.5 ft **41.** $\frac{1}{2}$ **43.** $\frac{-\sqrt{2}}{2}$ **45.** $-\sqrt{3}$ **47.** −1.57 **49.** 0.75 **51.** 1125°

# Chapter 14 Trigonometric Graphs and Identities

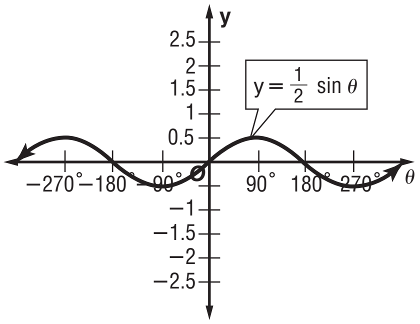
***Page 821 Chapter 14 Get Ready***

**1.** $\frac{\sqrt{2}}{2}$ **3.** 0 **5.** $-\frac{\sqrt{2}}{2}$ **7.** $-\frac{\sqrt{3}}{2}$ **9.** $\frac{2\sqrt{3}}{2}$

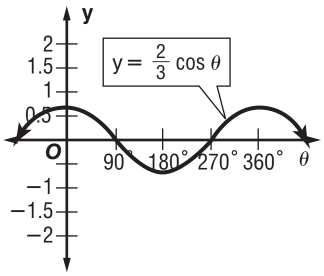
**11.** $-\sqrt{3}$ **13.** 60 ft **15.** $2x^2(x^2 - 2)$ **17.** $(2x + 1)(x - 2)$ **19.** 8, −3 **21.** 0, 12 **23.** −8, 5

***Pages 826–828 Lesson 14-1***

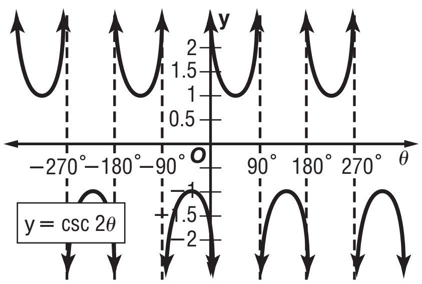
**1.** amplitude: $\frac{1}{2}$; period 360° or $2\pi$



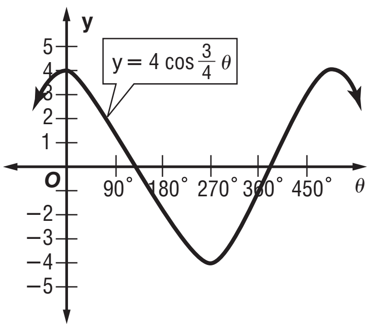

**3.** amplitude: $\frac{2}{3}$; period 360° or $2\pi$



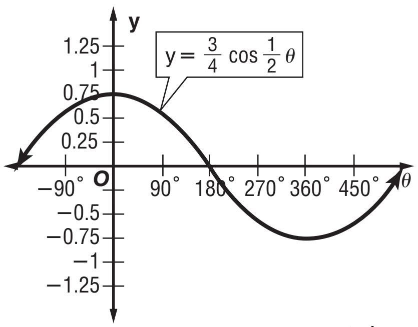

**5.** amplitude: does not exist; period: 180° or $\pi$




**7.** amplitude: 4; period: 480° or $\frac{8\pi}{3}$

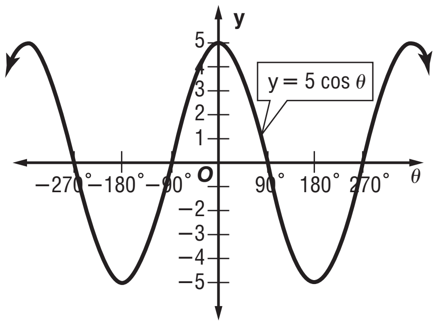



**9.** amplitude: $\frac{3}{4}$; period: 720° or $4\pi$

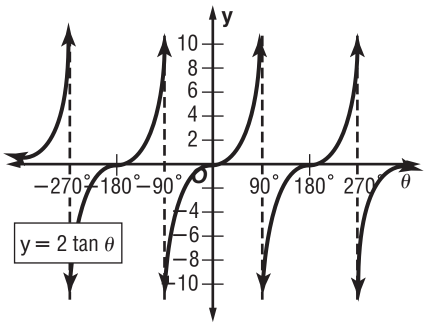


**11.** 4250; June 1

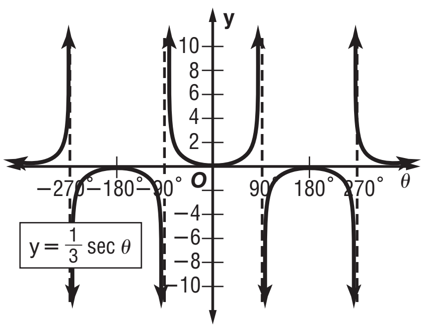
**13.** amplitude 5; period: 360° or $2\pi$



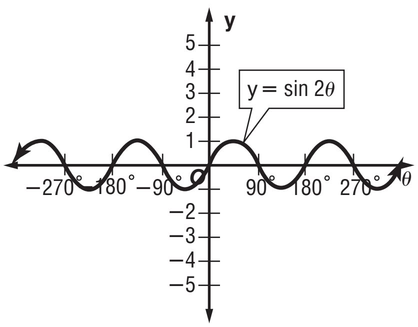

**15.** amplitude: does not exist; period: 180° or $\pi$



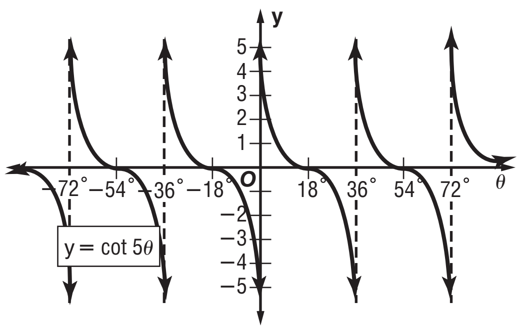

**17.** amplitude: does not exist; period: 360° or $2\pi$



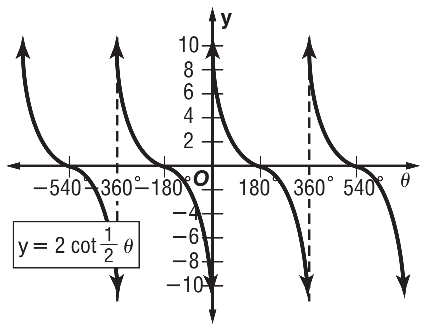

**19.** amplitude: 1; period: 180° or $\pi$




**21.** amplitude: does not exist; period: 36° or $\frac{\pi}{5}$

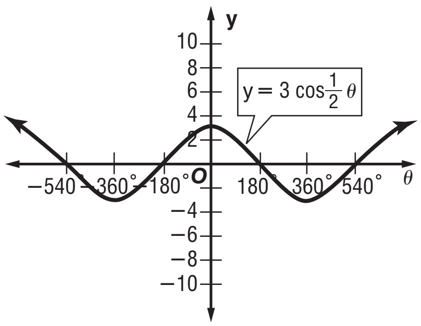



**23.** amplitude: does not exist; period: 360° or $2\pi$

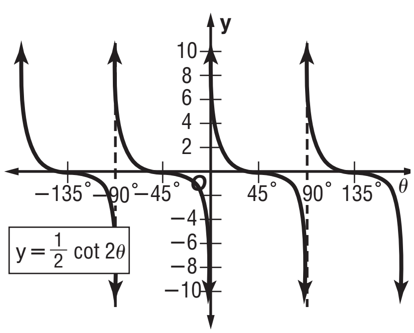



**25.** Sample answer: The amplitudes are the same. As the frequency increases, the period decreases.

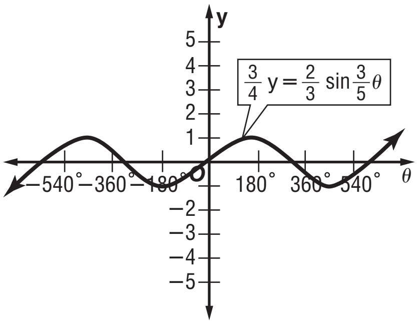
**27.** amplitude: 3; period: 720° or $4\pi$



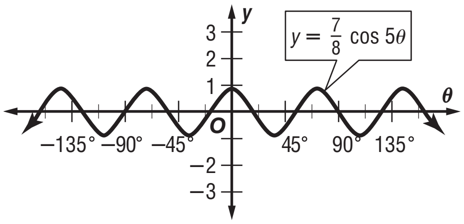

**29.** amplitude: does not exist; period: 90° or $\frac{\pi}{2}$



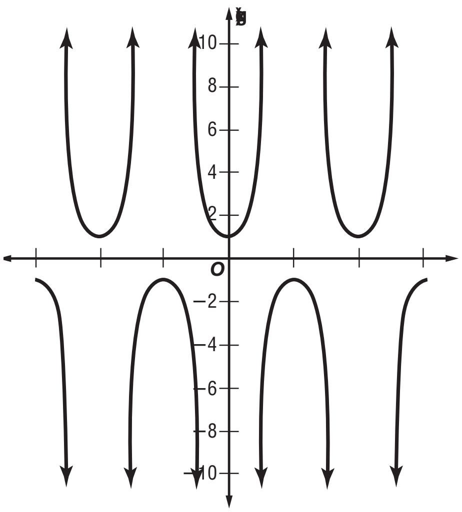
**31.** amplitude: $\frac{8}{9}$; period: 600° or $\frac{10\pi}{3}$



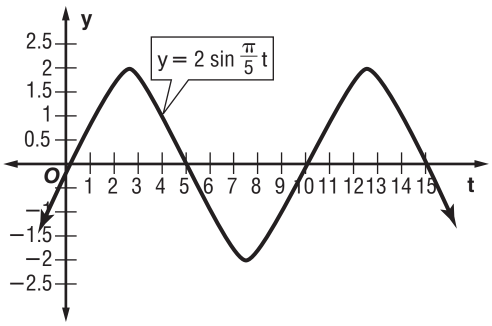

**33.** $y = \frac{7}{8} \cos 5\theta$



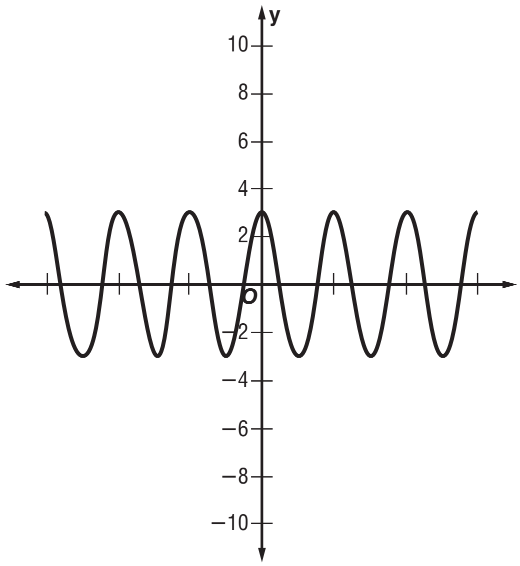

**35.** Vertical asymptotes located at $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\frac{7\pi}{2}$, etc. and $-\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\frac{7\pi}{2}$, etc.



**37.**



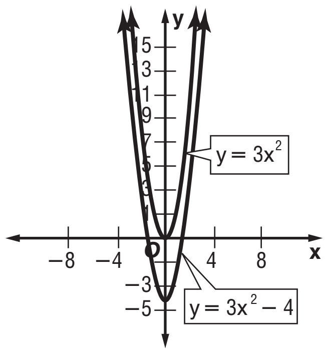

**39.** Sample answer: $y = 3 \cos (2\theta)$



**41.** Jamile; the amplitude is 3, and the period is $3\pi$. **43.** Sample answer: Tides display periodic behavior. This means that their pattern repeats at regular intervals. Tides rise and fall in a periodic manner, similar to the sine function.

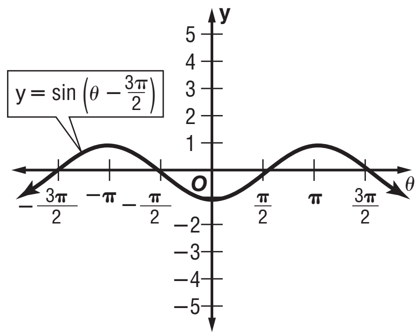
**45.** G **47.** −90° **49.** $\frac{1}{2}$ **51.** $\frac{\sqrt{2}}{2}$ **53.** 3, 11, 27, 59, 123

**55.**

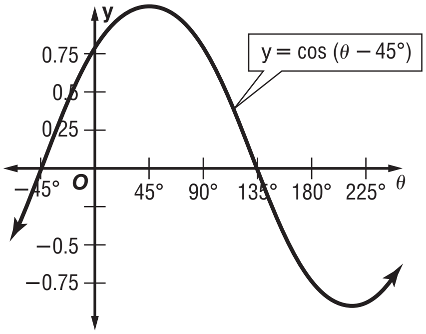



***Pages 834–836*** ***Lesson 14-2***

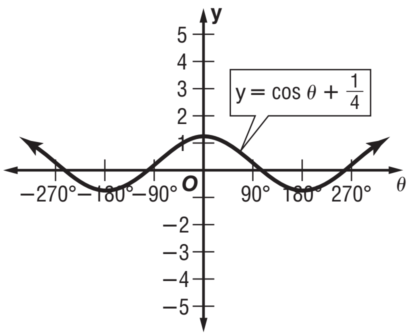
**1.** 1; $2\pi$; $\frac{\pi}{2}$



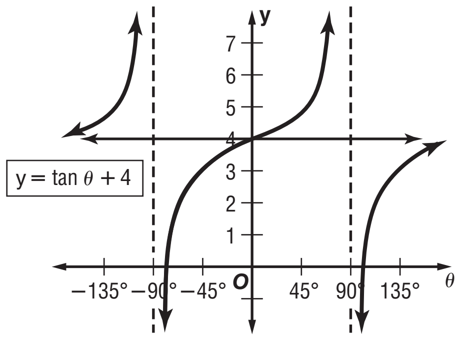
**3.** 1; 360°; 45°



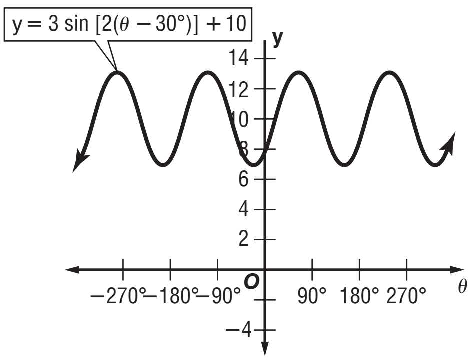

**5.** $\frac{1}{4}$; $y = \frac{1}{4}$; 1; 360°



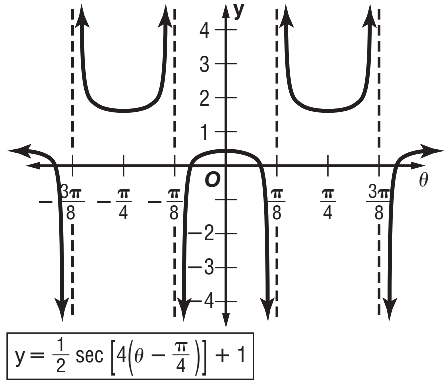

**7.** 4; $y = 4$; no amplitude; 180°




**9.** 10; 3; 180°; 30°

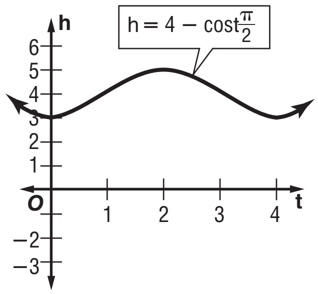



**11.** 1; no amplitude; $\frac{\pi}{2}$; $\frac{\pi}{4}$

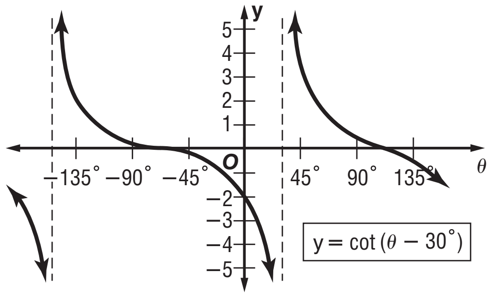



**13.** 4; 1; 4 s

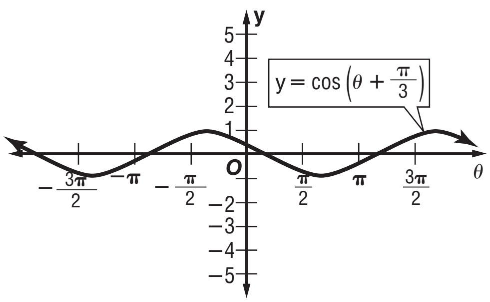
**15.**



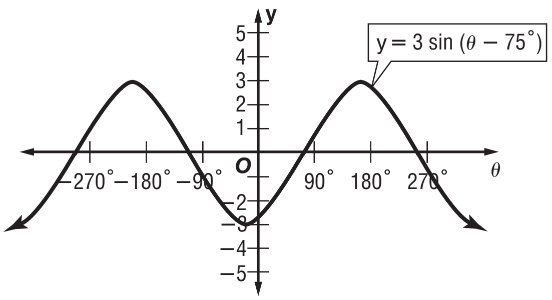

**17.** no amplitude; 180°; 30°



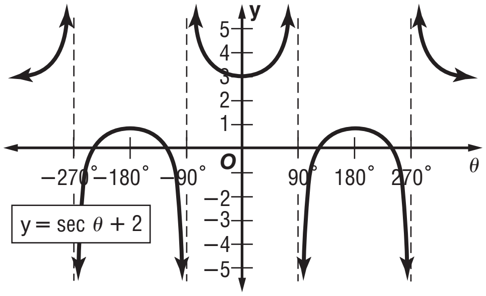

**19.** 1; $2\pi$; $-\frac{\pi}{3}$



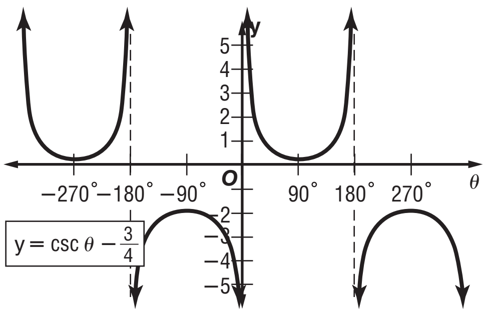
**21.** 3; 360°; 75°



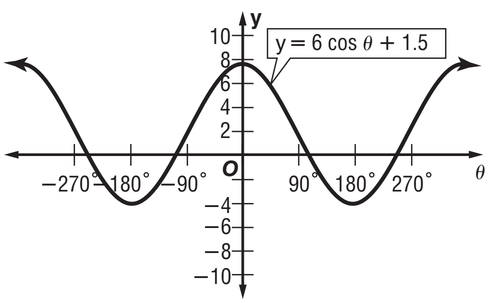

**23.** 2; $y = 2$; no amplitude; 360°



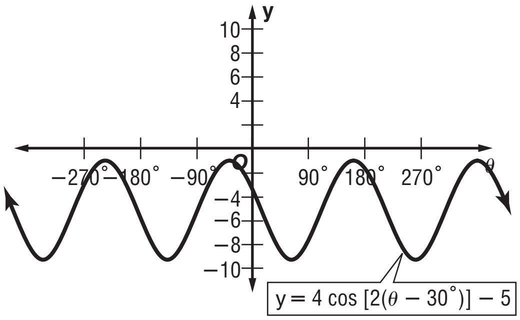

**25.** $-\frac{3}{4}$; $y = -\frac{3}{4}$; no amplitude; 360°



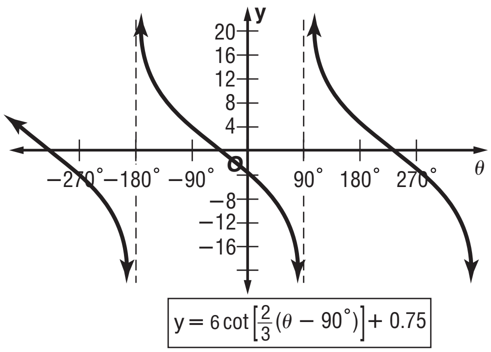

**27.** 1.5; $y = 1.5$; 6; 360°



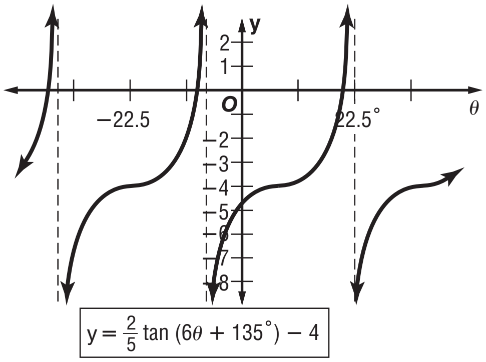

**29.** −5; 4; 180°; −30°



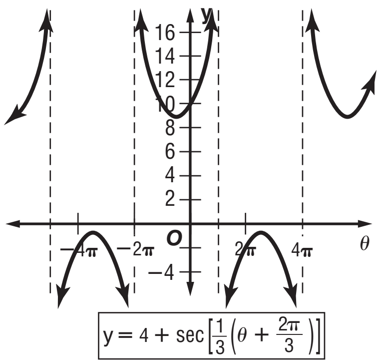

**31.** 0.75; does not exist; 270°; 90°




**33.** −4; does not exist; 30°; −22.5°



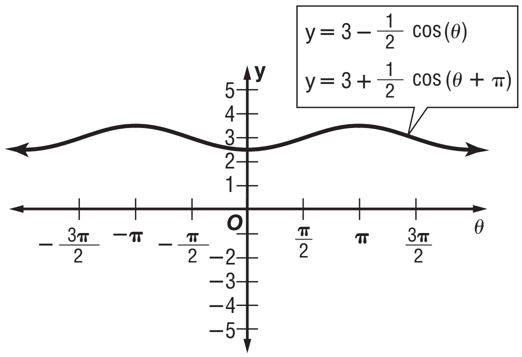

**35.** 4; does not exist; 6π; $-\frac{2\pi}{3}$



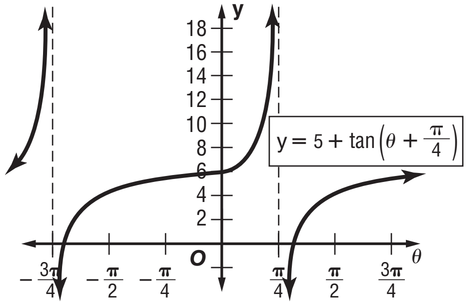

Selected Answers

**Selected Answers**

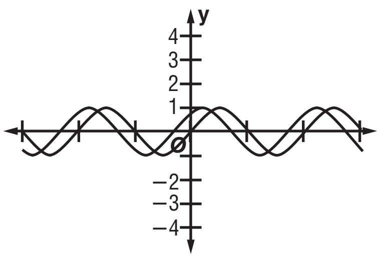
**37.** 300; 14.5 yr
**39.** The graphs are identical.



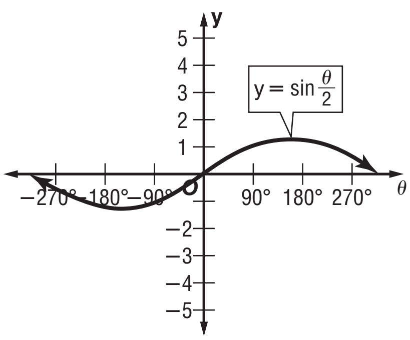

**41.** translation $\frac{\pi}{4}$ units left and 5 units up




**43.** c **45.** Sample answer: $y = \sin(\theta + 45°)$




**47.** Sample answer: You can use changes in amplitude and period along with vertical and horizontal shifts to show an animal population's starting point and display changes to that population over a period of time. The equation shows a rabbit population that begins at 1200, increases to a maximum of 1450, then decreases to a minimum of 950 over a period of 4 years. **49.** H **51.** amplitude: 1; period: 720° or $4\pi$




**53.** 0.75 **55.** 0.83 **57.** 35 **59.** 0.66

**61.** $\frac{5a - 13}{(a-2)(a-3)}$ **63.** $\frac{3y^2 + 10y + 5}{2(y-5)(y+3)}$ **65.** $-1$

**67.** $\frac{1}{2}$ **69.** $\frac{\sqrt{3}}{3}$ **71.** 1

***Pages 839–841*** ***Lesson 14-3***

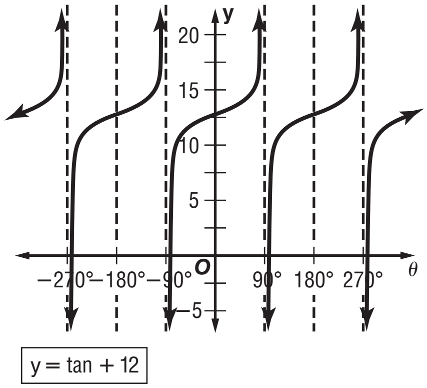
**1.** $-\frac{\sqrt{3}}{3}$ **3.** $\frac{3}{5}$ **5.** 1 **7.** $\sec\theta$ **9.** $\sin\theta = \cos\theta\,\frac{v^2}{gR}$

**11.** $\frac{\sqrt{5}}{3}$ **13.** $2\sqrt{2}$ **15.** $\frac{3}{5}$ **17.** $-\frac{3\sqrt{5}}{5}$ **19.** 1

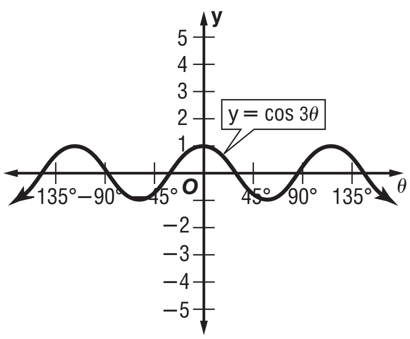
**21.** $\sin\theta$ **23.** $-3$ **25.** $\tan\theta$ **27.** $P = I^2R\sin^2 2\pi ft$

**29.** $\frac{3}{4}$ **31.** $-\frac{4\sqrt{7}}{7}$ **33.** $\cot^2\theta$ **35.** 1 **37.** about 4 m/s

**39.** $E = \frac{I\cos\theta}{R^2}$ **41.** Sample answer: The sine function is negative in the third and fourth quadrants. Therefore, the terminal side of the angle must lie in one of those two quadrants. **43.** $\frac{10\pi}{3}$
**45.** Sample answer: You can use equations to find the height and the horizontal distance of a baseball after it has been hit. The equations involve using the initial angle the ball makes with the ground with the sine function. Both equations are quadratic in nature with a leading negative coefficient. Thus, both are inverted parabolas which model the path of a baseball. **47.** F
**49.** 12; $y = 12$; no amplitude; 180°




**51.** amplitude: 1; period: 120° or $\frac{2\pi}{3}$




**53.** 93 **55.** 498 **57.** Subtraction (=)
**59.** Substitution (=)

***Pages 844–846*** ***Lesson 14-4***

**1.** $\tan\theta\,(\cot\theta + \tan\theta) \stackrel{?}{=} \sec^2\theta$

$1 + \tan^2\theta \stackrel{?}{=} \sec^2\theta$

$\sec^2\theta = \sec^2\theta$

**3.** $\dfrac{\cos^2\theta}{1-\sin\theta} \stackrel{?}{=} 1 + \sin\theta$

$\dfrac{1-\sin^2\theta}{1-\sin\theta} \stackrel{?}{=} 1 + \sin\theta$

$\dfrac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta} \stackrel{?}{=} 1 + \sin\theta$

$1 + \sin\theta = 1 + \sin\theta$

**5.** $\dfrac{\sin\theta}{\sec\theta} \stackrel{?}{=} \dfrac{1}{\tan\theta + \cot\theta}$

$\dfrac{\sin\theta}{\sec\theta} \stackrel{?}{=} \dfrac{1}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}$

$\dfrac{\sin\theta}{\sec\theta} \stackrel{?}{=} \dfrac{1}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$

$\dfrac{\sin\theta}{\sec\theta} \stackrel{?}{=} \dfrac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}$

$\dfrac{\sin\theta}{\sec\theta} \stackrel{?}{=} \dfrac{\sin\theta\cos\theta}{1}$

$\dfrac{\sin\theta}{\sec\theta} = \dfrac{\sin\theta}{\sec\theta}$

**7.** D

**9.** $\cot\theta\,(\cot\theta + \tan\theta) \stackrel{?}{=} \csc^2\theta$

$\cot^2\theta + \cot\theta\tan\theta \stackrel{?}{=} \csc^2\theta$

$\cot^2\theta + \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta} \stackrel{?}{=} \csc^2\theta$

$\cot^2\theta + 1 \stackrel{?}{=} \csc^2\theta$

$\csc^2\theta = \csc^2\theta$

**11.** $\sin\theta\sec\theta\cot\theta \stackrel{?}{=} 1$

$\sin\theta \cdot \dfrac{1}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta} \stackrel{?}{=} 1$

$1 = 1$

**13.** $\dfrac{1-2\cos^2\theta}{\sin\theta\cos\theta} \stackrel{?}{=} \tan\theta - \cot\theta$

$\dfrac{(1-\cos^2\theta) - \cos^2\theta}{\sin\theta\cos\theta} \stackrel{?}{=} \tan\theta - \cot\theta$

$\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} \stackrel{?}{=} \tan\theta - \cot\theta$

$\dfrac{\sin^2\theta}{\sin\theta\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta\cos\theta} \stackrel{?}{=} \tan\theta - \cot\theta$

$\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta} \stackrel{?}{=} \tan\theta - \cot\theta$

$\tan\theta - \cot\theta = \tan\theta - \cot\theta$

**15.** $\sin\theta + \cos\theta \stackrel{?}{=} \dfrac{1+\tan\theta}{\sec\theta}$

$\sin\theta + \cos\theta \stackrel{?}{=} \dfrac{1 + \dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta}}$

$\sin\theta + \cos\theta \stackrel{?}{=} \dfrac{\dfrac{\sin\theta + \cos\theta}{\cos\theta}}{\dfrac{1}{\cos\theta}}$

$\sin\theta + \cos\theta \stackrel{?}{=} \dfrac{\sin\theta + \cos\theta}{\cos\theta} \cdot \cos\theta$

$\sin\theta + \cos\theta = \sin\theta + \cos\theta$

**17.** $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{1-\cos\theta}{\sin\theta} \stackrel{?}{=} 2\csc\theta$

$\dfrac{\sin\theta}{\sin\theta} \cdot \dfrac{\sin\theta}{1-\cos\theta} + \dfrac{1-\cos\theta}{1-\cos\theta} \cdot \dfrac{1-\cos\theta}{\sin\theta} \stackrel{?}{=} 2\csc\theta$

$\dfrac{\sin^2\theta}{\sin\theta\,(1-\cos\theta)} + \dfrac{1-2\cos\theta+\cos^2\theta}{\sin\theta\,(1-\cos\theta)} \stackrel{?}{=} 2\csc\theta$

$\dfrac{\sin^2\theta + \cos^2\theta + 1 - 2\cos\theta}{\sin\theta\,(1-\cos\theta)} \stackrel{?}{=} 2\csc\theta$

$\dfrac{2-2\cos\theta}{\sin\theta\,(1-\cos\theta)} \stackrel{?}{=} 2\csc\theta$

$\dfrac{2\,(1-\cos\theta)}{\sin\theta\,(1-\cos\theta)} \stackrel{?}{=} 2\csc\theta$

$\dfrac{2}{\sin\theta} \stackrel{?}{=} 2\csc\theta$

$2\csc\theta = 2\csc\theta$

**19.** $\dfrac{\sin^2\theta}{1-\cos\theta} \stackrel{?}{=} 1 + \cos\theta$

$\dfrac{\sin^2\theta}{1-\cos\theta} \cdot \dfrac{1+\cos\theta}{1+\cos\theta} \stackrel{?}{=} 1 + \cos\theta$

$\dfrac{\sin^2\theta\,(1+\cos\theta)}{1-\cos^2\theta} \stackrel{?}{=} 1 + \cos\theta$

$\dfrac{\sin^2\theta\,(1+\cos\theta)}{\sin^2\theta} \stackrel{?}{=} 1 + \cos\theta$

$1 + \cos\theta = 1 + \cos\theta$

**21.** 598.7 m

**23.** $\dfrac{1+\tan\theta}{1+\cot\theta} \stackrel{?}{=} \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{1 + \dfrac{\sin\theta}{\cos\theta}}{1 + \dfrac{\cos\theta}{\sin\theta}} \stackrel{?}{=} \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\dfrac{\sin\theta + \cos\theta}{\cos\theta}}{\dfrac{\sin\theta + \cos\theta}{\sin\theta}} \stackrel{?}{=} \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\sin\theta + \cos\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\sin\theta + \cos\theta} \stackrel{?}{=} \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta}$

**25.** $1+\frac{1}{\cos\theta} \stackrel{?}{=} \frac{\tan^2\theta}{\sec\theta-1}$

$1+\frac{1}{\cos\theta} \stackrel{?}{=} \frac{\tan^2\theta}{\sec\theta-1}\cdot\frac{\sec\theta+1}{\sec\theta+1}$

$1+\frac{1}{\cos\theta} \stackrel{?}{=} \frac{\tan^2\theta(\sec\theta+1)}{\sec^2\theta-1}$

$1+\frac{1}{\cos\theta} \stackrel{?}{=} \frac{\tan^2\theta(\sec\theta+1)}{\tan^2\theta-1}$

$1+\frac{1}{\cos\theta} \stackrel{?}{=} \sec\theta+1$

$1+\frac{1}{\cos\theta} = 1+\frac{1}{\cos\theta}$

**27.** $\cos^4\theta-\sin^4\theta \stackrel{?}{=} \cos^2\theta-\sin^2\theta$

$(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta) \stackrel{?}{=} \cos^2\theta-\sin^2\theta$

$(\cos^2\theta-\sin^2\theta)\cdot 1 \stackrel{?}{=} \cos^2\theta-\sin^2\theta$

$\cos^2\theta-\sin^2\theta = \cos^2\theta-\sin^2\theta$

**29.** $\frac{\cos\theta}{1+\sin\theta}+\frac{\cos\theta}{1-\sin\theta} \stackrel{?}{=} 2\sec\theta$

$\frac{\cos\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}+\frac{\cos\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta} \stackrel{?}{=} 2\sec\theta$

$\frac{\cos\theta(1-\sin\theta)+\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} \stackrel{?}{=} 2\sec\theta$

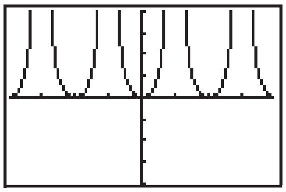
$\frac{\cos\theta-\sin\theta\cos\theta+\cos\theta+\sin\theta\cos\theta}{1-\sin^2\theta} \stackrel{?}{=} 2\sec\theta$

$\frac{2\cos\theta}{\cos^2\theta} \stackrel{?}{=} 2\sec\theta$

$\frac{2}{\cos\theta} \stackrel{?}{=} 2\sec\theta$

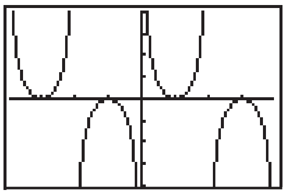
$2\sec\theta = 2\sec\theta$

**31.** $[-360, 360]$ scl: 90 by $[-5, 5]$ scl: 1; may be



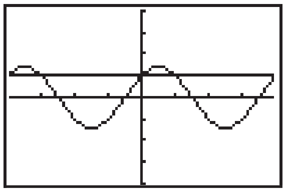
[−360, 360] scl: 90 by [−5, 5] scl: 1

**33.** $[-360, 360]$ scl: 90 by $[-5, 5]$ scl: 1; may be



[−360, 360] scl: 90 by [−5, 5] scl: 1

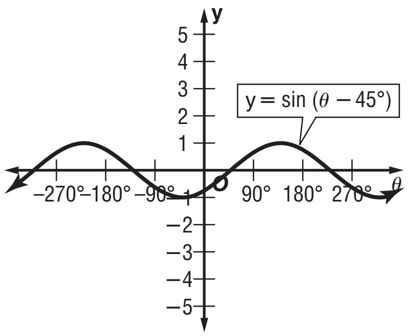
**35.** $[-360, 360]$ scl: 90 by $[-5, 5]$ scl: 1; is not



[−360, 360] scl: 90 by [−5, 5] scl: 1

**37.** $\sin^2\theta-\cos^2\theta=2\sin^2\theta$ does not belong with the others. The other equations are identities, but $\sin^2\theta-\cos^2\theta=2\sin^2\theta$ is not. $\sin^2\theta-\cos^2\theta=2\sin^2\theta-1$ would be an identity. **39.** Sample answer: The expressions have not yet been shown to be equal, so you could not use the properties of equality on them. Graphing two expressions could result in identical graphs for a set interval, that are different elsewhere. **41.** G

**43.** $-\frac{\sqrt{5}}{3}$ **45.** $-\frac{\sqrt{7}}{4}$ **47.** 1; 360°; 45°




**49.** $\frac{1}{10^7}$ **51.** −5, −1 **53.** −2 **55.** $\frac{\sqrt{2}}{4}$ **57.** $\frac{2-\sqrt{3}}{4}$

## *Pages 851–852* *Lesson 14-5*

**1.** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **3.** $\frac{\sqrt{2}-\sqrt{6}}{4}$ **5.** $\frac{\sqrt{3}}{2}$ **7.** $\frac{5-\sqrt{3}}{1+5\sqrt{3}}$

**9.** $\sin\left(\theta+\frac{\pi}{2}\right) \stackrel{?}{=} \cos\theta$

$\sin\theta\cos\frac{\pi}{2}+\cos\theta\sin\frac{\pi}{2} \stackrel{?}{=} \cos\theta$

$\sin\theta\cdot 0+\cos\theta\cdot 1 \stackrel{?}{=} \cos\theta$

$\cos\theta = \cos\theta$

**11.** $\frac{\sqrt{2}}{2}$ **13.** $\frac{-\sqrt{6}-\sqrt{2}}{4}$ **15.** $\frac{-\sqrt{6}-\sqrt{2}}{4}$

**17.** $-\frac{\sqrt{2}}{2}$ **19.** $\frac{\sqrt{2}-\sqrt{6}}{4}$ **21.** $-\frac{\sqrt{3}}{2}$

**23.** 0.3681 *E* **25.** 0.6157 *E*

**27.** $\sin(270°-\theta) \stackrel{?}{=} \sin 270°\cos\theta-\cos 270°$

$\stackrel{?}{=} -1\cos\theta-0$

$= -\cos\theta$

**29.** $\cos(90°-\theta) \stackrel{?}{=} \cos 90°\cos\theta+\sin 90°\sin\theta$

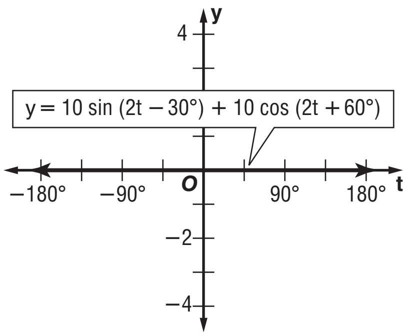
$\stackrel{?}{=} 0\cdot\cos\theta+1\cdot\sin\theta$

$= \sin\theta$

**31.** $$\sin\left(\theta+\frac{3\pi}{2}\right)\stackrel{?}{=}-\cos\theta$$
$$\sin\theta\cos\frac{3\pi}{2}+\cos\theta\sin\frac{3\pi}{2}\stackrel{?}{=}-\cos\theta$$
$$\sin\theta\cdot 0+\cos\theta\cdot(-1)\stackrel{?}{=}-\cos\theta$$
$$0+(-\cos\theta)\stackrel{?}{=}-\cos\theta$$
$$-\cos\theta=-\cos\theta$$

**33.** $$\cos(2\pi+\theta)\stackrel{?}{=}\cos\theta$$
$$\cos 2\pi\cos\theta-[\sin 2\pi\sin\theta]\stackrel{?}{=}\cos\theta$$
$$1\cdot\cos\theta-[0\cdot\sin\theta]\stackrel{?}{=}\cos\theta$$
$$1\cdot\cos\theta-0\stackrel{?}{=}\cos\theta$$
$$\cos\theta=\cos\theta$$

**35.**




**37.** $\sin(60°+\theta)+\sin(60°-\theta)$
$$\stackrel{?}{=}\sin 60°\cos\theta+\cos 60°\sin\theta+\sin 60°\cos\theta-\cos 60°\sin\theta$$
$$\stackrel{?}{=}\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta$$
$$=\sqrt{3}\cos\theta$$

**39.**
$$\sin(\alpha+\beta)\sin(\alpha-\beta)\stackrel{?}{=}\sin^2\alpha-\sin^2\beta$$
$$\stackrel{?}{=}(\sin\alpha\cos\beta+\cos\alpha\sin\beta)(\sin\alpha\cos\beta-\cos\alpha\sin\beta)$$
$$\stackrel{?}{=}\sin^2\alpha\cos^2\beta-\cos^2\alpha\sin^2\beta$$
$$\stackrel{?}{=}\sin^2\alpha(1-\sin^2\beta)-(1-\sin^2\alpha)\sin^2\beta$$
$$\stackrel{?}{=}\sin^2\alpha-\sin^2\alpha\sin^2\beta-\sin^2\beta+\sin^2\alpha\sin^2\beta$$
$$=\sin^2\alpha-\sin^2\beta$$

**41.** Sample answer: $\alpha=\frac{\pi}{4}$; $\beta=\frac{3\pi}{2}$

**43.** $$\tan(\alpha+\beta)\stackrel{?}{=}\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$
$$\stackrel{?}{=}\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$
$$\stackrel{?}{=}\frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$
$$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)\stackrel{?}{=}\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$
$$\stackrel{?}{=}\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$$
$$\stackrel{?}{=}\frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$
$$=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

**45.** A

**47.** $$\cot\theta+\sec\theta\stackrel{?}{=}\frac{\cos^2\theta+\sin\theta}{\sin\theta\cos\theta}$$
$$\cot\theta+\sec\theta\stackrel{?}{=}\frac{\cos^2\theta}{\sin\theta\cos\theta}+\frac{\sin\theta}{\sin\theta\cos\theta}$$
$$\cot\theta+\sec\theta\stackrel{?}{=}\frac{\cos\theta}{\sin\theta}+\frac{1}{\cos\theta}$$
$$\cot\theta+\sec\theta=\cot\theta+\sec\theta$$

**49.** $$\sin\theta(\sin\theta+\csc\theta)\stackrel{?}{=}2-\cos^2\theta$$
$$\sin^2\theta+1\stackrel{?}{=}2-\cos^2\theta$$
$$1-\cos^2\theta+1\stackrel{?}{=}2-\cos^2\theta$$
$$2-\cos^2\theta+1=2-\cos^2\theta$$

**51.** 1 **53.** $\sec\theta$ **55.** about 228 mi **57.** $\pm\frac{\sqrt{5}}{2}$

**59.** $\pm\frac{\sqrt{5}}{2}$

***Pages 857–859*** ***Lesson 14-6***

**1.** $\frac{24}{25}, -\frac{7}{25}, \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}$ **3.** $\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\sqrt{2-\sqrt{3}}}{2}, \frac{\sqrt{2+\sqrt{3}}}{2}$

**5.** $-\frac{\sqrt{2-\sqrt{3}}}{2}$ **7.** 1.64

**9.** $$\cos^2 2x+4\sin^2 x\cos^2 x\stackrel{?}{=}1$$
$$\cos^2 2x+\sin^2 2x\stackrel{?}{=}1$$
$$1=1$$

**11.** $-\frac{4\sqrt{6}}{25}, -\frac{23}{25}, \frac{\sqrt{10}}{5}, -\frac{\sqrt{15}}{5}$ **13.** $\frac{24}{25}, \frac{7}{25}, \frac{3\sqrt{10}}{10}, -\frac{\sqrt{10}}{10}$

**15.** $-\frac{\sqrt{15}}{8}, -\frac{7}{8}, \frac{\sqrt{10}}{4}, \frac{\sqrt{6}}{4}$ **17.** $\frac{\sqrt{2-\sqrt{2}}}{2}$ **19.** $-\frac{\sqrt{2-\sqrt{3}}}{2}$

**21.** $-\frac{\sqrt{2-\sqrt{3}}}{2}$

**23.** $$2\cos^2\frac{x}{2}\stackrel{?}{=}1+\cos x$$
$$2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2\stackrel{?}{=}1+\cos x$$
$$2\left(\frac{1+\cos x}{2}\right)\stackrel{?}{=}1+\cos x$$
$$1+\cos x=1+\cos x$$

**25.** $$\sin^2 x\stackrel{?}{=}\frac{1}{2}(1-\cos 2x)$$
$$\sin^2 x\stackrel{?}{=}\frac{1}{2}[1-(1-2\sin^2 x)]$$
$$\sin^2 x\stackrel{?}{=}\frac{1}{2}(2\sin^2 x)$$
$$\sin^2 x=\sin^2 x$$

**27.** $\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} \stackrel{?}{=} \tan x$

$\frac{1 - \cos^2 x}{\sin x \cos x} \stackrel{?}{=} \tan x$

$\frac{\sin^2 x}{\sin x \cos x} \stackrel{?}{=} \tan x$

$\frac{\sin x}{\cos x} \stackrel{?}{=} \tan x$

$\tan x = \tan x$

**29.** $\frac{1}{4} \tan \theta$ **31.** $\frac{120}{169}, \frac{119}{169}, \frac{5\sqrt{26}}{26}, -\frac{\sqrt{26}}{26}$

**33.** $\frac{\sqrt{15}}{8}, \frac{7}{8}, \frac{\sqrt{8 + 2\sqrt{15}}}{4}, -\frac{\sqrt{8 - 2\sqrt{15}}}{4}$

**35.** $-\frac{4\sqrt{21}}{5}, \frac{17}{25}, \frac{\sqrt{5\sqrt{2} + 10\sqrt{21}}}{10}, \frac{\sqrt{5\sqrt{10} - 10\sqrt{21}}}{10}$

**37.** $\dfrac{1 \pm \sqrt{\frac{1 - \cos L}{1 + \cos L}}}{1 \pm \sqrt{\frac{1 - \cos L}{1 + \cos L}}}$

**39.** Sample answer: If $x$ is in the third quadrant, then $\frac{x}{2}$ is between 90° and 135°. Use the half-angle formula for cosine knowing that the value is negative.
**41.** Sample answer: 45°; $\cos 2(45°) = \cos 90°$ or 0, $2 \cos 45° = 2 \cdot \frac{\sqrt{2}}{2}$ or $\sqrt{22}$

**43.** D **45.** $\frac{\sqrt{6} + \sqrt{2}}{4}$ **47.** $-\frac{\sqrt{2}}{2}$ **49.** $\frac{\sqrt{6} + \sqrt{2}}{4}$

**51.** $\cot^2 \theta - \sin^2 \theta \stackrel{?}{=} \frac{\cos^2 \theta \csc^2 \theta - \sin^2 \theta}{\sin^2 \theta \csc^2 \theta}$

$\cot^2 \theta - \sin^2\theta \stackrel{?}{=} \dfrac{\cos^2 \theta \frac{1}{\sin^2 \theta} - \sin^2\theta}{\sin^2 \theta \frac{1}{\sin^2 \theta}}$

$\cot^2 \theta - \sin^2\theta \stackrel{?}{=} \frac{\cot^2 \theta - \sin^2\theta}{1}$

$\cot^2 \theta - \sin^2\theta = \cot^2 \theta - \sin^2\theta$

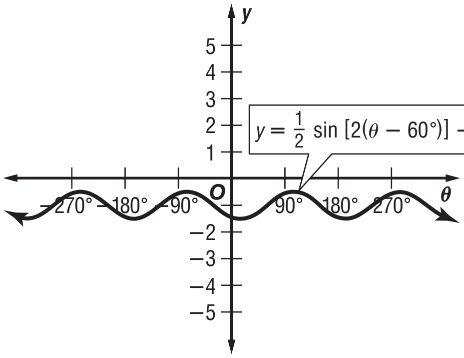
**53.** $10^1$ or 10 **55.** $(a^4)2 - 7(a^4) + 13$ **57.** $4(d^3)^2 + 2(d^3) + 104$ **59.** 5 **61.** $n^2 - 7n + 5f$ **63.** 1, −1
**65.** $\frac{5}{2}, -2$ **67.** $0, -\frac{1}{2}$

### Pages 864–866 Lesson 14-7

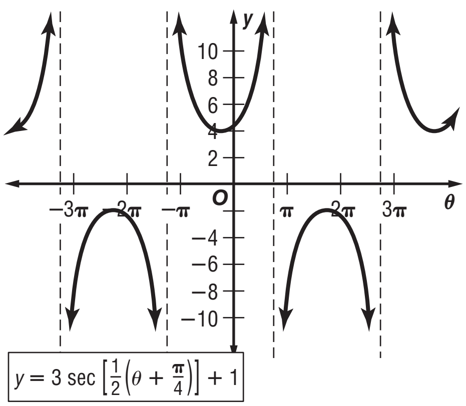
**1.** 60°, 120°, 240°, 300° **3.** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **5.** $0 + \frac{2k\pi}{3}$
**7.** $90° + k \cdot 360°, 180° + k \cdot 360°$ **9.** $\frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi$ or $210° + k \cdot 360°, 330° + k \cdot 360°$
**11.** 31.3° **13.** 240°, 300° **15.** 30°, 150°, 180°, 330°
**17.** $\pi + 2k\pi, \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$ **19.** $0 + 2k\pi$
**21.** $0° + k \cdot 180°$ **23.** $30° + k \cdot 360°, 150° + k \cdot 360°$
**25.** $\frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi$ or $210° + k \cdot 360°, 330° + k \cdot 360°$ **27.** $\frac{\pi}{2} + k\pi, \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$ or $90° + k \cdot 180°, 120° + k \cdot 360°, 240° + k \cdot 360°$ **29.** 10
**31.** $\frac{\pi}{2}$ **33.** $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$ **35.** $0 + k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$ **37.** $\frac{5\pi}{4} + k \cdot 2\pi, \frac{7\pi}{4} + k \cdot 2\pi, \frac{\pi}{6} + k \cdot 2\pi, \frac{5\pi}{6} + k \cdot 2\pi$ **39.** $120° + k \cdot 360°, 240° + k \cdot 360°$
**41.** $120° + k \cdot 360°, 240° + k \cdot 360°$ **43.** $\frac{\pi}{2} + 4k\pi$ or $90° + k \cdot 720°$ **45.** $\frac{\pi}{3} + k \cdot 2\pi, \frac{4\pi}{3} + k \cdot 2\pi, \frac{\pi}{4} + k \cdot 2\pi, \frac{5\pi}{4} + k \cdot 2\pi$, or $60° + k \cdot 360°, 240° + k \cdot 360°, 45° + k \cdot 360°, 225° + k \cdot 360°$
**47.** about 32° **49.** Sample answer: If $\sec \theta = 0$ then $\frac{1}{\cos \theta} = 0$. Since no value of $\theta$ makes $\frac{1}{\cos \theta} = 0$, there are no solutions. **51.** Sample answer: The function is periodic with two solutions in each of its infinite number of periods. **53.** D
**55.** $\frac{24}{25}, \frac{7}{25}, \frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10}$ **57.** $\frac{5\sqrt{11}}{18}, \frac{7}{18}, \frac{\sqrt{3}}{6}, \frac{\sqrt{33}}{6}$
**59.** $-\frac{\sqrt{3}}{2}$ **61.** $\frac{1}{2}$

### Pages 867–870 Chapter 14 Study Guide and Review

**1.** phase shift **3.** vertical shift **5.** double-angle formula **7.** trigonometric identity **9.** amplitude
**11.** amplitude: 4; period: 180° or $\pi$ **13.** amplitude: does not exist; period: 360° or $2\pi$ **15.** amplitude: does not exist; period: 45° or $\frac{\pi}{4}$
**17.** $-1, \frac{1}{2}$, 180°, 60°

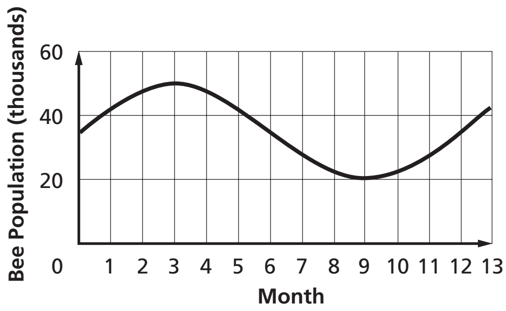



**19.** 1, does not exist, $4\pi - \frac{\pi}{4}$



**21.** $p = 35{,}000 + 15{,}000 \sin\left(\frac{\pi}{6}t\right)$

**Bee Population**




**23.** $-\frac{4}{3}$ **25.** $\cot\theta$ **27.** $\csc\theta$

**29.**
$$\frac{\sin\theta}{\tan\theta} + \frac{\cos\theta}{\cot\theta} \stackrel{?}{=} \cos\theta + \sin\theta$$
$$\frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta}{\frac{\cos\theta}{\sin\theta}} \stackrel{?}{=} \cos\theta + \sin\theta$$
$$\frac{\sin\theta}{} \cdot \frac{\cos\theta}{\sin\theta} + \cos\theta \cdot \frac{\sin\theta}{\cos\theta} \stackrel{?}{=} \cos\theta + \sin\theta$$
$$\cos\theta + \sin\theta = \cos\theta + \sin\theta$$

**31.**
$$\cot^2\theta \sec^2\theta \stackrel{?}{=} 1 + \cot^2\theta$$
$$\frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta} \stackrel{?}{=} 1 + \cot^2\theta$$
$$\frac{1}{\sin^2\theta} \stackrel{?}{=} 1 + \cot^2\theta$$
$$\csc^2\theta \stackrel{?}{=} 1 + \cot^2\theta$$
$$1 + \cot^2\theta = 1 + \cot^2\theta$$

**33.**
$$I_m \cos^2\theta = I_m\left(1 - \frac{1}{\csc^2\theta}\right)$$
$$I_m \cos^2\theta = I_m(1 - \sin^2\theta)$$
$$I_m \cos^2\theta = I_m \cos^2\theta$$

**35.** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **37.** $\frac{\sqrt{2} - \sqrt{6}}{4}$ **39.** $\frac{-\sqrt{6} - \sqrt{2}}{4}$

**41.**
$$\sin(30 - \theta) = \cos(60 + \theta)$$
$$\sin 30^\circ \cos\theta - \cos 30^\circ \sin\theta \stackrel{?}{=} \cos 60^\circ \cos\theta - \sin 60^\circ \sin\theta$$
$$\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta \stackrel{?}{=} \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$$
$$\sin(\theta + \pi) \stackrel{?}{=} -\sin\theta$$
$$\sin\theta\cos\pi + \cos\theta\sin\pi \stackrel{?}{=} -\sin\theta$$
$$(\sin\theta)(-1) + (\cos\theta)(0) \stackrel{?}{=} -\sin\theta$$
$$-\sin\theta = -\sin\theta$$

**43.**
$$-\cos\theta \stackrel{?}{=} \cos(\pi + \theta)$$
$$-\cos\theta \stackrel{?}{=} \cos\pi\cos\theta - \sin\pi\sin\theta$$
$$-\cos\theta \stackrel{?}{=} -1 \cdot \cos\theta - 0 \cdot \sin\theta$$
$$-\cos\theta = -\cos\theta$$

**45.** $\frac{120}{169}, \frac{119}{169}, \frac{5\sqrt{26}}{26}, \frac{-\sqrt{26}}{26}$ **47.** $-\frac{120}{169}, \frac{119}{169}, \frac{\sqrt{26}}{26}, -\frac{5\sqrt{26}}{26}$ **49.** $0^\circ$

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